Distributed Control and Optimization: from Robotics to Networking

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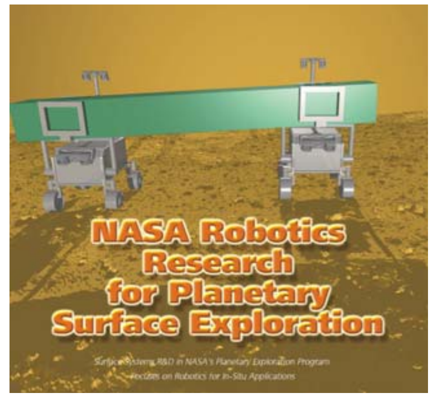
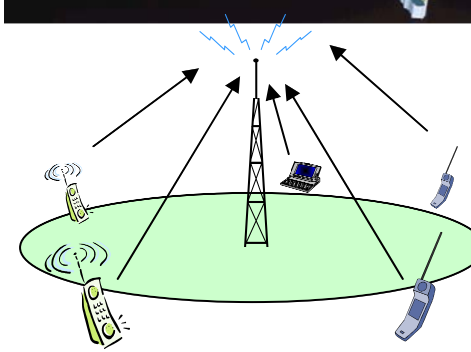
Distributed Control Systems

- Social insects, e.g., ants
- Team robots
- Internet
- Cellular network
- Power systems
- Networked control system

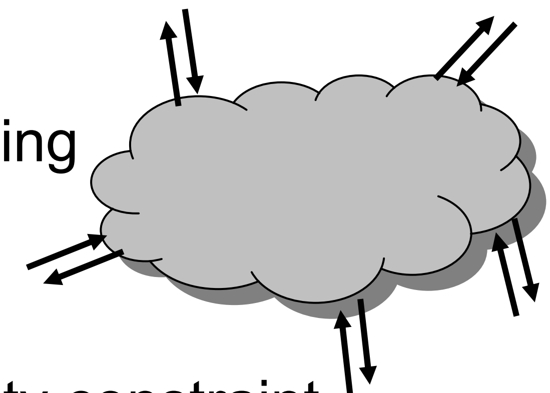




Today’s Talk

Decentralized action toward centralized goal through indirect communication (stigmergy: Grasse 1959 for social insects)

- **Collaborative load handling**:
  - Goal: carrying load to target without dropping
  - Communication: reaction force

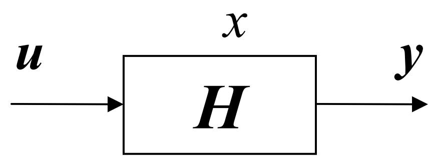


- **Internet:**
  - Goal: maximize overall utility under capacity constraint
  - Communication: packet loss, delay
- **Cellular network:**
  - Goal: maximum throughput subject to interference
  - Communication: interference, signal quality

Can we ensure stability and performance?

Common Tool: Passivity

Passivity: Energy conserving or dissipating



H is passive if there exists a *storage function* $V(x) \geq 0$ such that for some function $W(x) \geq 0$

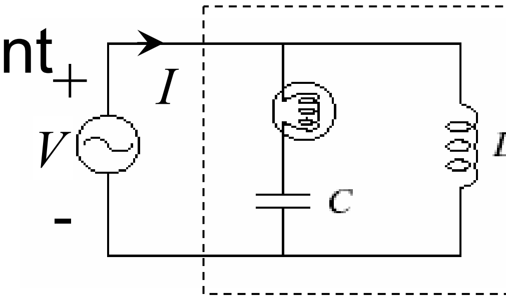
$$\dot{V} \leq -W(x) + u^T y$$

rate of energy dissipation

power into the system

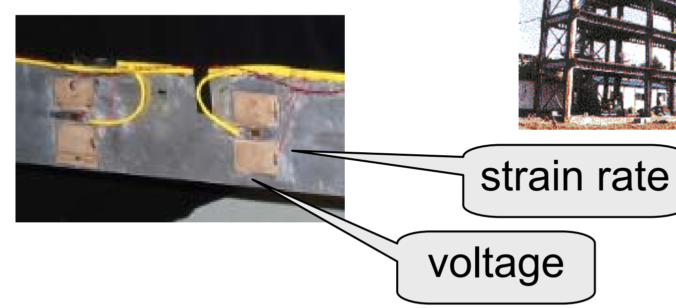
Passivity in Physical Systems

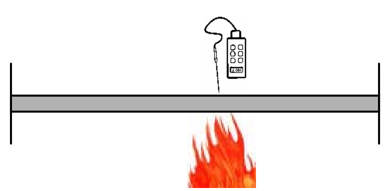
- RLC circuits: input=voltage, output=current (Anderson & Vongpanitlerd, 73)



- Structure with collocated sensor/actuator(Benhabib et al. 79, Joshi 89):
  force → velocity
  torque → ang. velocity
  voltage → strain rate (Hagood et al 91,Dosch et al. 92)

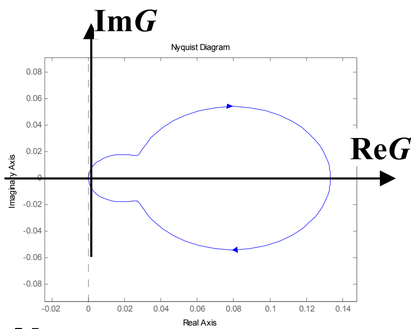
- Heat conduction with collocated heat input and temperature output

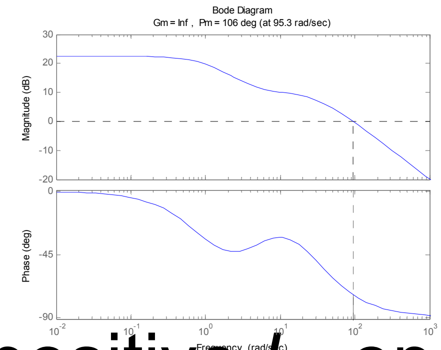




Characterization of Passivity Systems

LTI system: positive realness, $\pm$ 90° phase

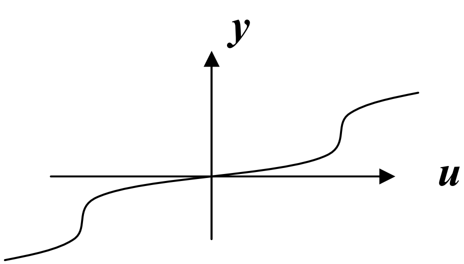




Nonlinear systems: positive L_2 operator

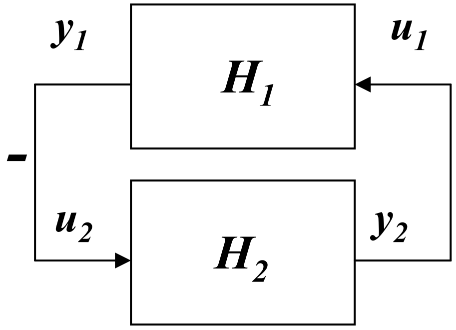
(Zames 66, Willems 72)

Memoryless nonlinearities: first/third quadrant function



Passivity Theorem

Negative feedback connection of passive systems with positive definite and radially unbounded storage functions is Lyapunov stable.



$$V(x_1 + x_2) = V_1(x_1) + V_2(x_2)$$

$$\dot{V}(x_1 + x_2) \le -W_1(x_1) - W_2(x_2) + u_1 y_1 + u_2 y_2$$

$$= -W_1(x_1) - W_2(x_2)$$

Strict passivity is important for

- asymptotic stability
- robustness analysis
- adaptive control

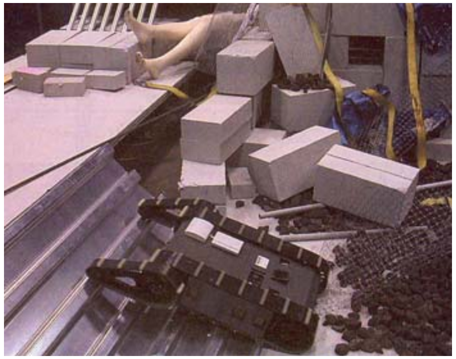
Applications

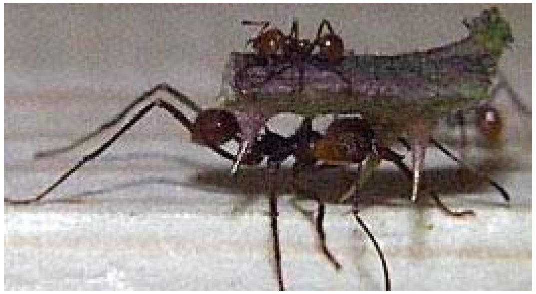
- LTI systems: robustness analysis and robust controller design
- Distributed parameter systems: structural systems, smart structure (PZT, SMA), thermal systems
- Nonlinear systems: e.g., robot control, attitude control
- Complex (large scale) systems: network, circuits, biological systems
- Fault tolerant systems: tight dependence on model information is inherently fragile

