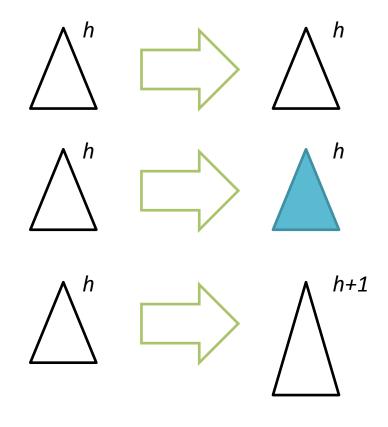
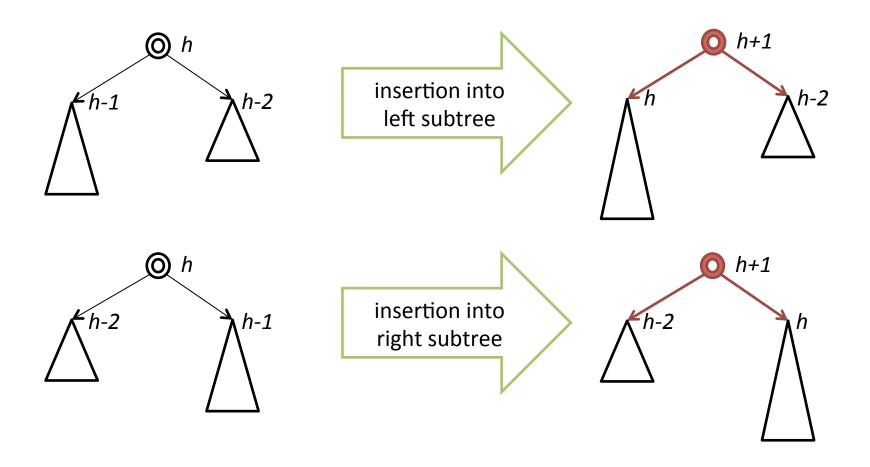
### **AVL** insertion postcondition

When you insert into an AVL tree, either

- you get a new tree with the same height (which may have had rotations performed), or
- you get a tree that hasn't had any rotations performed on it, with an increased height of at most one

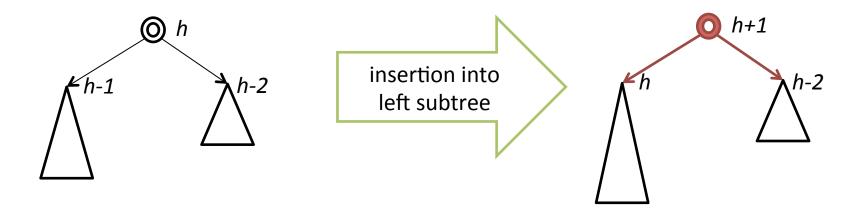


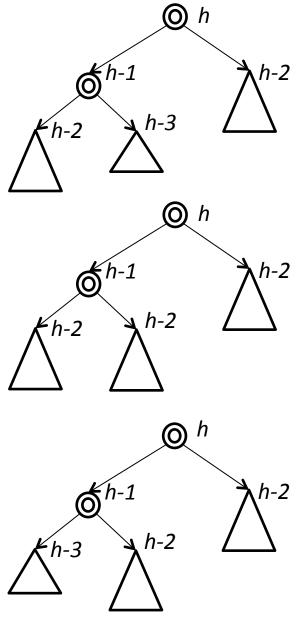
# how could a violation happen?



The red nodes oviolate the AVL height invariant. Two functions, rebalance\_left and rebalance\_right, will handle these two cases.

