Planning, Execution & Learning: Planning with POMDPs (II)

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Approximating Value Function

- Use Function Approximator with "Better" Properties than Piece-Wise Linear
  - Continuous (differentiable), non-linear
  - Typically use on the order of one vector per action

- Comparisons
  + Generally much more efficient
  – May poorly represent optimal solution (however, better function approximation usually implies better results)

SPOVA Algorithm (Parr, 1995)

- Approach
  - Use a small set of vectors to represent the value function
  - Approximate the value function by a smooth (differentiable) function

$$V(b) = \max_{v \in \Psi} (v \bullet b)$$
$$\approx \{\textstyle\sum_{v \in \Psi'} v \bullet b)^k\}^{1/k}$$

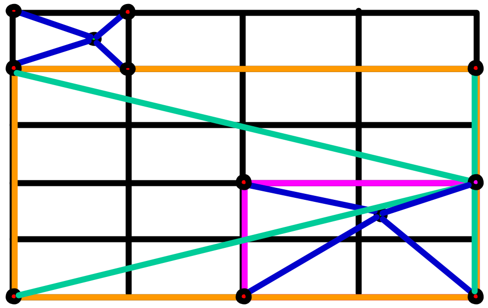
  - Use gradient descent to adjust components of the vectors

$$E(b) = V(b) - \beta\{\max_a\{R(a, b) + \gamma \textstyle\sum_{b'} p(b' \mid a, b)\, V(b')\}\}$$

$$v_{i,t+1}(s) = v_{i,t}(s) + \alpha E(b) b(s)(v_i \bullet b)^{k-1}/V(b)^{k-1}$$

Approximating Belief Space

- Use Grid-Based Approximation
  - Discretize belief space: Place finite grid over belief simplex
  - Evaluate value function at grid points
  - Interpolate

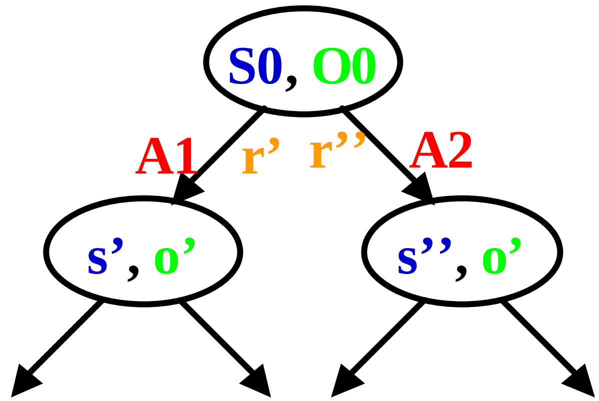


- Regular Grid (Lovejoy)
  + Simple method, easy interpolation
  – Exponential space needed
- Non-Regular Grid (Hauskrecht)
  + More accurate – tries to follow value contours
  – Interpolation is difficult
- Variable-Resolution Grid (Zhou & Hansen)
  + Fairly accurate – grid points added where distinctions are needed
  + Interpolation is fairly easy – add virtual grid points

Trajectory Trees (Kearns, et.al.)

- Choose Policy Based on Monte-Carlo Sampling
  - Restricted set of policies (Π)
  - Complexity depends on VC dimension of Π, rather than on state space
  - Assumes a ***generative model*** of POMDP
- Questions:
  - How many samples need generated to evaluate each policy?
  - How can you ***reuse*** samples from one policy to the next?
- Solution:
  - Generate trajectory ***tree***, rather than simple trajectory

