Reinforcement Learning and Plan Recognition

Manuela Veloso

Carnegie Mellon University
Computer Science Department

Planning - Fall 2001

Chapter 13 - Machine Learning, Tom Mitchell

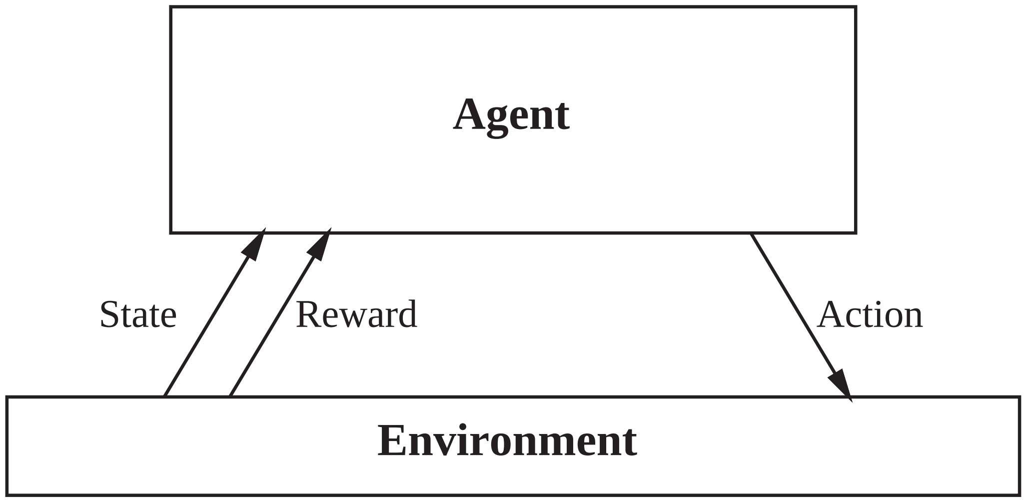
Han & Veloso - Behavior HMMs

Reinforcement Learning

- Assume the world is a Markov Decision Process - transition and rewards unknown; states and actions known.

- Two objectives:
  - learning the *model*
  - converging to the *optimal* plan.

Reinforcement Learning Problem



$$s_0 \xrightarrow[r_0]{a_0} s_1 \xrightarrow[r_1]{a_1} s_2 \xrightarrow[r_2]{a_2} \ldots$$

Goal: Learn to choose actions that maximize

$$r_0 + \gamma r_1 + \gamma^2 r_2 + \ldots \text{, where } 0 \leq \gamma < 1$$

Reinforcement Learning

- A variety of successful algorithms
  - Mitchell's book "Machine Learning" (chapter 13)
  - Sutton and Barto's book "Reinforcement Learning"
  - Kaebling, Moore, Littman: JAIR survey
- If we can do reinforcement learning, then:
  - outcome: for *every* state, *optimal* action is known
  - **Universal plan!**

Learning Conditions

- Assume world can be modeled as a Markov Decision Process, with rewards as a function of state and action.

- *Markov assumption:*
  New states and rewards are a function only of the current state and action, i.e.,
  - $s_{t+1} = \delta(s_t, a_t)$
  - $r_t = r(s_t, a_t)$

- *Unknown and uncertain environment:*
  Functions δ and r may be nondeterministic and are not necessarily known to learner.

Markov Decision Processes

- Finite set of states, S
- Finite set of actions, A
- Probabilistic state transitions, $\delta(s, a)$
- Reward for each state and action, $R(s, a)$

Model: states, actions, probabilistic transitions, rewards

Control Learning Task

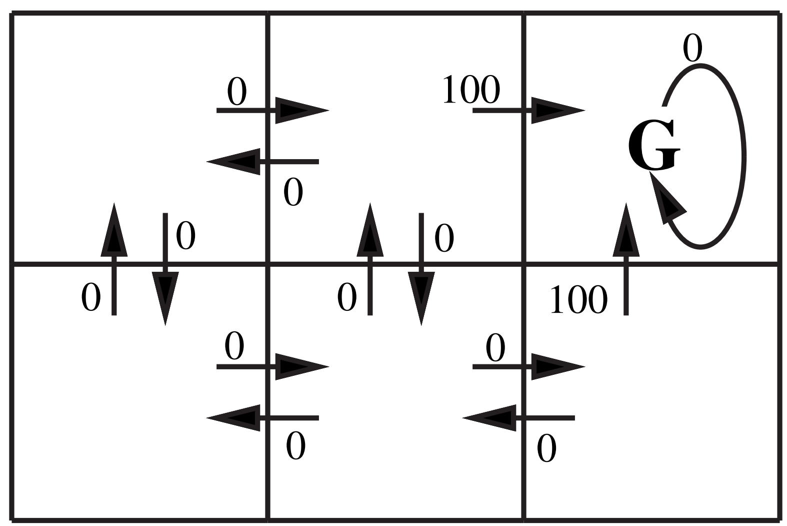
- Execute actions in world,
- Observe state of world,
- Learn action policy $\pi : S \rightarrow A$
  - Maximize expected reward

$$E[r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \ldots]$$

    from any starting state in S.
  - $0 \leq \gamma < 1$, discount factor for future rewards