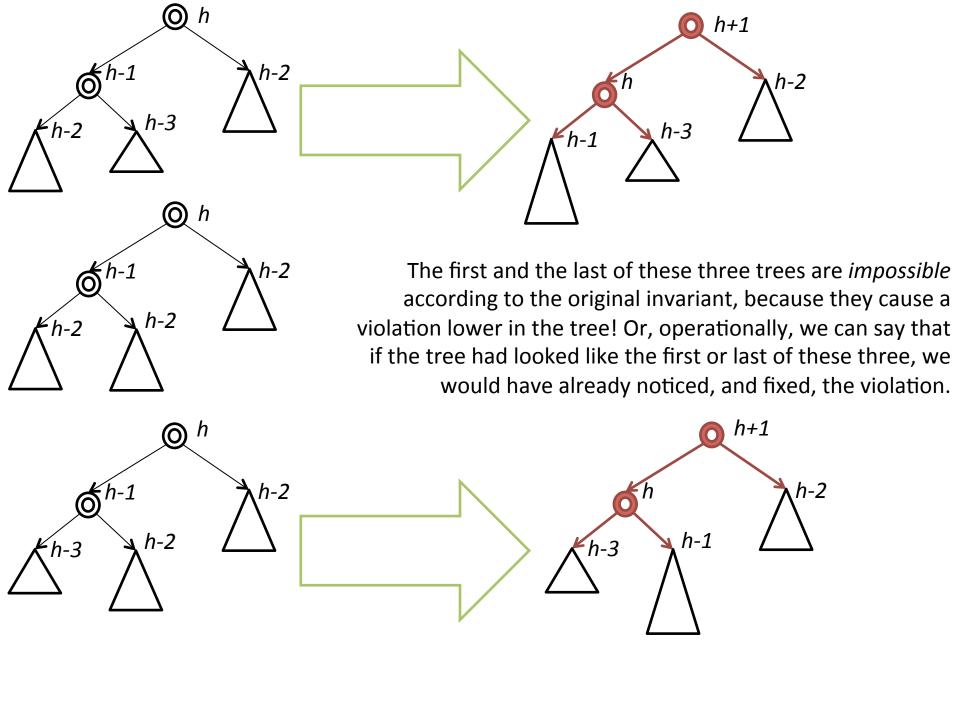
#### what does that left subtree look like?

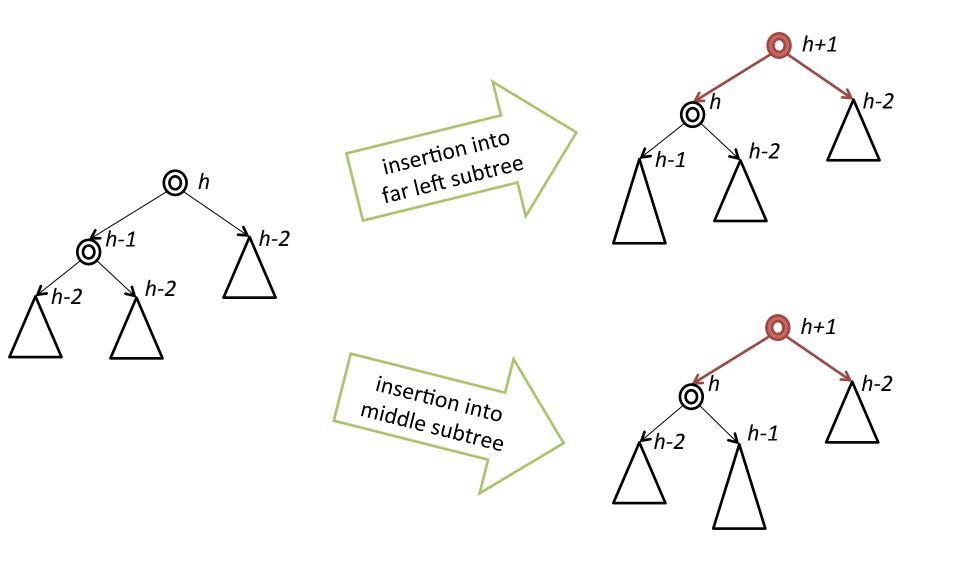




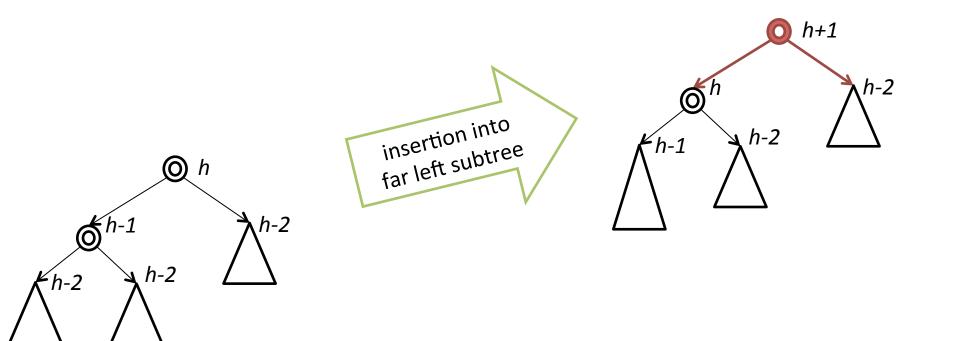
We know the AVL tree invariant held before, so the left subtree can only take one of three forms.