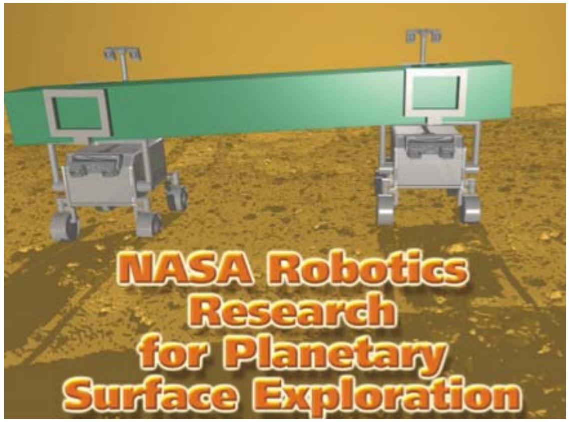
Distributed Cooperative Load Carrying

Motivation

- Material transport of hazardous materials, e.g., nuclear power plant, chemical/toxic substance removal
- Transportation of wounded personnel, search and rescue of victims.
- Robot colony for planetary exploration
- Biological inspiration: Ants





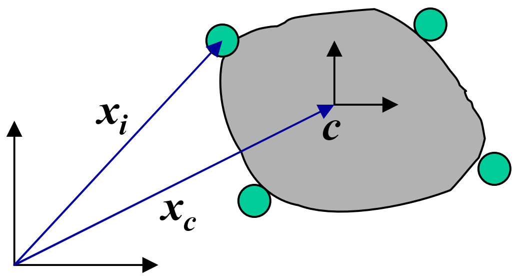


Decentralized Load Transport by Ants

- Ant sends recruiting signal if load is too heavy.
- # of ants depend on weight and resistance of load, not on size
- Ants reorient and re-grasp until load starts moving.
- If load is stuck, ants reorient and re-grasp until load gets unstuck.

A Simple Model

Point robots transporting a load. Assume rigid grasp and rigid load.



Goal: Find F_i based on x_i and f_i so that $x_c \rightarrow x_{cdes}$ and $f \rightarrow f_{des}$

$$
\begin{aligned}
m_i \ddot{x}_i &= F_i - f_i, \quad i = 1, \ldots, N \\
M_c \ddot{x}_c &= \sum_{i=1}^{N} A_{ic}^T f_i \\
x_i &= \phi_i(x_c) \\
\dot{x}_i &= A_{ic} \dot{x}_c \\
\ddot{x}_i &= A_{ic} \ddot{x}_c.
\end{aligned}
$$

Dynamics of each robot

Dynamics of load

Kinematic constraints

$$
\begin{aligned}
M \ddot{x} &= F - f \\
M_c \ddot{x}_c &= A^T f \\
x &= \phi(x_c), \quad \dot{x} = A \dot{x}_c, \quad \ddot{x} = A \ddot{x}_c.
\end{aligned}
$$

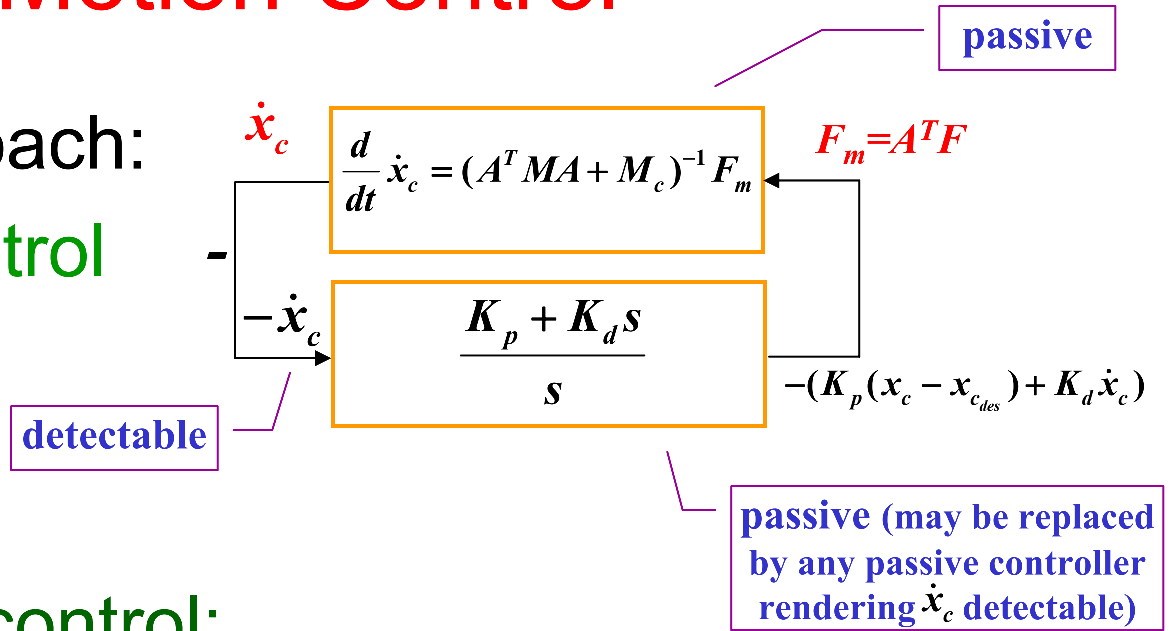
vectorize

$$
\begin{aligned}
\ddot{x}_c &= (A^T M A + M_c)^{-1} A^T F \\
f &= (M A M_c^{-1} A^T + I)^{-1} F
\end{aligned}
$$

Separate motion & force

Motion Control

- Passivity approach:
  centralized control

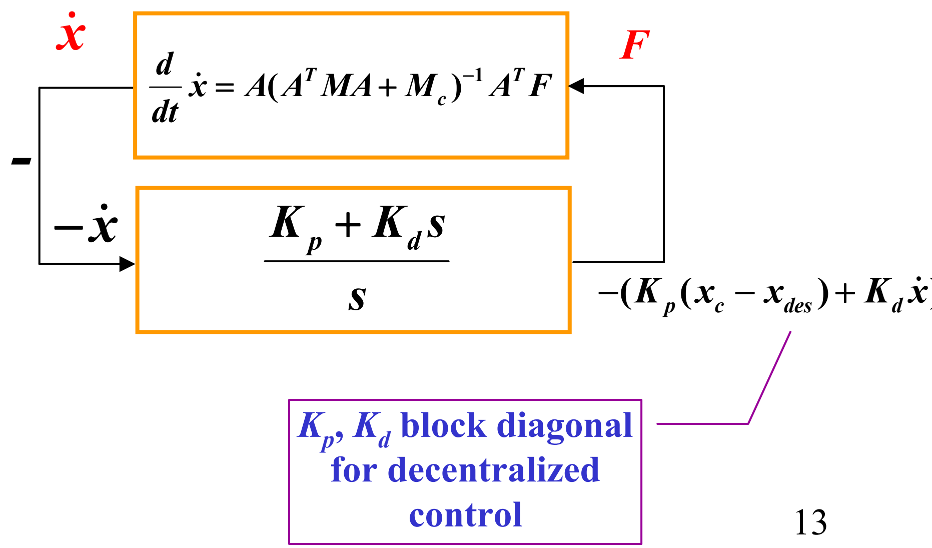


- Decentralized control:

Questions:

1. Is $\dot{x}$ detectable if x_{des} is kinematically feasible?

2. What if x_{des} is kinematically infeasible?



Asymptotic Behavior

From passivity, $\dot{x} \to 0$ asymptotically. Therefore, $A^T K_p (x - x_{des}) \to 0$ or

$\mathcal{R}(A)$

$$x^* = x_{des} + K_p^{-1} \tilde{A}^T \mu.$$

x^* must satisfy the constraint:

$$\widetilde{\mathcal{O}} \underbrace{\phi^{-1}(x_{des} + K_p^{-1} \tilde{A}^T \mu)}_{O\,x_c} = 0.$$

$$O = \begin{bmatrix} I \\ \vdots \\ I \end{bmatrix}$$

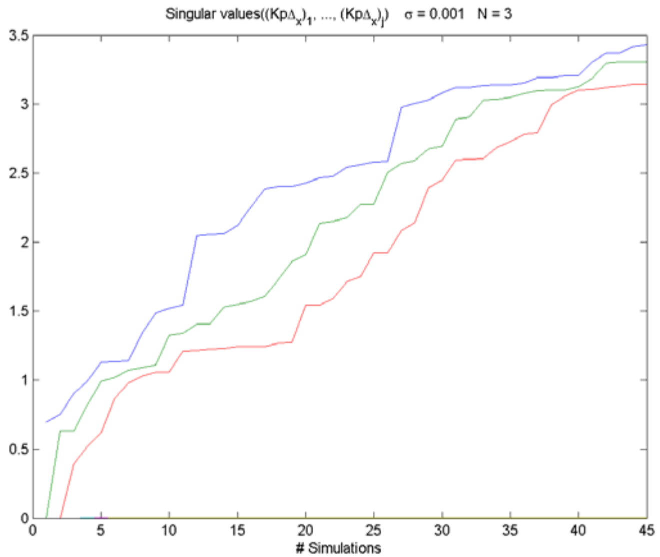
Since $\nabla_\mu (\widetilde{\mathcal{O}} \phi^{-1}(x_{des} + K_p^{-1} \tilde{A}^T \mu)) = \tilde{A} K_p^{-1} \tilde{A}^T > 0$, x^* is unique.