Statement of Learning Problem

- We have a target function to learn $\pi : S \rightarrow A$
- We have **no** training examples of the form $\langle s, a \rangle$
- We have training examples of the form $\langle \langle s, a \rangle, r \rangle$ (rewards can be *any* real number)



immediate reward values $r(s, a)$

Policies

Assume deterministic world

- There are *many possible* policies, of course not necessarily *optimal*, i.e., with maximum expected reward

- There can be also *several OPTIMAL* policies.

Value Function

- For each possible policy π, define an *evaluation function over states*

$$V^\pi(s) \equiv r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \ldots$$
$$\equiv \sum_{i=0}^{\infty} \gamma^i r_{t+i}$$

  where $r_t, r_{t+1}, \ldots$ are generated by following policy π starting at state s

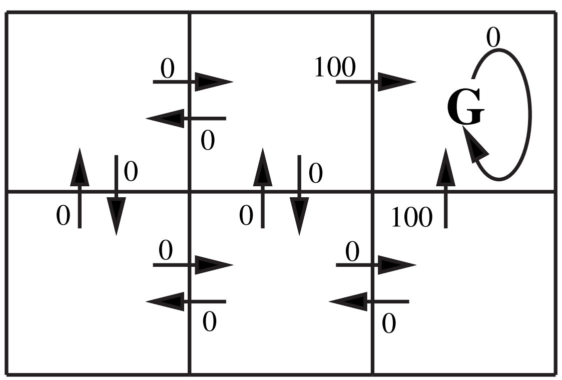
- Learning task: Learn OPTIMAL policy

$$\boxed{\pi^* \equiv \mathrm{argmax}_\pi V^\pi(s), (\forall s)}$$

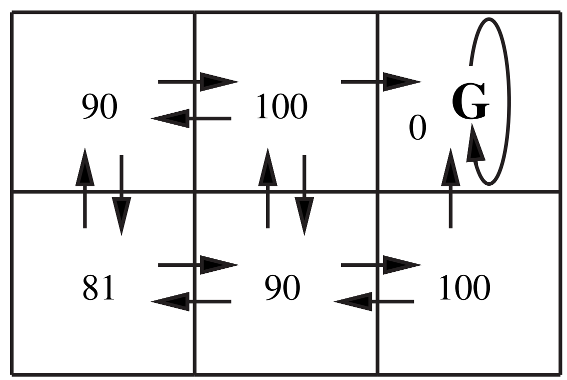
Learn Value Function

- Learn the evaluation function V^{π^*} – V^*.
- Select the optimal action from any state s, i.e., have an optimal policy, by using V^* with one step lookahead:

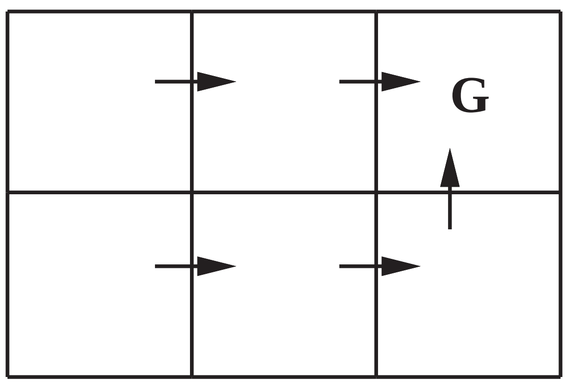
$$\pi^*(s) = \underset{a}{\operatorname{argmax}}[r(s,a) + \gamma V^*(\delta(s,a))]$$



rewards



$V^*(s)$ values



ONE optimal policy

Optimal Value to Optimal Policy

$$\boxed{\pi^*(s) = \text{argmax}_a [r(s,a) + \gamma V^*(\delta(s,a))]}$$

A problem:

- This works well if agent knows $\delta : S \times A \rightarrow S$, and $r : S \times A \rightarrow \Re$
- When it doesn't, it can't choose actions this way