But that's not all we know!

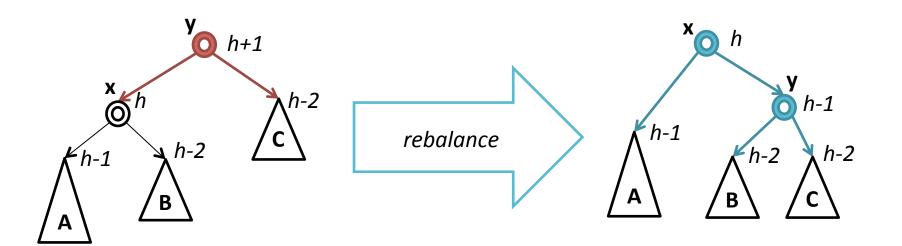


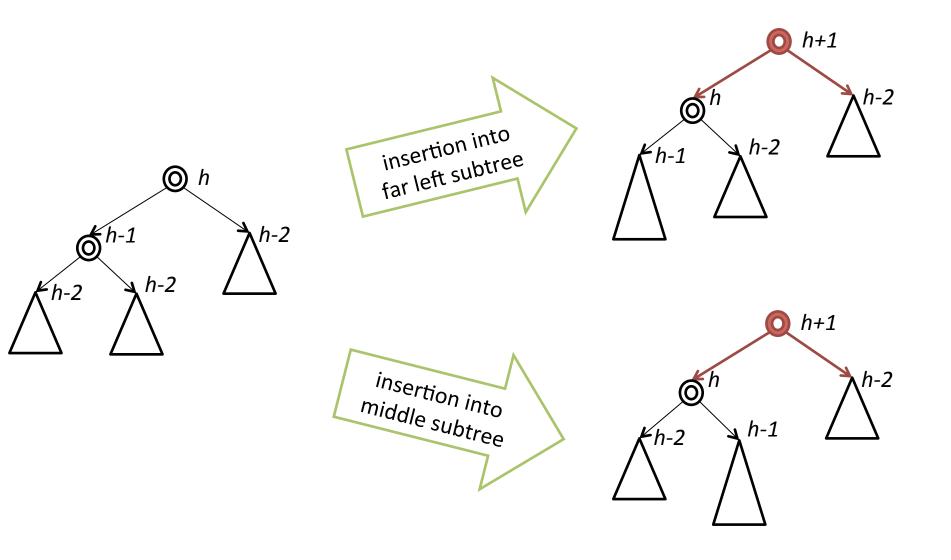


So we're left with this picture for rebalance\_left: two possible ways that BST insertion could have violated the AVL height invariant at the root.

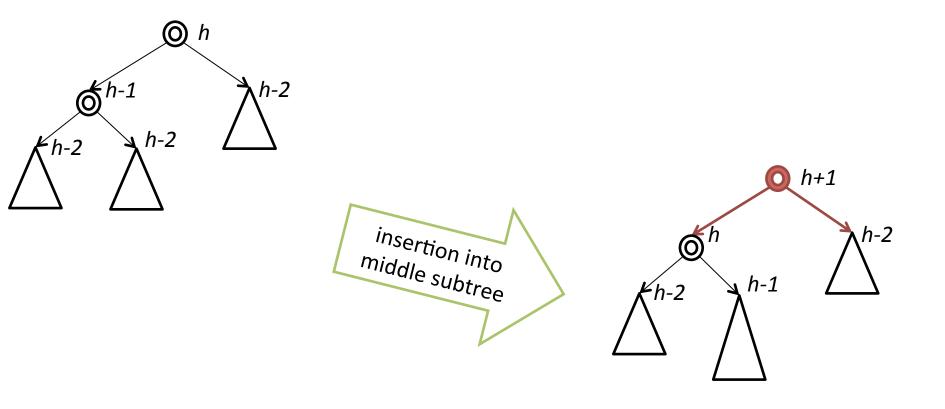


# fixup: single rotation



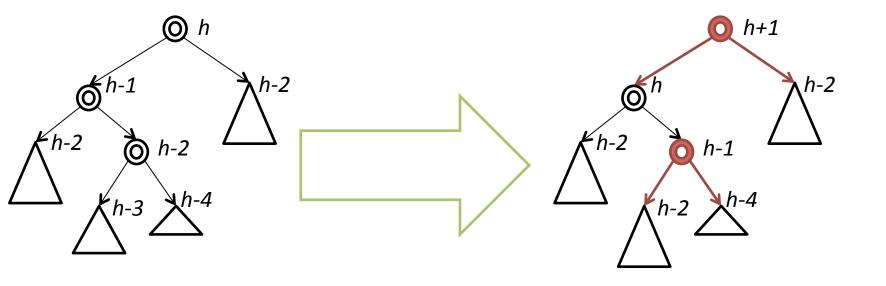


That takes care of the first case...