$(x^*, \dot{x})$ is globally asymptotically stable

Feasibility of Set Point

If x_{des} is feasible, $\mu = 0$ is the unique solution. Therefore, $(x_{des}, \dot{x})$ is globally asymptotically stable.

If x_{des} is infeasible, steady state error $x^* - x_{des}$ my be used to identify the grasp map, $\mathcal{N}(A^T)$.
Define $g(x_{des}) := K_p(x^* - x_{des})$, x^* is the unique solution of $A^T K_p(x^* - x_{des}) = 0$, then $\mathcal{R}(g) = \mathcal{N}(A^T)$.



A three-robot example: singular values of $[x_1^* - x_{1_{des}}, x_2^* - x_{2_{des}}, x_3^* - x_{3_{des}}]$.

Extension: Exponential Stability

Passivity only provides asymptotic stability which does not guarantee robustness (e.g., imprecise $\mathcal{N}(A^T)$). We can modify the combined storage functions to show exponential stability:

$$V = \underbrace{\frac{1}{2}\dot{x}_c^T M_c \dot{x}_c + \frac{1}{2}\dot{x}^T M \dot{x} + \frac{1}{2}\Delta x^T \hat{K}_p \Delta x}_{\text{storage functions}} + c\dot{x}^T \hat{K}_p \Delta x$$

Additional cross term

$\hat{K}_p := K_p A (A^T K_p A)^{-1} A^T K_p$ **Note that** $A^T \hat{K}_p = A^T K_p$

$$\dot{V} = -\dot{x}_c^T A^T (K_d - cK_p) A \dot{x}_c - c(A^T K_p \Delta x)^T (M_c + A^T M A)^{-1} (A^T K_p \Delta x)$$
$$- c\dot{x}_c^T (A^T K_d A)(M_c + A^T M A)^{-1} (A^T K_p \Delta x)$$

For c sufficiently small, $V > 0$ and $\dot{V} < 0$, and both are quadratic, therefore, $(x^*, 0)$ is globally exponentially stable .

Motion and Force Control

Move/squeeze decomposition (Wen, Kreutz '92):

$$F = F_m + F_s, \qquad F_s \in \mathcal{N}(A^T)$$

Rigid load assumption ➔

motion control affects force,
but force control does not affect motion

Internal force:

$$f_s = F_s - \underbrace{\widetilde{A}^T(\widetilde{A}\widetilde{A}^T)^{-1}\widetilde{A}(MAM_c^{-1}A^T + I)^{-1}MAM_c^{-1}A^T F_m}_{\text{motion induced force, } \overline{\gamma}}$$

Force Control Law

$$F_s = -C(s)(f_s - f_{s_{des}}) + f_{s_{des}}.$$

Complete force and $N(A^T)$ required at each robot – not decentralized!

Choice of $C(s)$:

- Stability: $(1+C(s))$ has all the zeros in the open left half plane.
- Zero steady state error: $C(s)$ has at least a pole at 0 (integral control).
- Disturbance rejection: $C(s)$ should have high gains over the spectrum of disturbance.
- Robustness: $C(s)$ has sufficient phase lead to ensure large phase margin for delay robustness.

A common choice: $C(s) = \frac{k_f}{s}$, integral force control.

(Open loop force control: $C(s) = 0$)

Decentralized Force Control

Replace squeeze force control law with a fully decentralized force control law

$$F_s = -C(s)(f - f_{s_{des}}) + f_{s_{des}}.$$

No longer in *N*(A^T)

Motion and force loops are now coupled --- exponential stability of decoupled system can now be used to show motion and force loops remain exponentially stable (large force feedback gain does adversely affect rate of convergence).

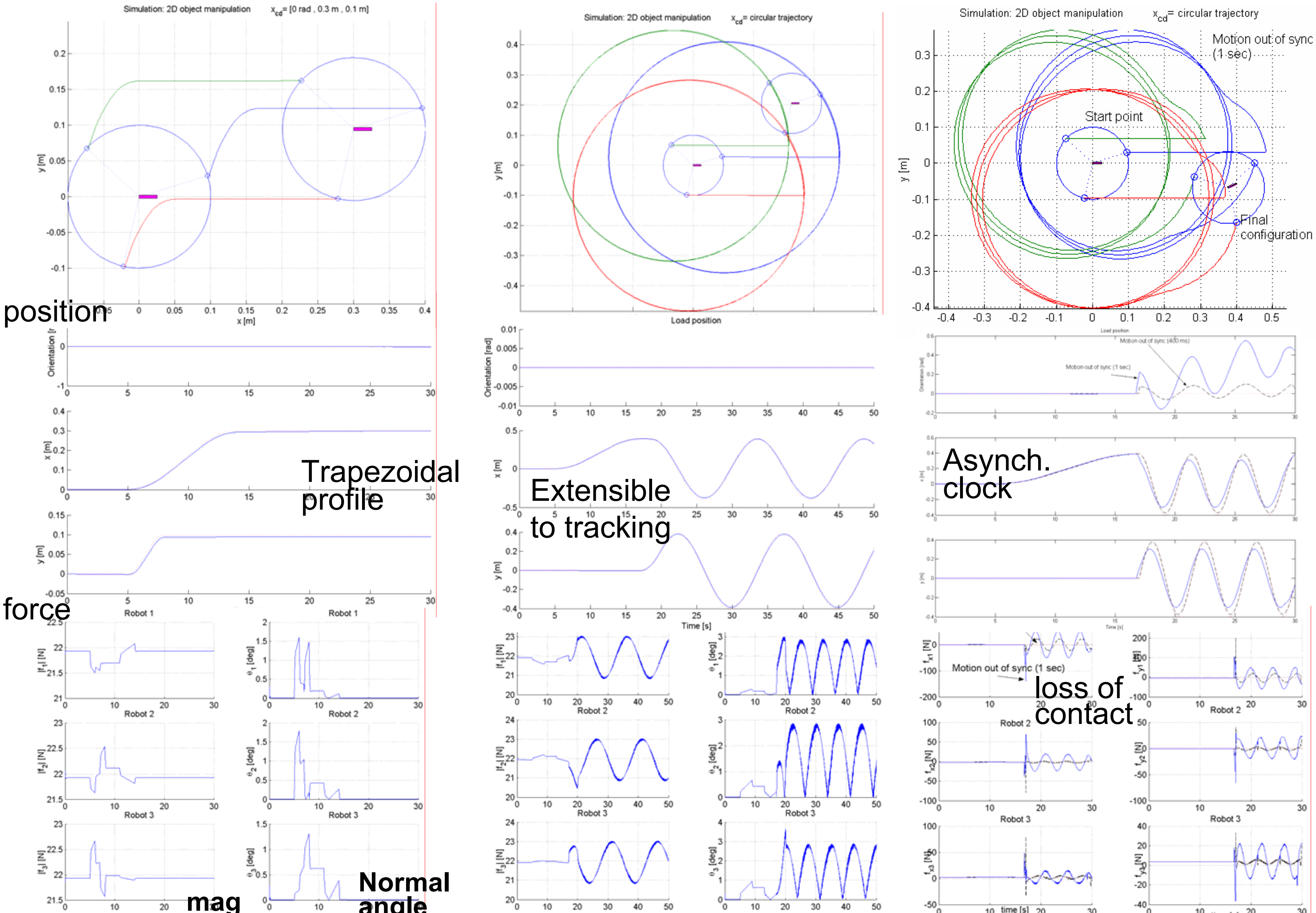
Summary of Decentralized Collaborative Load Transport

- Apply random x_{des} to decentralized motion control to obtain direction for force set point $f_{s,des}$
- Apply decentralized motion and force control to move load to goal while maintaining squeeze force.