Q Function

- Define new function very similar to V^*

$$Q(s,a) \equiv r(s,a) + \gamma V^*(\delta(s,a))$$

Learn Q function - Q-learning

- If agent learns Q, it can choose optimal action even without knowing δ or r.

$$\pi^*(s) = \underset{a}{\operatorname{argmax}}[r(s,a) + \gamma V^*(\delta(s,a))]$$

$$\pi^*(s) = \underset{a}{\operatorname{argmax}}\, Q(s,a)$$

Q-Learning

Note that Q and V^* are closely related:

$$V^*(s) = \max_{a'} Q(s, a')$$

Which allows us to write Q recursively as

$$\begin{aligned} Q(s_t, a_t) &= r(s_t, a_t) + \gamma V^*(\delta(s_t, a_t))) \\ &= r(s_t, a_t) + \gamma \max_{a'} Q(s_{t+1}, a') \end{aligned}$$

Q-learning actively generates examples. It "processes" examples by updating its Q values. *While* learning, Q values are approximations.

Training Rule to Learn Q

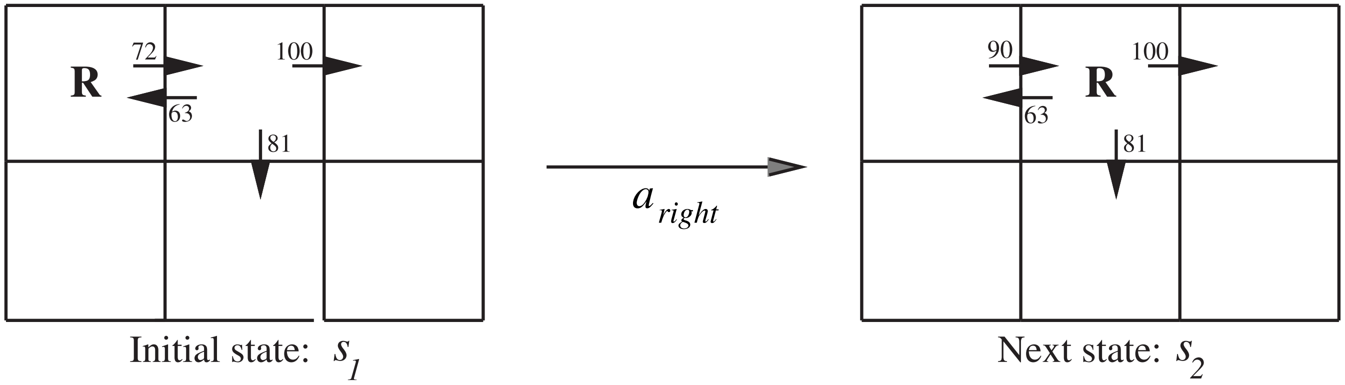
Let $\hat{Q}$ denote current approximation to Q.

Then Q-learning uses the following **training rule**:

$$\boxed{\hat{Q}(s,a) \leftarrow r + \gamma \max_{a'} \hat{Q}(s',a')}$$

where s' is the state resulting from applying action a in state s,
and r is the reward that is returned.

Example - Updating $\hat{Q}$



$$
\begin{aligned}
\hat{Q}(s_1, a_{right}) &\leftarrow r + \gamma \max_{a'} \hat{Q}(s_2, a') \\
&\leftarrow 0 + 0.9 \ \max\{63, 81, 100\} \\
&\leftarrow 90
\end{aligned}
$$

Q Learning for Deterministic Worlds

For each s, a initialize table entry $\hat{Q}(s,a) \leftarrow 0$

Observe current state s

Do forever:

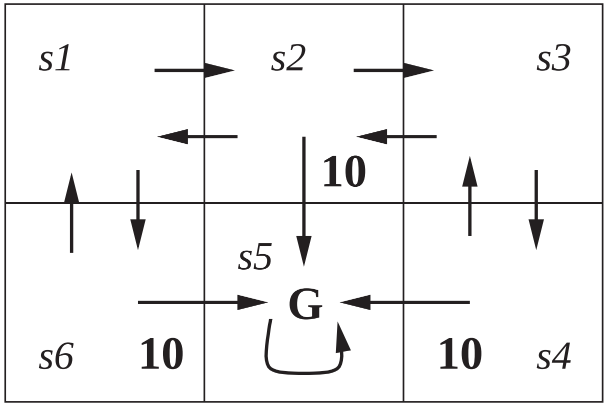
- Select an action a and execute it
- Receive immediate reward r
- Observe the new state s'
- Update the table entry for $\hat{Q}(s,a)$ as follows:

$$\hat{Q}(s,a) \leftarrow r + \gamma \max_{a'} \hat{Q}(s',a')$$

- $s \leftarrow s'$

Q Learning Iterations

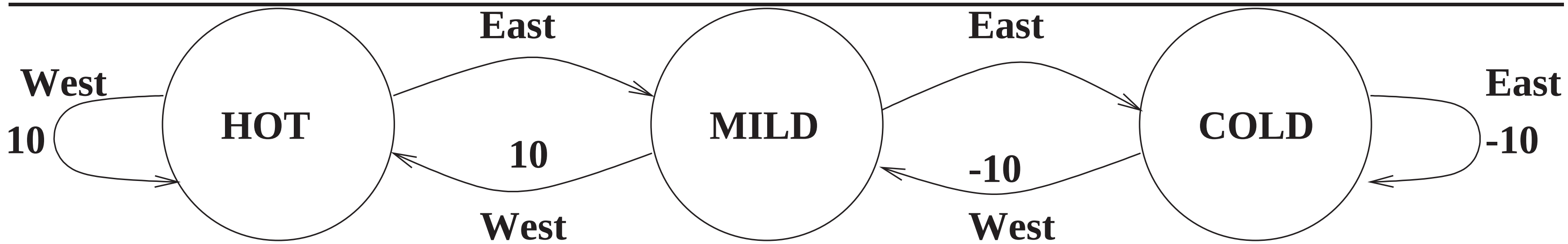
Starts at bottom left corner - moves clockwise around perimeter;
Initially $Q(s,a) = 0$; $\gamma = 0.8$



$$\hat{Q}(s,a) \leftarrow r + \gamma \max_{a'} \hat{Q}(s',a')$$

Q(s1,E)	Q(s2,E)	Q(s3,S)	Q(s4,W)
0	0	0	r + γ max{Q(s5,loop)}= 10 + 0.8 . 0 = 10
0	0	r + γ max{Q(s4,W),Q(s4,N)}= 0 + 0.8 max{10,0}= 8	10
0	r + γ max{Q(s3,W),Q(s3,S)}= 0 + 0.8 max{0,8}= 6.4	8	10

Problem - Deterministic

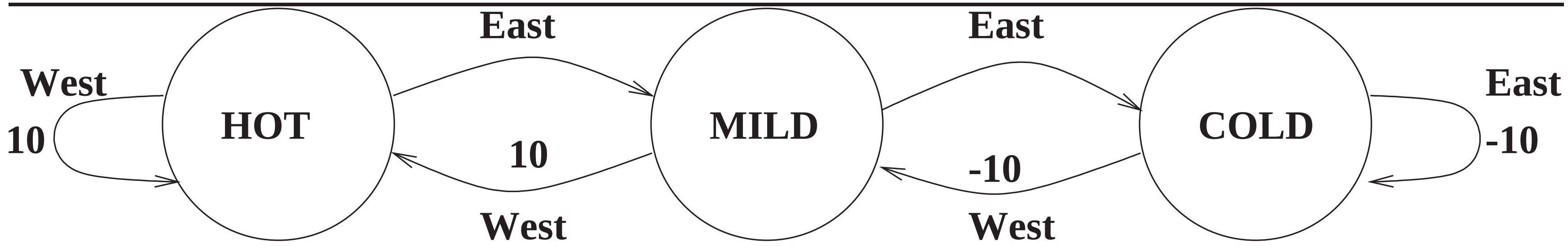


How many possible **policies** are there in this 3-state, 2-action deterministic world?

A robot starts in the state Mild. It moves for 4 steps choosing actions **West, East, East, West**. The initial values of its Q-table are 0 and the discount factor is $\gamma = 0.5$.

	Initial State: MILD		Action: West New State: HOT		Action: East New State: MILD		Action: East New State: COLD		Action: West New State: MILD	
	East	West	East	West	East	West	East	West	East	West
HOT	0	0	0	0	**5**	0	5	0	5	0
MILD	0	0	0	**10**	0	10	**0**	10	0	10
COLD	0	0	0	0	0	0	0	0	0	**-5**

Problem - Deterministic



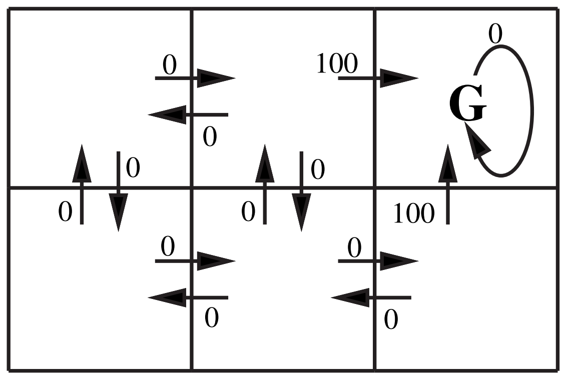
Why is the policy $\pi(s) =$ West, for all states, *better than* the policy $\pi(s) =$ East, for all states?

