# 15-122: Principles of Imperative Computation

## Recitation 27 Solutions

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## Exercises

1. Verify that the relation for union-find ($u$ and $v$ are related if and only if there's a path from $u$ to $v$) is an equivalence relation by showing that it is reflexive, symmetric and transitive. (You can assume we're working with undirected graphs.)

*Solution:* Reflexive: There is a path from any vertex to itself: the zero-length path.

Symmetric: Since we're working with undirected graphs, if there is a path from $u$ to $v$, there is a path from $v$ to $u$ — simply go in the opposite direction.

Transitive: If there's a path from $u$ to $v$ and one from $v$ to $s$, there's one from $u$ to $s$ — just go from $u$ to $v$ and then from there to $s$.

2. What's the worst case cost of adding an edge, assuming we never update canonical representations of the vertices in the larger equivalence class and that we have $n$ vertices total?

*Solution:* If we have $\frac{n}{2}$ vertices in each equivalence class, we'll have to update $\frac{n}{2}$ vertices no matter what. So, at worst we must do $O(n)$ updates to the array of canonical representations.