

Exercises

1. Verify that the relation for union-find (u and v are related if and only if there's a path from u to v) is an equivalence relation by showing that it is reflexive, symmetric and transitive. (You can assume we're working with undirected graphs.)

Solution: Reflexive: There is a path from any vertex to itself: the zero-length path.

Symmetric: Since we're working with undirected graphs, if there is a path from u to v , there is a path from v to u — simply go in the opposite direction.

Transitive: If there's a path from u to v and one from v to s , there's one from u to s — just go from u to v and then from there to s .

2. What's the worst case cost of adding an edge, assuming we never update canonical representations of the vertices in the larger equivalence class and that we have n vertices total?

Solution: If we have $\frac{n}{2}$ vertices in each equivalence class, we'll have to update $\frac{n}{2}$ vertices no matter what. So, at worst we must do $O(n)$ updates to the array of canonical representations.