- $\pi_1(s) =$ West, for all states, $\gamma = 0.5$

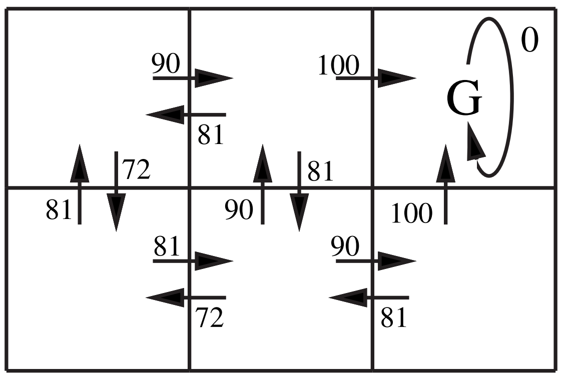
$$V^{\pi_1}(\text{HOT}) = 10 + \gamma V^{\pi_1}(\text{HOT}) = 20.$$

- $\pi_2(s) =$ East, for all states, $\gamma = 0.5$
  - $V^{\pi_2}(\text{COLD}) = -10 + \gamma V^{\pi_2}(\text{COLD}) = -20,$
  - $V^{\pi_2}(\text{MILD}) = 0 + \gamma V^{\pi_2}(\text{COLD}) = -10,$
  - $V^{\pi_2}(\text{HOT}) = 0 + \gamma V^{\pi_2}(\text{MILD}) = -5.$

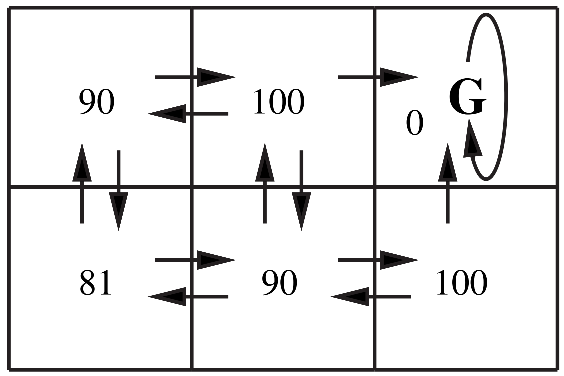
Another Deterministic Example



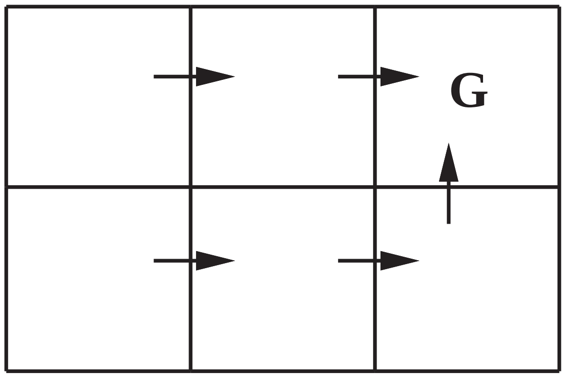
$r(s, a)$ values



$Q(s, a)$ values



$V^*(s)$ values



One optimal policy

Nondeterministic Case

What if reward and next state are non-deterministic?

We redefine V, Q by taking expected values

$$
\begin{aligned}
V^{\pi}(s) &\equiv E[r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \ldots] \\
&\equiv E[\sum_{i=0}^{\infty} \gamma^i r_{t+i}]
\end{aligned}
$$