Trajectory Trees



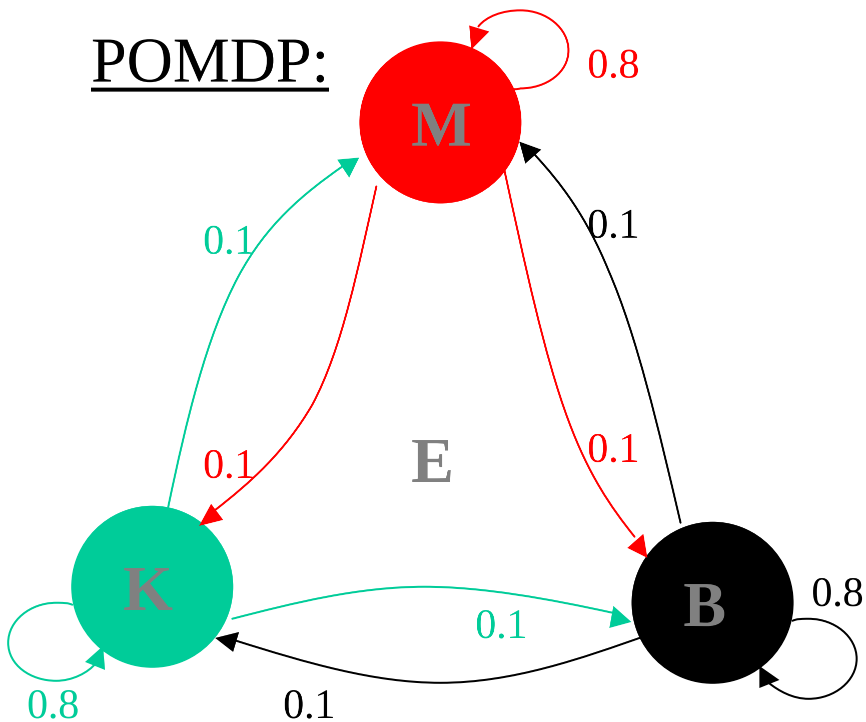
- Generate Tree Stochastically
  - Fixed Horizon H_ε: H_ε'th step can contribute at most $\varepsilon/2$ to total discounted return
  - Can be used to evaluate ***any*** policy
- Provable Bounds
  - $V^\pi(S0) = (\sum_{i=1,m} R(\pi, T_i))/m$
  - $m = O((V_{max}/\varepsilon)^2 \cdot H_\varepsilon \cdot VC(\Pi) + \log(1/\delta))$
    - With probability $(1-\delta)$, you are within ε of the true value of the policy

Hierarchical POMDPs (Pineau)

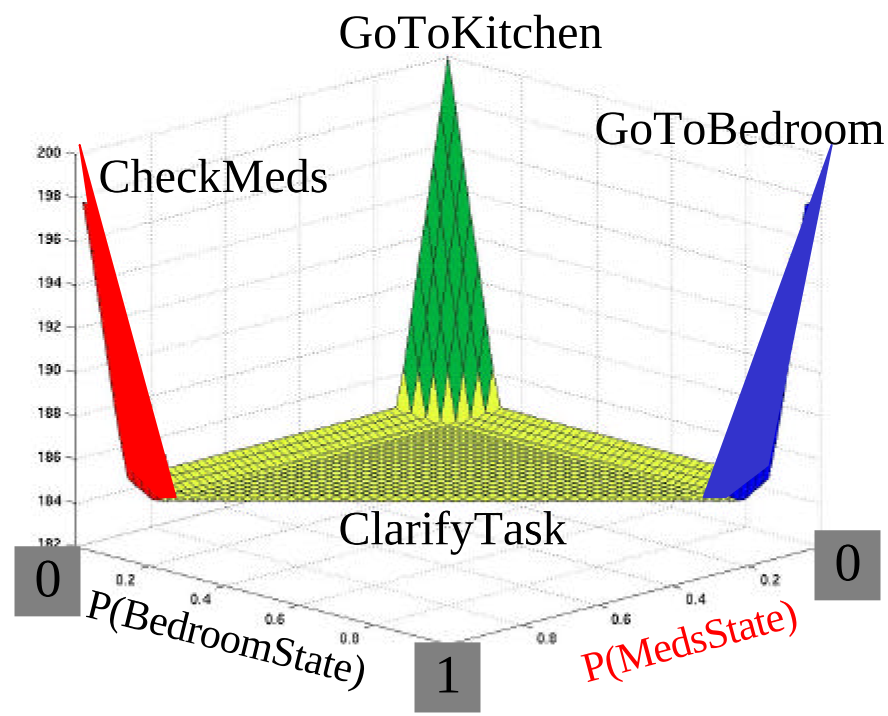
- ***Basic Idea:***
  - Break the problem into many “related” POMDPs
  - Each smaller POMDP has only a subset of ***actions*** (and, possibly, observations)
  - Value iteration has exponential run time: $O((|S|^2|A|\Gamma_{n-1}^{|O|})$

