Recitation 10:
Cost Dynamics and Parallelism

15-312: Principles of Programming Languages

Wednesday, March 26, 2014

1 Cost Dynamics

Theorem 1 (7.7b). *For any closed expresion e and value v of the same type, if $e \mapsto^k v$ then $e \Downarrow^k v$.*

In class we proved the converse of this theorem. You will need the following lemmas to prove the other direction:

Lemma 2. *If e val then $e \Downarrow e$.*

Lemma 3. *If $e \mapsto e'$ and $e' \Downarrow v$ then $e \Downarrow v$.*

Task   Prove Theorem 7.7.b using the provided lemmas. (Hint: start by induction on k)

2 Parallelism and Sequentialism

Parallel computation is a powerful tool which allows computations to be carried out simultaneously and can be used to drastically reduce the running time of certain progams. While there are many models for parralelism, today we will focus on nested parallelism. This form of parallelism creates a hierarchy of computations which may contain parallel computations within themselves. Once computations carried out simultaneously are complete, we usually want to combine the results in some way. For this recitation, we will use the following rules to capturethese concepts:

$$\frac{\Gamma \vdash e_1 : \tau_1 \qquad \Gamma \vdash e_2 : \tau_2 \qquad \Gamma, x_1 : \tau_1, x_2 : \tau_2 \vdash e : \tau}{\Gamma \vdash \mathtt{par}(e_1; e_2; x_1.x_2.e) : \tau}(40.1)$$

$$\frac{e_1 \mapsto_{\mathtt{seq}} e_1'}{\mathtt{par}(e_1; e_2; x_1.x_2.e) \mapsto_{\mathtt{seq}} \mathtt{par}(e_1'; e_2; x_1.x_2.e)}(40.2a)$$

$$\frac{e_1 \ \mathsf{val} \qquad e_2 \mapsto_{\mathtt{seq}} e_2'}{\mathtt{par}(e_1; e_2; x_1.x_2.e) \mapsto_{\mathtt{seq}} \mathtt{par}(e_1; e_2'; x_1.x_2.e)}(40.2b)$$

$$\frac{e_1 \ \mathsf{val} \qquad e_2 \ \mathsf{val}}{\mathtt{par}(e_1; e_2; x_1.x_2.e) \mapsto_{\mathtt{seq}} [e_1, e_2/x_1, x_2]e}(40.2c)$$

$$\frac{e_1 \mapsto_{\mathtt{par}} e_1' \qquad e_2 \mapsto_{\mathtt{par}} e_2'}{\mathtt{par}(e_1; e_2; x_1.x_2.e) \mapsto_{\mathtt{par}} \mathtt{par}(e_1'; e_2'; x_1.x_2.e)} (40.3a)$$

$$\frac{e_1 \mapsto_{\mathtt{par}} e_1' \qquad e_2 \; \mathsf{val}}{\mathtt{par}(e_1; e_2; x_1.x_2.e) \mapsto_{\mathtt{par}} \mathtt{par}(e_1'; e_2; x_1.x_2.e)} (40.3b)$$

$$\frac{e_1 \; \mathsf{val} \qquad e_2 \mapsto_{\mathtt{par}} e_2'}{\mathtt{par}(e_1; e_2; x_1.x_2.e) \mapsto_{\mathtt{par}} \mathtt{par}(e_1; e_2'; x_1.x_2.e)} (40.3c)$$

$$\frac{e_1 \; \mathsf{val} \qquad e_2 \; \mathsf{val}}{\mathtt{par}(e_1; e_2; x_1.x_2.e) \mapsto_{\mathtt{par}} [e_1, e_2/x_1, x_2]e} (40.3d)$$

$$\frac{e_1 \Downarrow v_1 \qquad e_2 \Downarrow v_2 \qquad [v_1, v_2/x_1, x_2]e \Downarrow v}{\mathtt{par}(e_1; e_2; x_1.x_2.e) \Downarrow v} (40.4)$$

There are several interesting theorems which accompany these rules:

Theorem 4 (40.1). *For all v val, $e \mapsto_{\mathtt{seq}}^* v$ if, and only if, $e \Downarrow v$.*

Theorem 5 (40.2). *For all v val, $e \mapsto_{\mathtt{par}}^* v$ if, and only if, $e \Downarrow v$.*

Theorem 6 (40.3/Implicit Parallelism). *For all v val, $e \mapsto_{\mathtt{seq}}^* v$ if, and only if, $e \mapsto_{\mathtt{par}}^* v$.*

Task Prove Theorem 40.3. You can use any theorems or lemmas from this handout.