$$Q(s,a) \equiv E[r(s,a) + \gamma V^*(\delta(s,a))]$$

Nondeterministic Case

Q learning generalizes to nondeterministic worlds

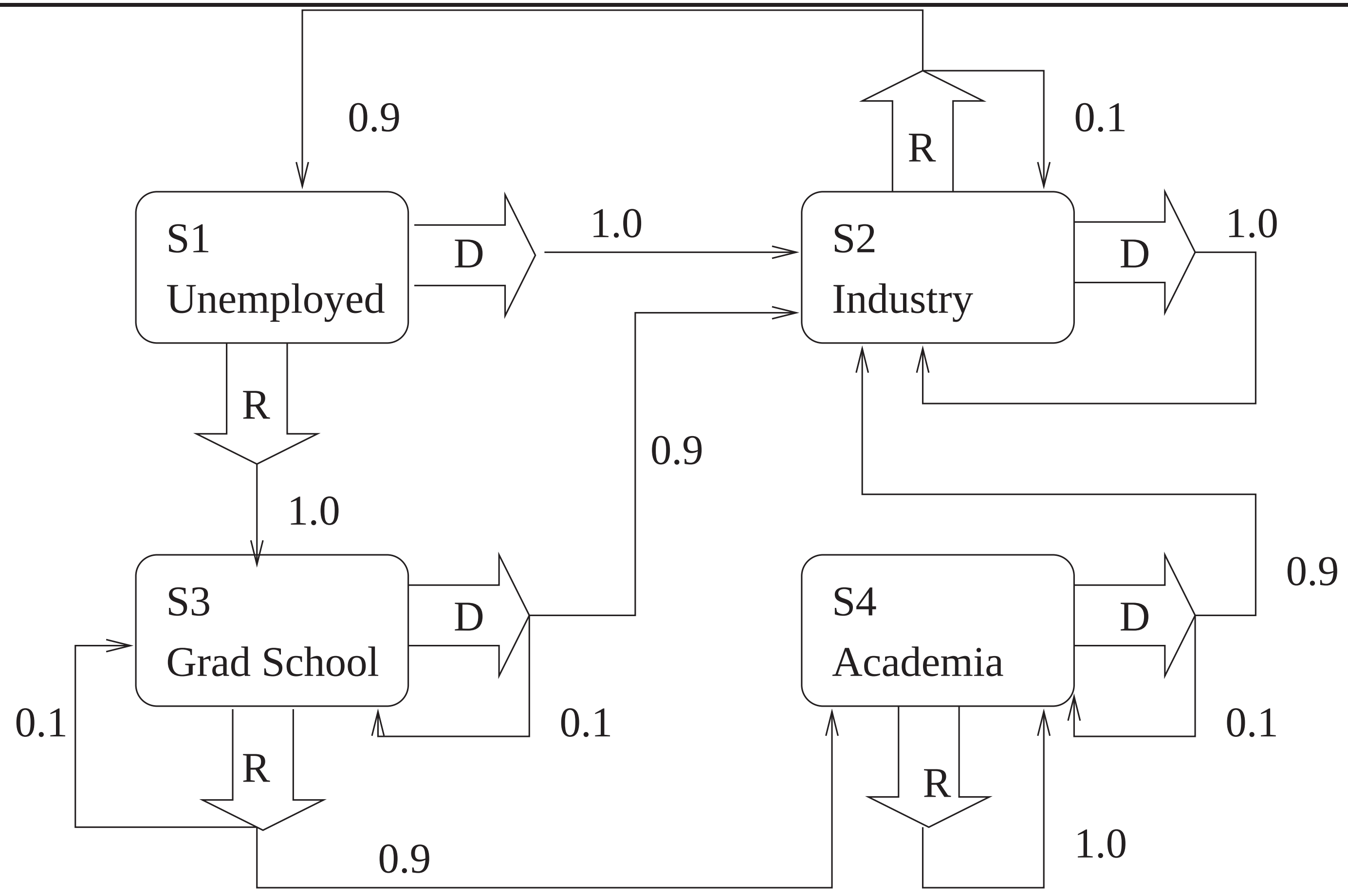
Alter training rule to

$$\begin{aligned}\hat{Q}_n(s,a) \leftarrow \quad & (1-\alpha_n)\hat{Q}_{n-1}(s,a) + \\ & \alpha_n[r + \gamma \max_{a'} \hat{Q}_{n-1}(s',a')],\end{aligned}$$

where $\alpha_n = \frac{1}{1+visits_n(s,a)}$, and $s' = \delta(s,a)$.

$\hat{Q}$ still converges to Q^* (Watkins and Dayan, 1992)

Nondeterministic Example



REWARD

S1	S2	S3	S4
0	100	0	10

Nondeterministic Example

π^*(s) = D, for any s= S1, S2, S3, and S4, $\gamma = 0.9$.

```
------------------------------------------------------------
V*(S2) = r(S2,D) + 0.9 (1.0 V*(S2))
V*(S2) = 100 + 0.9 V*(S2)
V*(S2) = 1000.

V*(S1) = r(S1,D) + 0.9 (1.0 V*(S2))
V*(S1) = 0 + 0.9 x 1000
V*(S1) = 900.

V*(S3) = r(S3,D) + 0.9 (0.9 V*(S2) + 0.1 V*(S3))
V*(S3) = 0 + 0.9 (0.9 x 1000 + 0.1 V*(S3))
V*(S3) = 81000/91.

V*(S4) = r(S4,D) + 0.9 (0.9 V*(S2) + 0.1 V*(S4))
V*(S4) = 40 + 0.9 (0.9 x 1000 + 0.1 V*(S4))
V*(S4) = 85000/91.
```