Example

POMDP:



Value Function:

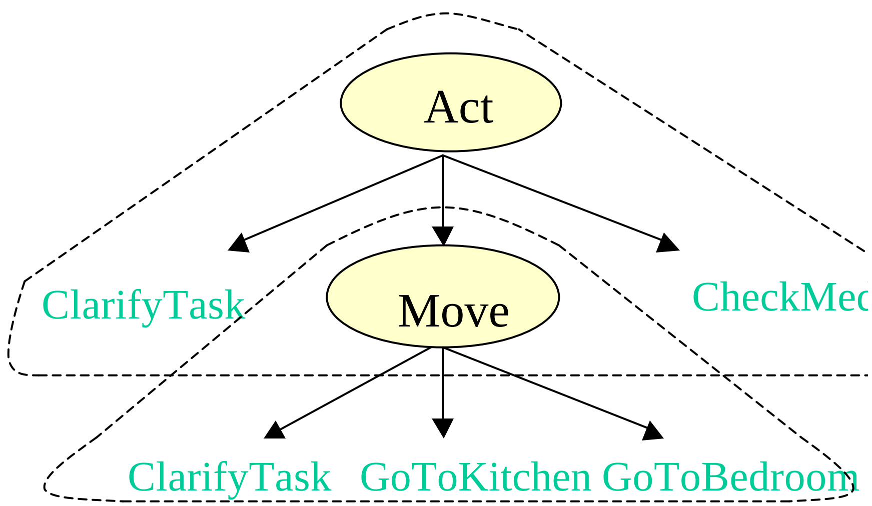


S_o= {Meds, Kitchen, Bedroom}

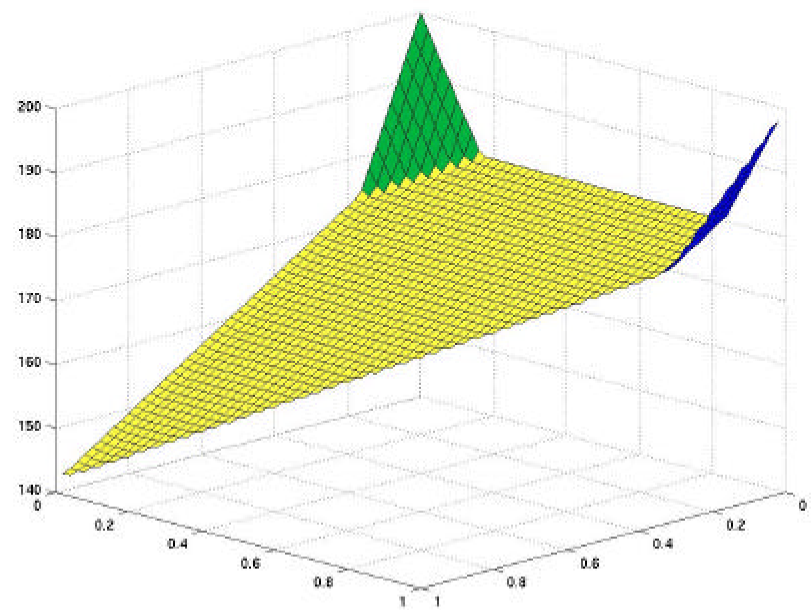
A_o = {ClarifyTask, CheckMeds, GoToKitchen, GoToBedroom}

