THE UNIVERSITY OF TEXAS AT AUSTIN
Department of Statistics and Data Sciences
College of Natural Sciences

SDS 321: Introduction to Probability and Statistics

Lecture 7: Counting II

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Announcements

- Homework is due today 4pm!
- HW3 is out already and HW1 solutions are available.

Combinations

So far we have been talking only about ordered samples. For many applications we only need an unordered sample. In particular we want to figure out how many ways one can form groups of size r from a set of size n.

- Q: How many groups of size r are there of a given population of size n?

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- 12,23,13. So $\binom{3}{2}=3$.

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- Earlier we learned about n permute r. This is how we choose r elements without replacement, but the **order matters**.
- Let consider all ordered samples of size 2 picked without replacement from 3 numbers.
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- Say we are picking 3 out of 4 numbers. Consider all ordered arrangements.
  $\underbrace{123, 132, 231, 213, 312, 321}_{\text{(1,2,3) appears 6 times}}, \underbrace{143, 134, 431, 413, 314, 341}_{\text{(1,3,4) appears 6 times}}, \ldots$

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- This also sometimes written as $C(n, r)$.
- These are also known as the binomial coefficients.

Binomial coefficients: fun stuff

- $\binom{n}{k} = \binom{n}{n-k}$
  - Number of ways to choose k out of n things is the same as choosing $(n-k)$ things out n and removing them.

- $\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}$

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    - Just choose $k+1$ elements from S! So in $\binom{n}{k+1}$ ways.
  - How many ways can you choose a $k+1$ elements so that x is included?
    - You can choose k elements from S elements in $\binom{n}{k}$ ways and add x to that pile.

Example I

- How many binary sequences of length n are there with k ones?

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- Same as choosing k out of n positions in the sequence. We put a one in these positions and zeros in the rest. So the answer is $\binom{n}{k}$.

Example II

- What is $\sum_{k=0}^{n} \binom{n}{k}$?

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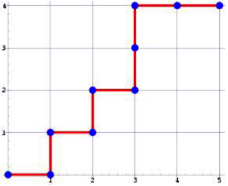
- What is $\sum_{k=0}^{n} \binom{n}{k}$?
- Well this is just the total number of possible binary strings, so 2^n.

Practice Problems

- 

  You are walking on a grid. You can either go right or up by one step. You start from (0,0). How many paths are there to $(5, 10)$?

- How many of the above paths go via $(4, 4)$?

Practice Problem: Path counting

- Think of each right as a 1 and each up as a 0. Now you have a bijection with every binary sequence of length 15 with 5 "1"s. A bijection is essentially saying that each path you construct can be written as a length 15 binary sequence with 5 "1"s and for any such binary sequence you have a path that takes you to $(5, 10)$.
- How many such sequences are there? $\binom{15}{5}$.
- Now you need to go via $(4, 4)$. So first count paths from $(0, 0)$ to $(4, 4)$. There are $\binom{8}{4}$ such paths. How many paths go from $(4, 4)$ to $(5, 10)$? Change you origin! This is the same as counting paths from $(0, 0)$ to $(1, 6)$. There are $\begin{pmatrix} 7 \\ 1 \end{pmatrix} = 7$ such paths. So the total number of paths is $\begin{pmatrix} 8 \\ 4 \end{pmatrix} \times 7$ using the multiplication rule.

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  - Think Binomial coefficient!
- How many configurations of length 10 strings are there with three 0's, four 1's and three 2's?

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  - First choose positions of the 0's in $\binom{10}{3}$ ways.

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  - Now choose positions of the 1's from the remaining 7 in $\binom{7}{4}$ ways.

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  - The remaining three positions are given to the 2's.

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  - Now choose positions of the 1's from the remaining 7 in $\binom{7}{4}$ ways.
  - The remaining three positions are given to the 2's.
  - So a total of $\frac{10!}{3!7!} \times \frac{7!}{4!3!} = \frac{10!}{3!4!3!}$ ways.

Multinomial coefficients

- $\binom{n}{k}$:= #ways to divide n elements into two disjoint groups, where the first group has size k and the second size $n-k$.
- $\binom{n}{n_1, n_2, n_3}$:= #ways to divide n elements into 3 disjoint groups of sizes n_1, n_2 and $n_3 = n - n_1 - n_2$ respectively.