Nondeterministic Example

What is the Q-value, Q(S2,R)?

```
Q(S2,R) = r(S2,R) + 0.9 (0.9 V*(S1) + 0.1 V*(S2))
Q(S2,R) = 100 + 0.9 (0.9 x 900 + 0.1 x 1000)
Q(S2,R) = 100 + 0.9 (810 + 100)
Q(S2,R) = 100 + 0.9 x 910
Q(S2,R) = 919.
```

Discussion

- How should the learning agent use the *intermediate* Q values?
  - Exploration
  - Exploitation

- Scaling up in the size of the state space
  - Function approximator (neural net instead of table)
  - Generalization
  - Reuse, use of macros
  - Abstraction, learning substructure

Ongoing Research

- Partially observable state
- Continuous action, state spaces
- Learn state abstrations
- Optimal exploration strategies
- Learn and use $\hat{\delta} : S \times A \rightarrow S$
- Multiple learners - Multi-agent reinforcement learning

Behavior Recognition

If someone is acting according to a Markov Model, then can we *follow* its "behavior" - state / actions transitions??

- What is known?
- What is observable?
- Real data and abstracted model
- Speech recognition

$\boxed{\text{Hidden Markov Models}}$

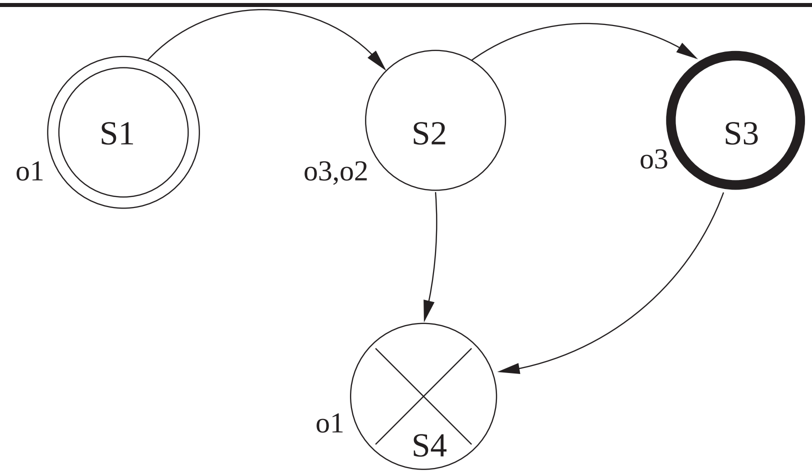
Behavior Recognition

State the problem as a *behavior membership decision.*

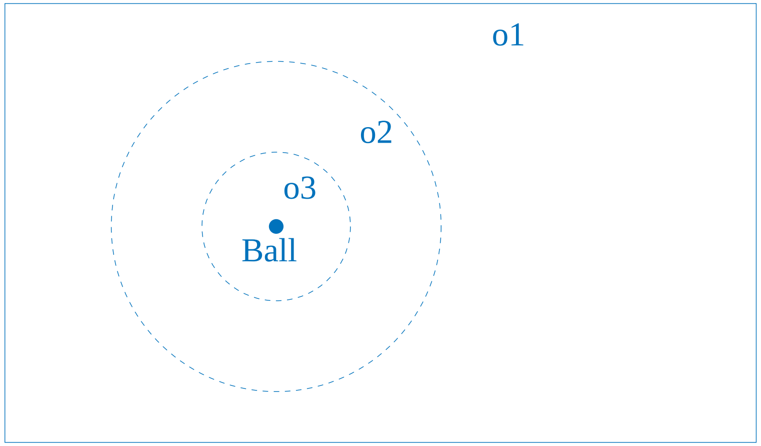
- R acts according to a set of behaviors $B(i)$,
- O has a model of the set of possible behaviors,
- O *recognizes,* which $B(i)$ R is performing.

We can extend HMMs to behavior recognition.

HMM Behavior Representation



- Observations are defined in terms of state features.