- Passivity is useful in initial motion and force control, but modification is required for exponential stability.

- Extensions: gain tuning and adaptation, asynchronous clock, quantization in force feedback, finger contact, limited communication

Simulation

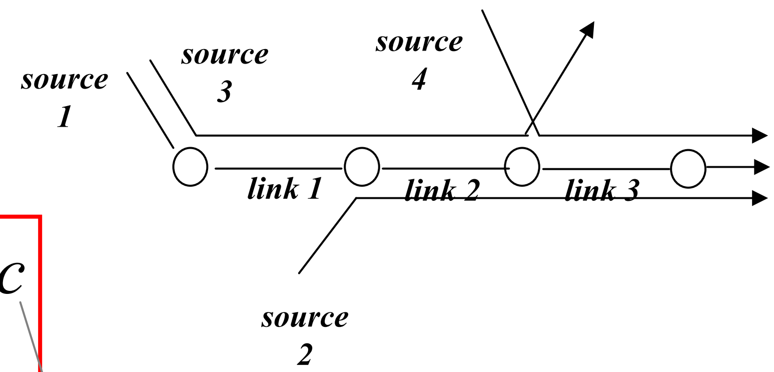


Simulation: 2D object manipulation
x_{cd}= [0 rad , 0.3 m , 0.1 m]
Simulation: 2D object manipulation
x_{cd}= circular trajectory
Simulation: 2D object manipulation
x_{cd}= circular trajectory
Motion out of sync (1 sec)
Start point
Final configuration
position
Load position
Trapezoidal profile
Extensible to tracking
Asynch. clock
force
Robot 1
Robot 2
Robot 3
Motion out of sync (1 sec)
loss of contact
mag
Normal angle
time [s]
Time [s]

Distributed Network Flow Regulation

Resource Allocation Problem

- Multiple users sharing limited communication resources



Source rate

$$\max_{x>0} \sum_i U_i(x_i) \quad \text{subject to} \quad \underbrace{Rx}_{y} \le c$$

Concave utility functions

Link rate

Link capacity

Optimality Condition: $q_i^* = U_i'(x_i^*)$ $\quad p_\ell^* \begin{cases} = 0 & \text{if } y_\ell^* < c_\ell \\ \ge 0 & \text{if } y_\ell^* = c_\ell \end{cases}$

$q = R^T p$

Lagrange multiplier

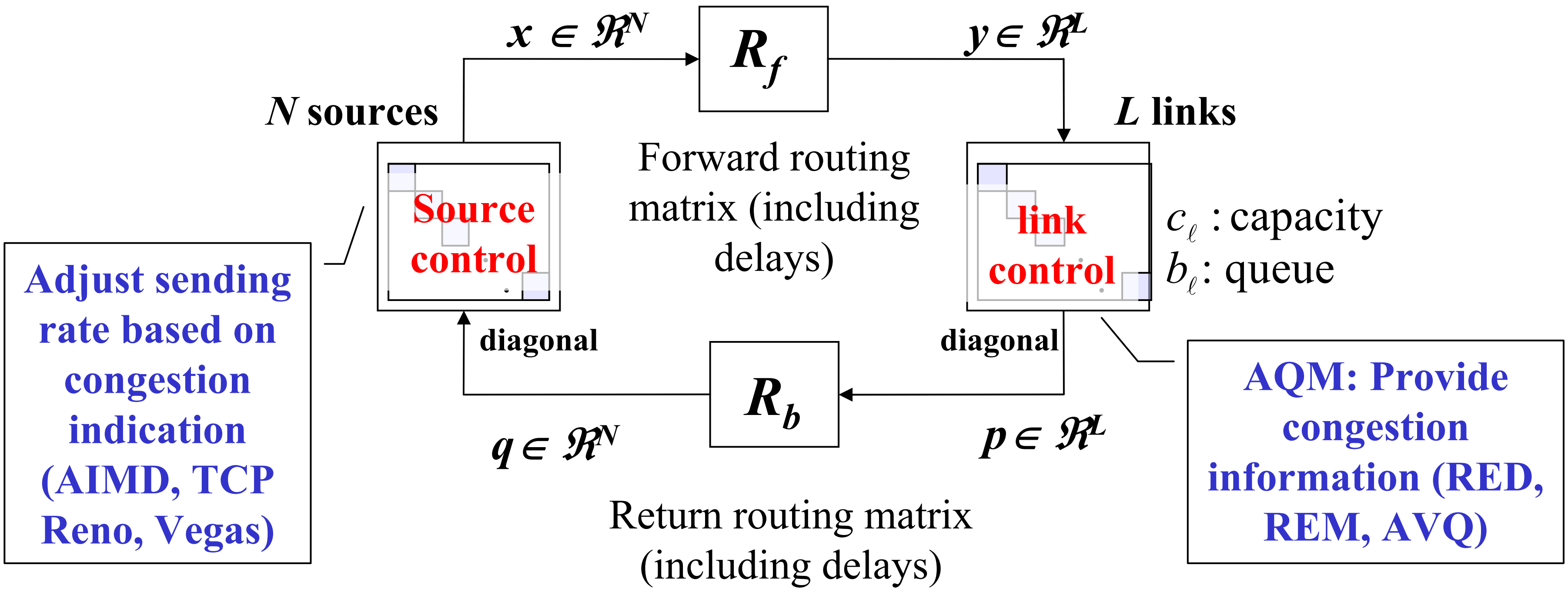
Examples:

TCP Reno: $U(x) = \frac{\sqrt{2}}{\tau} \tan^{-1}\left(\frac{\tau x}{\sqrt{2}}\right)$

TCP Reno (Variant): $U(x) = -\frac{a^2}{\tau^2 x}$

TCP Vegas (proportional fair): $U(x) = k \ln x$

Distributed Feedback Implementation



Network Flow Control Problem

Design source and link control laws to achieve:

- Stability: all signals are bounded and converge to equilibrium values.
- Utilization: maximize throughput (keep y_l close to c_l).
- Fairness: all sources have "equitable" shares of capacity.
- Robustness: maintain stability and performance under model variation and disturbances: unmodeled flows, time delays, capacity variation.

Problem Decomposition

- Find equilibrium values, x_i^*, y_l^*, p_l^*, q_i^*, based on maximizing specified utility function subject to the capacity constraint (addressing utilization and fairness).
- Design source and link dynamic controllers to stabilize about equilibrium (addressing stability and robustness).

Control Problem

- Design source and link control algorithms so that network converges to global optimum, i.e., $x_i \rightarrow x_i^*$, $y_\ell \rightarrow y_\ell^*$, $q_i \rightarrow q_i^*$, $p_\ell \rightarrow p_\ell^*$.

Challenges:

- Decentralization: x_i can only depend on q_i, p_ℓ can only depend on y_ℓ.
- No routing information: R cannot be used in control design.
- No explicit coordination among sources and links.

Stabilizing Flow Control: Primal

- Primal approach (Kelly, Mauloo, Tan 98)

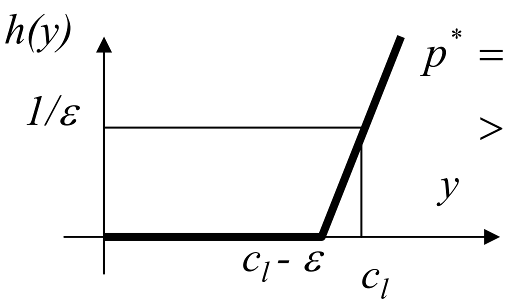
  Source: First order gradient law

  Link: Static penalty function

static

$$\dot{x}_i = k_i \left(U_i'(x_i) - q_i\right)_{x_i}^{+}, \qquad p_\ell = h(y_\ell)$$

First order



$$p^* = 0 \text{ if } y^* \le c_\ell - \varepsilon$$
$$> 0 \text{ if } y^* > c_\ell - \varepsilon$$

Stabilizing Flow Control: Dual

- Dual approach: Kelly, Mauloo, Tan 98, Low 99, Paganini 00.