O_o = {Noise, Meds, Kitchen, Bedroom}

Hierarchical Action Partitioning



Local Value Function and Policy
***Move* Controller**



Modeling Abstract Actions

Problem: *Need parameters for abstract action Move*

Solution: *Use the local policy of corresponding low-level controller*

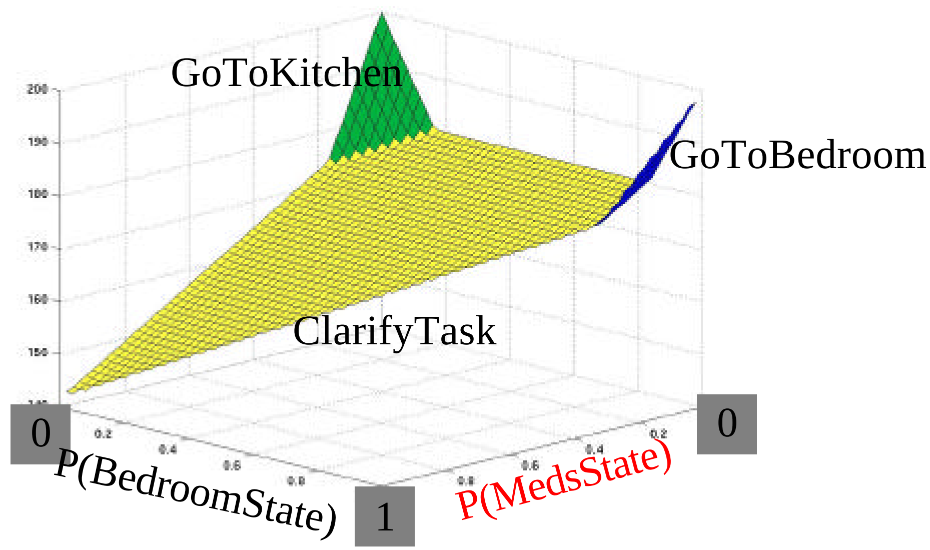
General form: $\Pr(s_j \mid s_i, a_k^{\text{abstract}}) = \Pr(s_j \mid s_i, \underbrace{\text{Policy}(a_k^{\text{abstract}}, s_i)})$

Example:

$\Pr(s_j \mid$ MedsState, Move $)$

$= \Pr(s_j \mid$ MedsState, ClarifyTask $)$

Policy (Move, s_i):



Greedy Approaches to POMDP Planning

- Solve POMDP as if it were an MDP
- Choose Action Based on Current Belief State
  - **“most likely”** – $\text{argmax}_a(Q(\text{argmax}_s(b(s)), a)$
  - **“voting”** – $\text{argmax}_a(\sum_{s \in S,\, a = \text{argmax}_{a'} Q(s, a')} b(s))$
  - **“Q-MDP”** – $\text{argmax}_a(\sum_{s \in S,} b(s)\, Q(s, a))$

- Essentially, try to act optimally as if the POMDP were to become observable after the next action
  - Cannot plan to do actions just to gain information

Greedy Approaches to POMDP Planning

- Extensions to Allow Information-Gathering Actions (Cassandra 1996)
  - Compute entropy ***H(b)*** of belief state
  - If entropy is below a threshold, use a greedy method ***Z(a, b)*** for choosing action
  - If entropy is above a threshold, choose the action that reduces expected entropy the most

$$EE(a, b) = \sum_{b'} \mathrm{p}(b' \mid a, b)\, H(b')$$

$$\pi(\mathrm{s}) = \begin{cases} \mathrm{argmax}_a\, Z(a, b) & \text{if } H(b) < t \\ \mathrm{argmin}_a\, EE(a, b) & \text{otherwise} \end{cases}$$