A Behavior HMM

- $N = \{s_{initial}\} \cup \{s_{intermediate}\} \cup \{s_{accept}\} \cup \{s_{reject}\}$
- $M = \{o_i\}$ – the observation space
- $A = \{a_{ij}\}$ – The state transition matrix, where:

$$a_{ij} = Pr(S_{t+1} = s_j | S_t = s_i), 1 \leq i, j \leq N$$

- $B = \{b_i(o_k)\}$ – The observation probabilities

$$b_i(o) = Pr(o | S_t = s_i), 1 \leq i \leq N$$

- $\pi = \{\pi_i\}$ – The initial state distribution

$$\pi_i = Pr(S_1 = s_i), 1 \leq i \leq N$$

Behavior Recognition

How to perform behavior recognition?

- The state is not directly observable.
- We infer the probability of being at state s_i,

$$\sum_i Pr(S_t = s_i) = 1, \forall i$$

- This probability gives the likelihood of s_i being the actual *behavioral* state of the robot.

Multiple Behaviors, Multiple BHMMs

- A different BHMM is used for each type of behavior.
- Several BHMMs may show high accepting probabilities at the same time, as behaviors are not necessarily mutually exclusive.
- Given an observation sequence $O = o_1, o_2, \ldots, o_t$, what is the probability that we are in s_i?

$$Pr(S_t = s_i | O = o_1, o_2, \ldots, o_t, \lambda),$$

where λ is the Hidden Markov Model parameters.

Orchestrating Multiple BHMMs

$$Pr(S_t = s_i | O = o_1, o_2, \ldots, o_t, \lambda) =$$

$$\frac{Pr(S_t = s_i \wedge O = o_1, o_2, \ldots, o_t, \lambda)}{Pr(O = o_1, o_2, \ldots, o_t, \lambda)}$$

Let $\alpha_i(t) = Pr(S_t = s_i \wedge O = o_1, o_2, \ldots, o_t, \lambda)$, then:

$$Pr(S_t = s_i | O = o_1, o_2, \ldots, o_t, \lambda) = \frac{\alpha_i(t)}{\sum_i \alpha_i(t)}$$

$\alpha_i(t)$ is computed recursively:

$$\alpha_i(t+1) = \sum_j \alpha_j(t) a_{ji} b_i(o_{t+1})$$

Behavior Recognition: Issues

- Our assumption requires each behavior to be a sequence of state traversals.

- We assume that each behavior starts from the initial state and completes at the accept state.

- Our Behavior Hidden Markov Model can recognize a single execution of a behavior, *provided it is instantiated at the time when the real behavior starts executing.*

- Otherwise, it will be "off phase" with the actually behavior and reliable recognition has low guarantees.

Behavior Segmentation and Restart

Instantiate a new recognizer at regular intervals.

- No effort searching for segmentation points between successive executions of two behaviors, and possible fail: we ignore such points.
- Probabilistic segmentation: One of the recognizers will have high probability of instantiating at a point in time close to the behavior start time.
- Granularity: important, too *sparse* results in high recognizer's probability of missing the beginning of the behavior; how *frequent*?

Granularity of Behavior Restart

Two separate schemes to determine the removal of a BHMM.

- A *timeout* is set for each type of behavior.
  - Assumption: each behavior takes some estimated amount of time to execute (e.g. Go-To-Ball 30s)
- A BHMM is removed when it reaches a high probability for a reject state.
  - The robot's behavior is not doing what the BHMM is trying to recognize.

BHMM Recognition Algorithm

- For all behavior recognizer type
  - Instantiate initial copy
  - Record start time and mark this instance as active
- Forever until done
  - Obtain object locations from vision system
  - For each active behavior recognizer
    - Compute current observation from vision data
    - Update current state probabilities using Most Like State update.
    - Find most likely state (= $\texttt{mls}$)
    - If ($\texttt{mls} \in \{$accept state$\}$)
      - signal
      - Continue to next recognizer instance
    - If ($\texttt{mls} \in \{$reject state$\}$ AND $Pr(\texttt{mls}) >$ reject threshold)
      - signal and mark instance as inactive
      - Continue to next recognizer instance
    - Compute elapse time for this instance. If larger than timeout threshold mark instance as inactive
  - if ((current-time - last-instantiate-time) $>$ instantiate threshold)
    - For all behavior recognizer type
      - Instantiate initial copy
      - Record start time and mark this instance as active
    - last-instantiate-time = current-time

Summary

- Markov model for state/action transitions.
- Markov systems with reward - goal achievement
- Markov decision processes - added actions
- Value, policy iteration
- Q-learning
- Hidden Markov models - observation and recognition.
- Later: POMDPs - Viterbi and Multi/Markov Viterbi