  Source: Static optimality condition

  Link: First (queue) or second order law

First order

static

$$x_i = U_i^{'-1}(q_i),\ \dot{b}_\ell = (y_\ell - c_\ell)^+_{b_\ell},\ p_\ell = b_\ell$$

Second order

$$x_i = U_i^{'-1}(q_i),\ \dot{b}_\ell = (y_\ell - c_\ell)^+_{b_\ell},\ \dot{p}_\ell = \gamma_\ell (y_\ell - c_\ell + \alpha_\ell b_\ell)^+_{p_\ell}$$

Stabilizing Flow Control: Primal/Dual

- Primal/dual approach: Altman/Basar/Srikant (98), Hollot/Chait (01), Kunniyur/Srikant (02)

  Source: First order gradient law

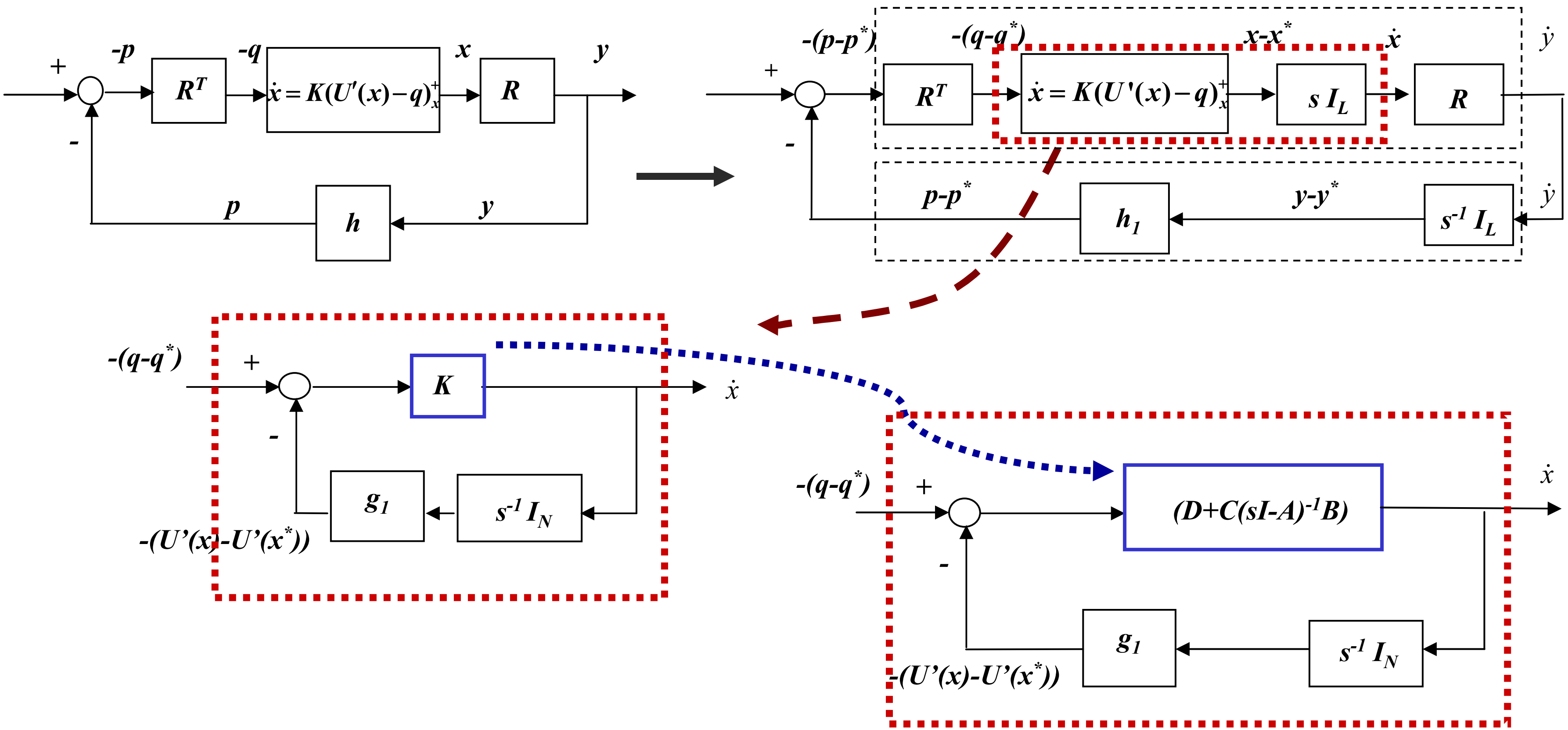
  Link: First order queue dynamics

$$\dot{x}_i = k_i(U_i'(x_i) - q_i),\ \dot{b}_\ell = (y_\ell - c_\ell)^+_{b_\ell},\ p_\ell = f_\ell(y_\ell, b_\ell)$$

First order

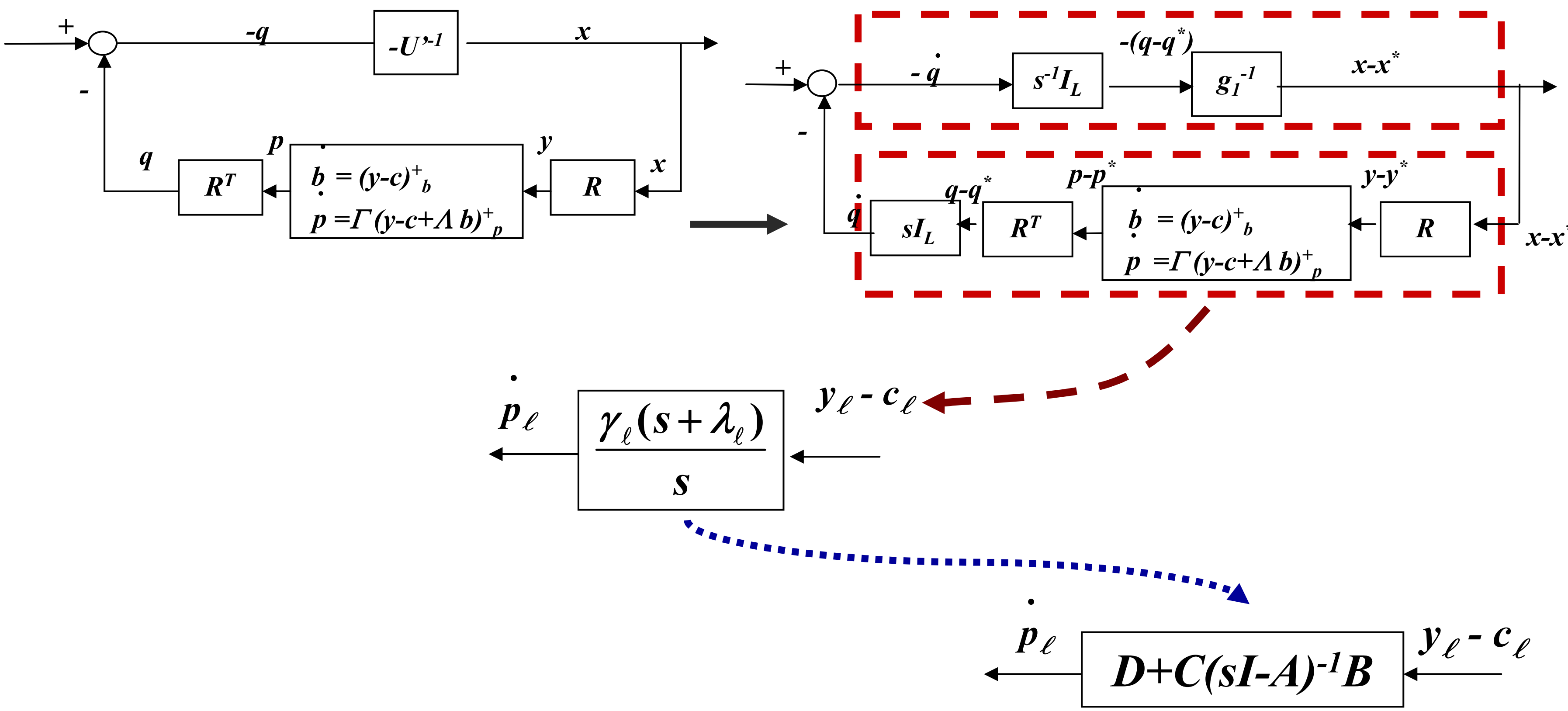
Passivity Perspective: Primal

Kelly's Primal Controller



Passivity Approach: Dual

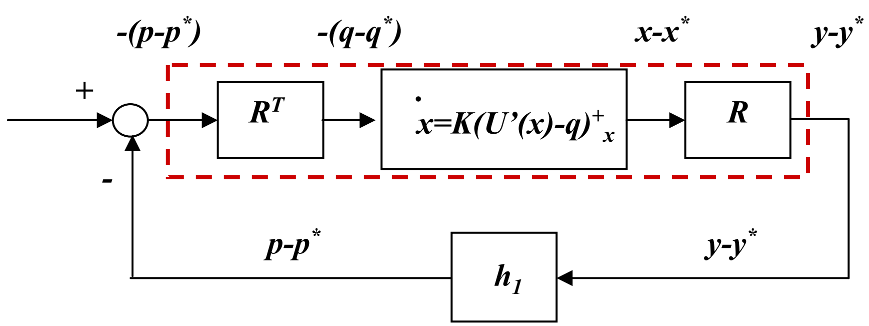
Low's Dual Controller



Extension

Passive decomposition is not unique:

- For first order source controller, the system between $-(p-p^*)$ and $(y-y^*)$ is strictly passive.