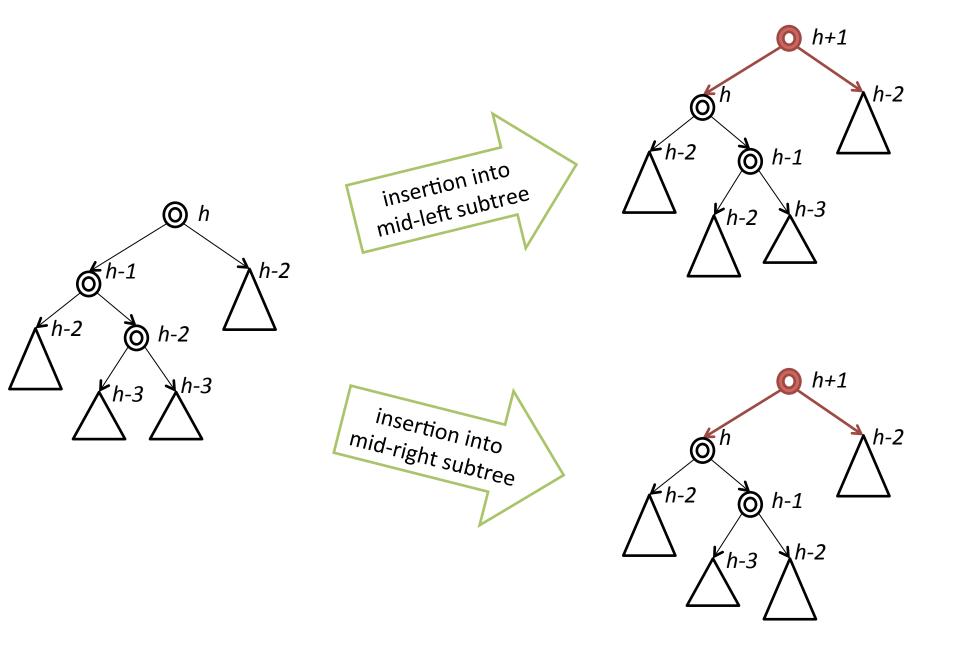


...now let's look at the second case.

To see how this works, we'll need to examine the structure of the middle tree.

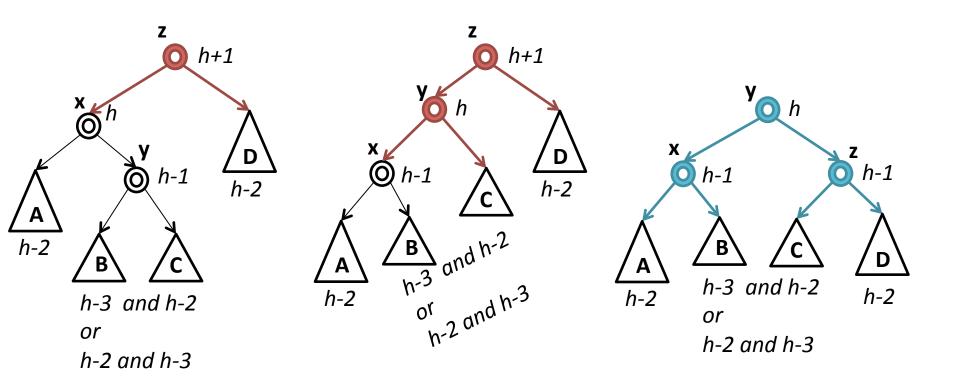


By the same reasoning as before, we know that the middle subtree can't itself have subtrees with different heights, or there would be a lower violation of the height invariant.



So both middle subtrees have height h-3, and one of them has height h-2 after the insertion. Again we have two cases! Luckily, we can deal with both the same way.

# fixup: double rotation



The solution is the same regardless of whether the insertion went into **B** or **C**. We rotate *twice*: first, left, at T->left, and then, right, at T.