15-122: Principles of Imperative Computation

Recitation 6 Solutions

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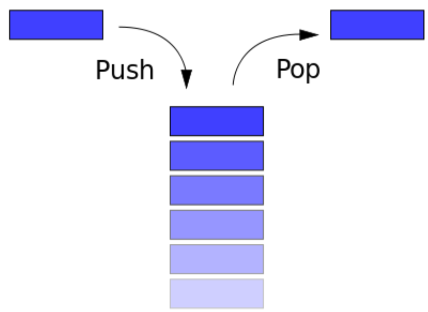
Stacks and Queues

Stacks are a LIFO (last in, first out) data structure. This is just like ordinary references to stack e.g. a stack of books.

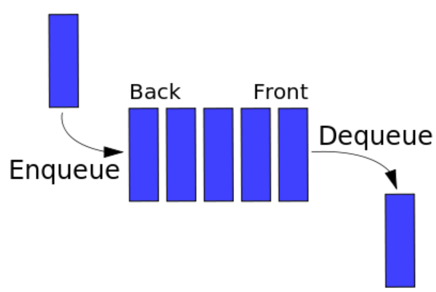


(Picture courtesy of Vegpuff/Wikipedia)

Here is the interface for stacks, as discussed in lecture:

```
stack stack_new();
bool stack_empty(stack S);
void push(stack S, string x);
string pop(stack S)
  /*@requires !stack_empty(S); @*/ ;
```

Queues are a FIFO (first in, first out) data structure. This is just like ordinary references to a queue e.g the queue (or line) at package pickup in the UC.



(Picture courtesy of Wikipedia)

Here is the interface for queues, as discussed in lecture:

```
queue queue_new();
bool queue_empty(queue Q);
void enq(queue Q, string x);
string deq(queue Q)
  /*@requires !queue_empty(Q); @*/ ;
```

Checkpoint 0

Write a function to reverse a queue, using only the functions from the interface.
Below is the general structure of this function. You may not need to fill in all the blanks.

```
/* Assume that you have data types stack and queue as described in lecture */
void reverse(queue Q)
{
  stack S = stack_new(); //Hint : Allocate a temporary data structure
  while(!queue_empty(Q))
  {
    string temp = deq(Q);
    push(S, temp);
  }
  while(!stack_empty(S))
  {
    string temp = pop(S);
    enq(Q, temp);
  }
}
```

Checkpoint 1

Why did we **NOT** need contracts in the function above?

Solution: Since we are clients using a given stack/queue interface, we can assume that, as long as we respect the interface, all stacks/queues are valid and that any operations involving them (which are allowed by the interface) are safe.

Checkpoint 2

Write a function to calculate the size of a stack without allocating any new data structures. At the end of the function, the stack must be unmodified (Hint : You will need to modify the stack within the function, but must ensure it is the same at the end)

Solution:

```
//recursive without allocating data structures
int stack_size(stack S)
//@requires is_stack(S);
//@ensures
{
  if (stack_empty(S)) return 0;
  string x = pop(S);
  int i = 1 + stack_size(S);
  push(S, x);
  return i;
}
```

Clac

clac is a relatively simple *postfix*-based programming language. As we read in numbers from the input (which we represent as a queue), we push operands onto a stack and act on them based on the instructions that are in the queue.

Here's an example of *clac* processing some input (you can get this yourself when working on the clac assignment by running clac-ref).

```
$ clac-ref -trace
Clac top level
clac>> 5 9 2 7 3 + - / dup * %
                                 stack || queue
                                       || 5 9 2 7 3 + - / dup * %
                                     5 || 9 2 7 3 + - / dup * %
                                   5 9 || 2 7 3 + - / dup * %
                                 5 9 2 || 7 3 + - / dup * %
                               5 9 2 7 || 3 + - / dup * %
                             5 9 2 7 3 || + - / dup * %
                              5 9 2 10 || - / dup * %
                                5 9 -8 || / dup * %
                                  5 -1 || dup * %
                               5 -1 -1 || * %
                                   5 1 || %
                                     0 ||
0
```

What's happening here? Well, we push all of the numbers onto the stack after reading them out of the queue. Then, we get to the +, so we pop two items (the 7 and the 3) off of the stack, add them, and push their sum, 10, back on. Next, we get to the -, pop off the 2 and 10 and subtract them, and get -8, which we push on to the stack. Then, we get to the /. We pop 9 and -8 and divide them. 9/-8 rounds to -1, so we push that onto the stack. Next, we execute the dup, which simply makes the top element of the stack appear twice. We get to the *, which multiplies the top two elements, giving us 1. Finally, we get to the %. 5 % 1 == 0, so we push 0. Then, we're out of instructions, so we end and pop the top item off of the stack and print it.

A common source of confusion with clac is if statements and else statements.

When we get to an if statement, we pop the top item off of the stack. If it is 0, we skip the next *two* tokens in the queue – we just ignore them. Otherwise (if it's non-zero), we continue processing tokens as normal.

When we get to an else statement, we *always* skip the next token in the queue.

So, why are these if/else statements? Let's take a look at some clac code

NOTE: In clac code below, we're using x it to mean any arbitrary int – you should fill in an int, like 1, -1, 0, etc, if you're actually running the code.

```
$ clac-ref -trace
Clac top level
clac>> 0 if 2 else 3
                                  stack || queue
                                        || 0 if 2 else 3
                                      0 || if 2 else 3
                                        || 3
                                      3 ||
3
clac>> 1 if 2 else 3
                                  stack || queue
                                      3 || 1 if 2 else 3
                                    3 1 || if 2 else 3
                                      3 || 2 else 3
                                    3 2 || else 3
                                    3 2 ||
2
```

Next, let's write a simple clac program: one that calculates absolute value. We can define $|x|$ as follows:

$$|x| = \begin{cases} x * 1 & \text{if } x \geq 0 \\ x * -1 & \text{if } x < 0 \end{cases}$$

So, if x is less than 0, we want to multiply it by -1 and otherwise we want to multiply it by 1. If we run the clac command `x 0 <`, then it will result in `1` being on the top of the stack if $x < 0$ and `0` being on the top of the stack otherwise.

We eventually want to multiply by either 1 or -1, so we should push the appropriate one of them onto the stack: If $x < 0$ we multiply by -1, otherwise we multiply by 1.

So, we add `if -1 else 1` to our command. Now we have

```
x 0 < if -1 else 1
```

This says "if $x < 0$, push -1 onto the stack. Otherwise, push 1 onto the stack." This works because when `x 0 <` evaluates to `0` (so $x \geq 0$), we ignore the tokens `-1` and `else`, so we just push `1` onto the stack. If `x 0 <` evaluates to `1` (so $x < 0$), then we push `-1` onto the stack and ignore the token `1`.

Next, we want to multiply by `x`, so we add `*` to the end:

```
x 0 < if -1 else 1 *
```

This doesn't work, though! We popped `x` off of the stack when we did the comparison. If we run the above command, we get:

```
Error: Error: not enough elements on stack
```

So, we need to duplicate `x` before we compare, so we can still use it later:

```
x dup 0 < if -1 else 1 *
```

That will compute the absolute value of `x`.