Lyapunov Function:

$$V = \frac{1}{2}(x - x^*)^T K^{-1}(x - x^*)$$

If $U''<0$ uniformly (strictly concave), $\dot{V}$ contains a negative definite term in x-x^* --- important for robustness!

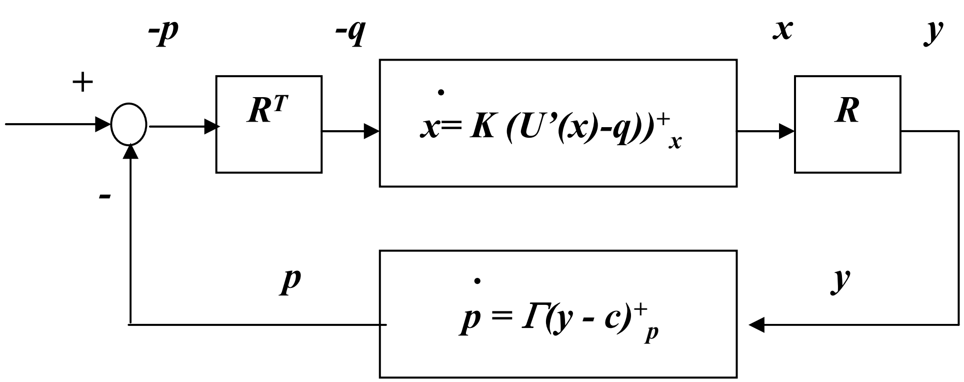
- Dual: $(y-y_\ell^*)$ to $(p-p_\ell^*)$ is also strictly passive

- Implementable using delay and loss

$$\tau = \frac{b}{c} + \tau_p \qquad \dot{\rho} = \begin{cases} \nu \\ y - c + \nu \end{cases} \qquad \dot{b} = \begin{cases} (y-c)_b^+ & b \le b_{\max} \text{ or } (b = b_{\max} \text{ and } y \le c) \\ 0 & b = b_{\max} \text{ and } y > c \end{cases}$$

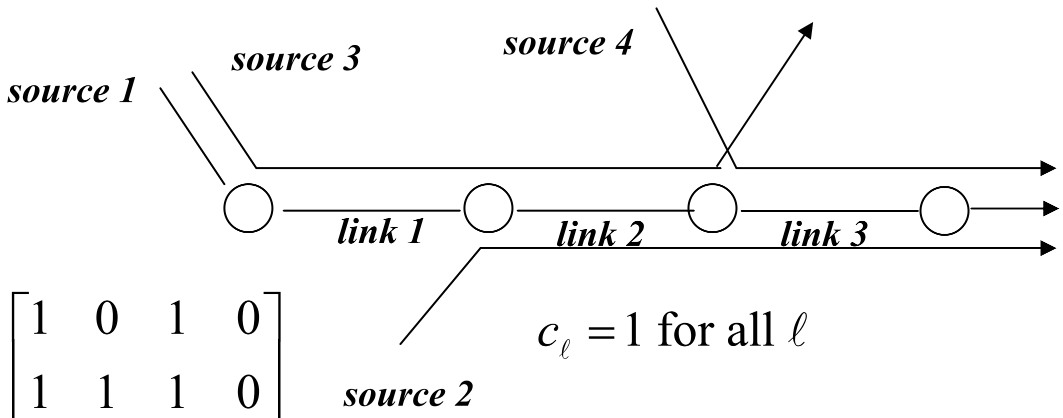
Passivity Approach: Primal/Dual Controller

- Consequence of passivity of first order source controller and first order link controller: combined dynamic controller is also stable.
- Generalizes Hollot/Chait controller and easily extended to Kunniyur/Srikant controller.



Infocom ’03, IEEE Trans. Automatic Control 2/04

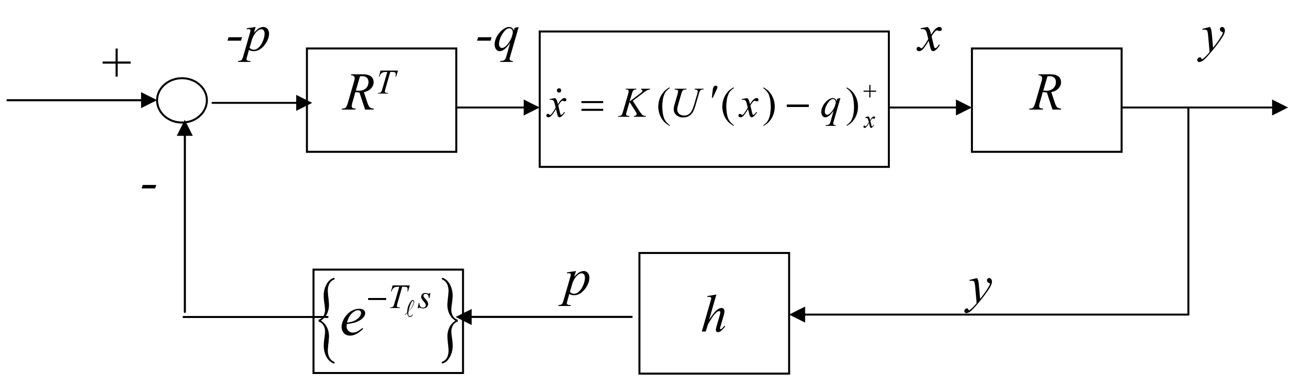
Simulation



$c_\ell = 1$ for all ℓ

$$R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

$$x^* = [0.25, 0.25, 0.5, 0.5]^T$$
$$q^* = [4, 4, 2, 2]^T$$
$$y^* = [0.75, 1, 1]^T$$
$$p^* = [0, 2, 2]^T$$

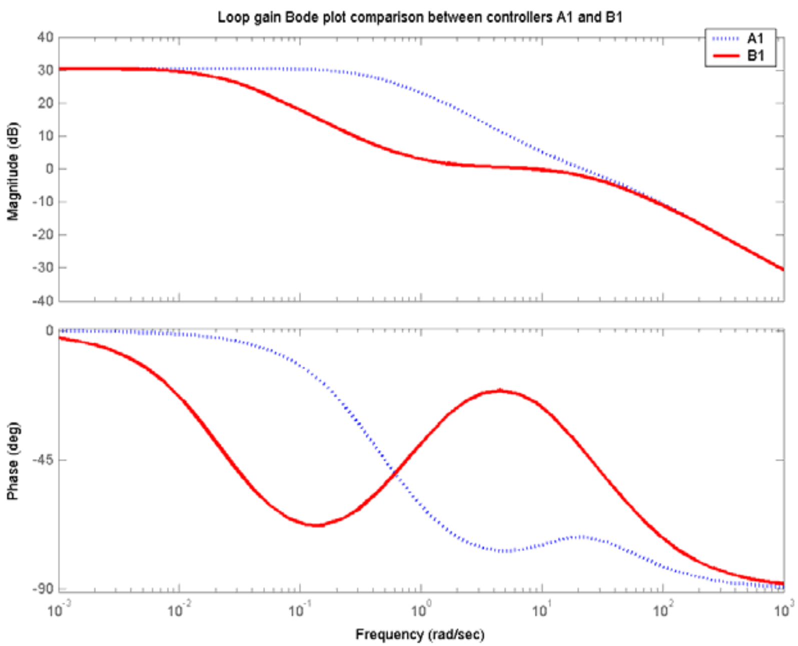


Simulation: Primal Controller

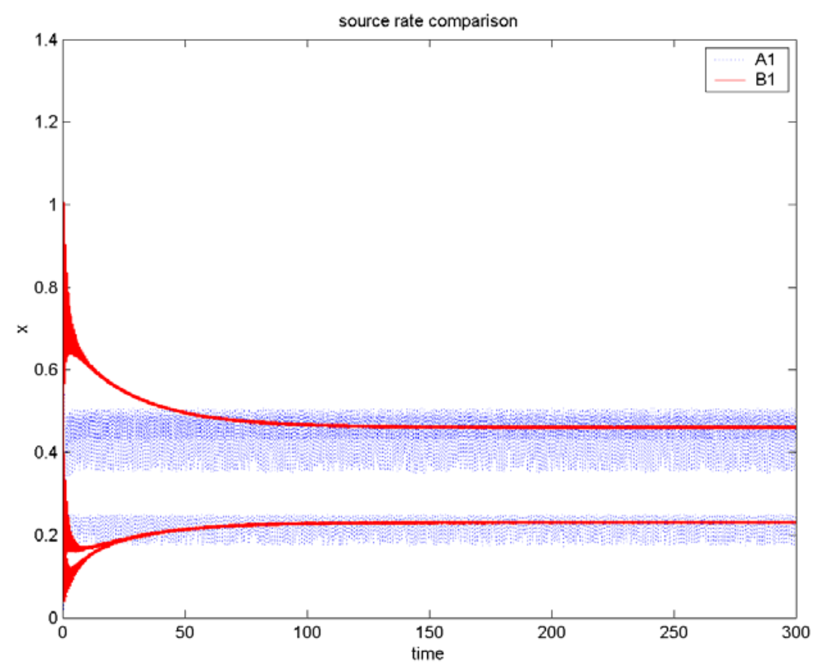
loop gain: $-Rh'(R\bar{x})(sI - W(s)U''(\bar{x}))^{-1}W(s)R^T$

$W(s) = k_i \boxed{(0.1)}$ or $D_i + C_i(sI - A_i)^{-1}B_i \boxed{\left(\frac{0.1(s+1)}{s+20}\right)}$ **(B1:Passive)**

(A1:Kelly)



.25 sec delay



A1 : ω_{gc} = 21.9rad/s, PM = 108.2^o

B1: ω_{gc} = 8.4rad/s, PM = 155.5^o

LTI $T_{\max} = \mathrm{PM}/\omega_{gc}$

A1 : $T_{\max} = 0.086$ sec

B1: $T_{\max} = 0.322$ sec

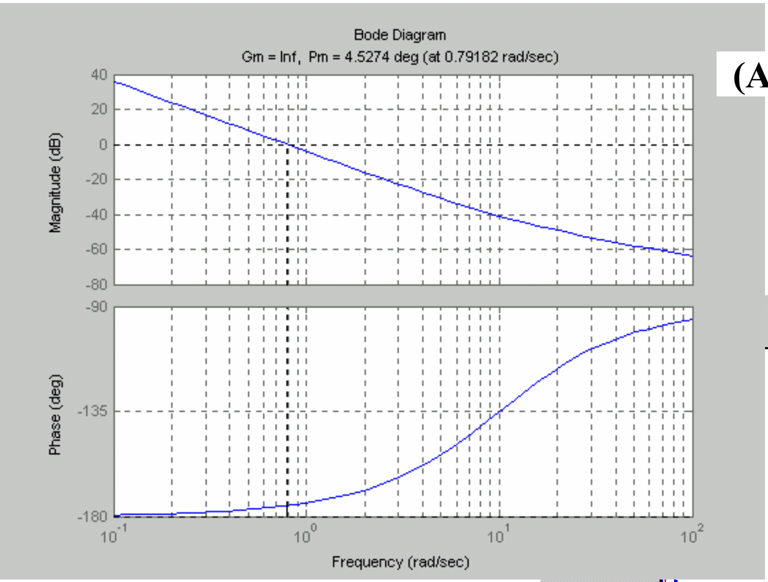
Simulation: Dual Controller

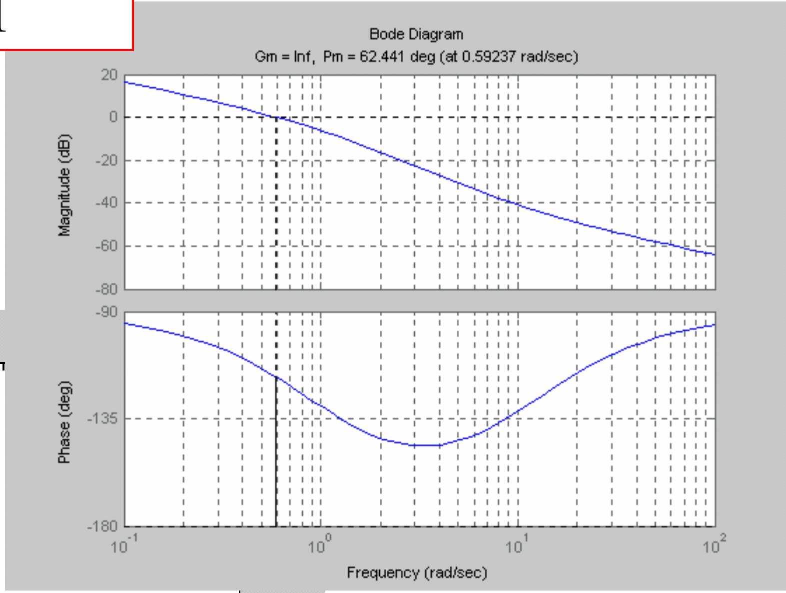
loop gain: $-RU''(\bar{x})^{-1}R^T s^{-1}(D_\ell + C_\ell(sI - A_\ell)^{-1}B_\ell)$

$$D_\ell + C_\ell(sI - A_\ell)^{-1}B_\ell = \boxed{\frac{2}{s}+2} \text{ or } \boxed{\frac{2}{s+1}+2}$$

(B2:Passive)

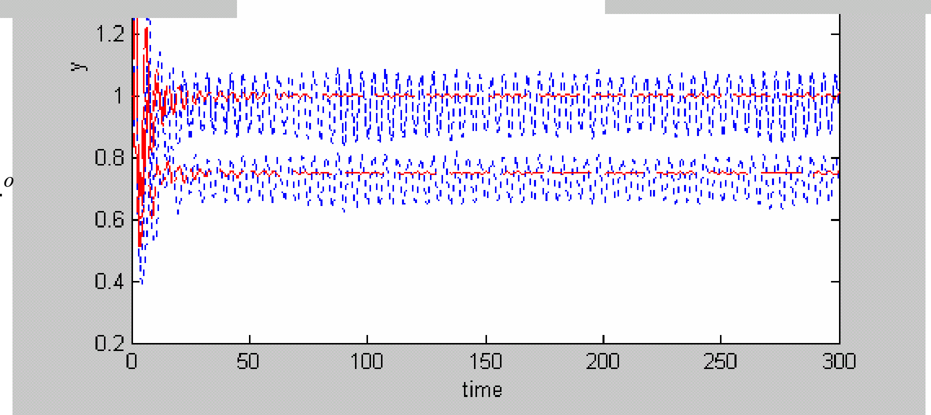
(A2:Low/Paganini)





link rate comparison

1 sec delay



A2 : $\omega_{gc} = 0.79\text{rad/s, PM} = 4.5^o$

B2: $\omega_{gc} = 0.59\text{rad/s, PM} = 62.4^o$

LTI $T_{\max} = \text{PM}/\omega_{gc}$

A2 : $T_{\max} = 0.1\sec$

B2: $T_{\max} = 1.85\sec$

Extensions

- L_p stability in the presence of L_p disturbances
- Delay robustness (gain may be scaled by $1/\tau$ for delay-invariant stability)
- Non-cooperative flows

CDC 03, Systems & Control Letters 04

Distributed Uplink Power Control in CDMA

Problem Formulation

Distributed power control: *i* th user minimizes its power p_i while maximizing signal-to-interference ratio (SIR)

SIR:

$$\gamma_i(p) = \frac{L h_i p_i}{\sum_{k \neq i} h_k p_k + \sigma^2}$$

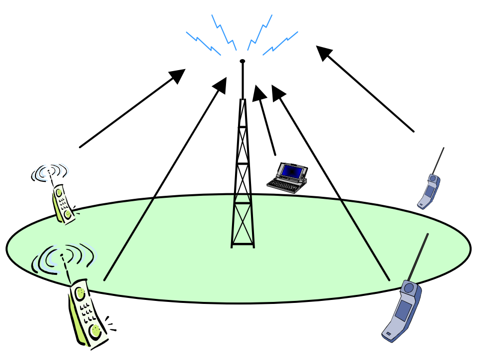
Signal power

Interference due to other users

Background noise power

L = spreading gain

h_i = channel gain



Game-Theoretic Approach

Non-cooperative game (Alpcan, Basar, Srikant, Altman '02)

$$\max_i J_i = U_i(\gamma_i(p)) - P_i(p_i)$$

U_i = utility function for ith user (concave)

P_i = cost of power (convex)

Asymptotically stability of Nash equilibrium (under certain assumptions) using gradient update law:

$$\dot{p}_i = -\lambda_i \frac{\partial J_i}{\partial p_i} = \frac{dU_i}{d\gamma_i}\frac{\partial \gamma_i}{\partial p_i} - \lambda_i \frac{dP_i(p_i)}{dp_i}, \quad \lambda_i > 0$$

For $U_i = u_i \log(\gamma_i + L)$

$$\frac{dU_i(\gamma_i)}{d\gamma_i} = \frac{u_i}{\gamma_i + L} = \frac{u_i\left(\sum_{k\neq i} h_k p_k + \sigma^2\right)}{L\left(\sum_i h_i p_i + \sigma^2\right)}$$

$$\frac{L\lambda_i h_i}{\sum_{k\neq i} h_k p_k + \sigma^2}$$

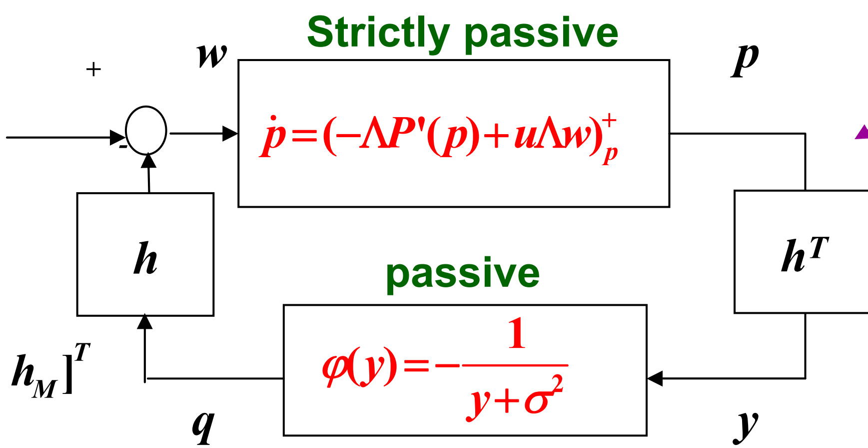
$$\dot{p}_i = -\lambda_i \frac{dP_i(p_i)}{dp_i} + \frac{u_i \lambda_i h_i}{\left(\sum_i h_i p_i + \sigma^2\right)}$$

Passivity Perspective

Write the gradient update law as

$$\dot{p}_i = -\lambda_i \frac{dP_i(p_i)}{dp_i} + \frac{u_i \lambda_i h_i}{\left(\sum_i h_i p_i + \sigma^2\right)} \quad \Rightarrow \quad \dot{p}_i = \left(-\lambda_i \frac{dP_i(p_i)}{dp_i} + u_i \lambda_i w_i\right)^+_{p_i}$$

$$w_i = \frac{h_i}{\left(\sum_i h_i p_i + \sigma^2\right)} = \frac{h_i}{\left(y + \sigma^2\right)} = -h_i \underbrace{\varphi(y)}_{q}$$



$$h = [h_1 \quad h_2 \quad \cdots \quad h_M]^T$$

Same structures as in flow control

Global asymptotic stability follows from passivity analysis.

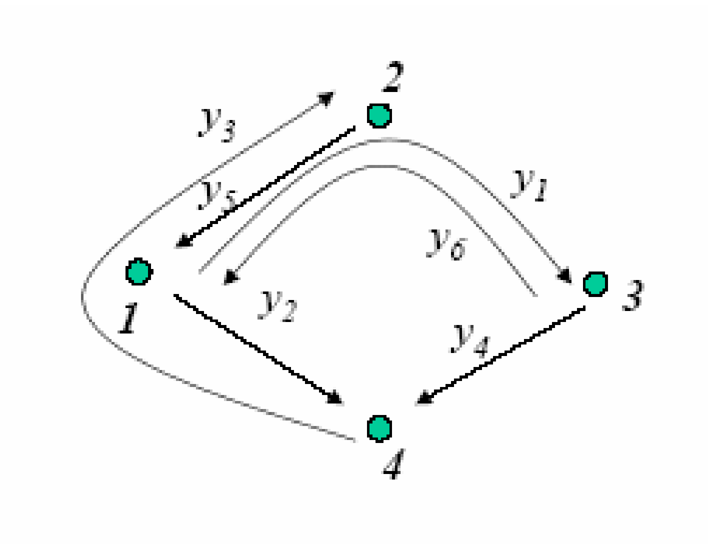
Extension

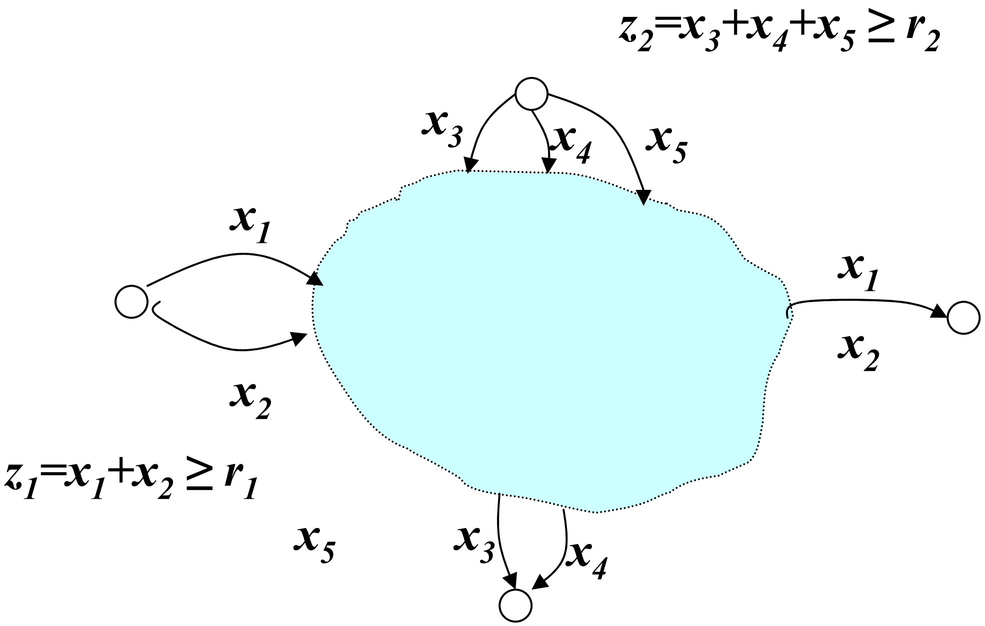
- Passive controller augmentation
- L_p stability in the presence of L_p disturbances
- Delay robustness (λ may be scaled by $1/\tau$ for delay-invariant stability)
- Robustness w.r.t. fading channel gain

ACC 04

Further Extensions in Networking

- Routing as multi-path flow regulation
- Mobile ad-hoc network (combined routing, power, flow, and position control)



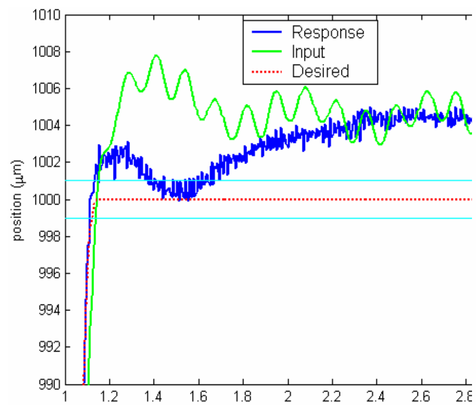


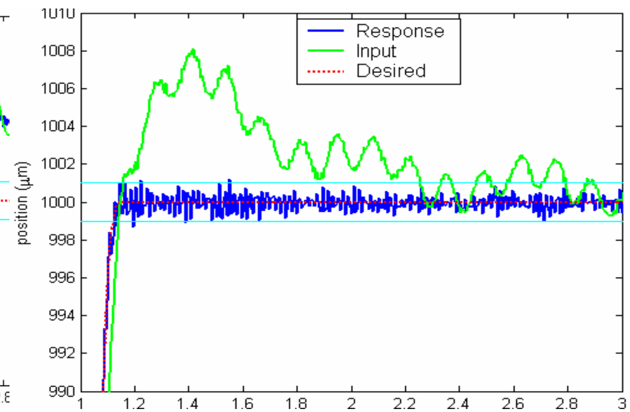
Summary

- Passivity is a good starting point for analysis and design of distributed control systems. Examples: distributed load transport, distributed resource allocation, distributed Nash game (CDMA power control)
- Performance and robustness requires further modification and optimization

Extensions

- Iterative learning: passivity in iteration (cca04)





- Receding horizon control: combining iteration and time evolution --- passivity in 2D (acc04, ASME JDSMC 04)