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- $\binom{n}{n_1, n_2, n_3} = \binom{n}{n_1} \times \binom{n-n_1}{n_2} = \frac{n!}{n_1!n_2!n_3!}$
- Generalizing to r groups of sizes $n_1, \dots, n_r$ with $n_1 + \cdots + n_r = n$ we have:

$$\binom{n}{n_1, \dots n_r} := \frac{n!}{n_1! \dots n_r!}$$

Sum up

- Here are all the things we have learned so far.
- There are $n!$ ways to permute n distinguishable objects.
- $(n)_r$ is the # ways one can pick r **ordered** objects from n distinguishable objects. $(n)_r = n!/(n-r)!$
- $\binom{n}{r}$ is the # ways one can pick r **unordered** objects from n distinguishable objects.
- $\frac{n!}{n_1!n_2!n_3!}$ is the # ways one can label n distinguishable objects with n_1 labels of one type, n_2 labels of a second type, and $n_3 = n - n_1 - n_2$ elements of the third type.
- Today we will learn more about occupancy problems.

Occupancy numbers

- So far we have been talking about distinguishable objects. In all these problems, which object was assigned to which group matters.
- Sometimes however we are interested in counting frequencies.
- How many ways can you divide 10 fruits among 4 children so that everyone gets at least one?
- Here, we are not interested in which fruit went to which child. We only care about how many of the fruits went to a child.
- So we think of the fruits as indistinguishable objects and the children are distinguishable bins.

Occupancy numbers (stars and bars)

- Say x_i is the number of fruits going to child i. So we are looking for ways to write $x_1 + x_2 + x_3 + x_4 = 10$, where $x_i > 0$.
- $1 + 1 + 4 + 4$ is not the same as $1 + 4 + 1 + 4$.
- How will you represent $x_1 = 1, x_2 = 3, x_3 = 2$, and $x_4 = 4$. Write 10 stars for 10 fruits. Put bars to represent bins/children.
- In order to divide 10 stars into 4 parts you will need 3 bars.
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- In order to divide 10 stars into 4 parts you will need 3 bars.
- $$\underbrace{*}_{1}|\underbrace{***}_{3}|\underbrace{**}_{2}|\underbrace{****}_{4}$$
- There are 9 spaces between 10 stars. We want to put 3 bars in these places. There are total $\binom{9}{3}$ ways of doing this.

Occupancy numbers (stars and bars)

- Say you want to distribute n fruits among k children so that everyone gets at least 1.
- Say x_i is the number of fruits going to child i. So we are looking for ways to write $x_1 + x_2 + \cdots + x_k = n$, where $x_i > 0$.
- Writing in stars and bars, you want to place $(k-1)$ bars in $(n-1)$ spaces between the stars.
- So the answer is $\binom{n-1}{k-1}$.

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- So the answer is $\binom{13}{3}$.

Occupancy numbers (stars and bars)

- How many ways can I write n as an sum of k *non-negative* integers? $1 + 2 + 4$ is different from $1 + 4 + 2$.
- We want to enumerate the ordered set of k-tuplets $\{x_1, \ldots, x_k\}$, such that $x_1 + \ldots x_k = n$ and $x_i \geq 0$, for $i = 1, \ldots, k$.

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- Define $y_i = x_i + 1$. Now for each r-tuplet of x_i's we have an r-tuplet of y_i's such that $y_1 + \cdots + y_k = n + k$, and $y_i > 0$ for $i = 1, \ldots, k$.

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- But this is the same as the former problem, with $n+k$ stars and k bars! So the answer is $\binom{n+k-1}{k-1}$.

Probability and counting: example 1a

- Our population consists of ten digits $\{0, 1, \ldots, 9\}$.
- I pick a 5 digit number uniformly at random. Such a number can start with zero, and may have repetitions.
- What is the probability p that the five digits are all different?

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- $\#$ all 5 digit numbers is 10^5.
- $\#$ 5 digit numbers without repetition is $(10)_5$.
- $p = (10)_5/10^5$.

Probability and counting: example 1b

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- What is the probability p that no two passengers get down at the same stop?

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- $p = (10)_5/10^5$.

Probability and counting: example 1c

- The birthdays of $r \leq 365$ people form a sample of size r from the population of all birthdays (365 days in the year). We assume that a person is equally likely to be born on any of the 365 days and no one was born on Feb 29^{th}.
- What is the probability that no two people will have the same birthday?

$$p = \frac{(365)_r}{365^r} = \frac{365!}{(365-r)!365^r} \tag{1}$$

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- What is the probability if $r = 366$?
- Then at least two people must have the same birthday. This is also called the Pigeonhole Principle. So $p = 0$.

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- What is the probability if $r = 366$?
- Then at least two people must have the same birthday. This is also called the Pigeonhole Principle. So $p = 0$.
- How will you calculate this for large r, say $r = 30$?

Birthdays

- Problem... my calculator can't handle 365! or 365^{30}.
- Take logarithms! $365^{30} = 10^{76.8688} = 7.392 \times 10^{76}$.
- We can *approximate* factorials using Stirling's approximation:

$$n! \sim \sqrt{2\pi} n^{n+1/2} e^{-n}$$

- The $\sim$ symbol means the ratio of the two sides tend to 1 as $n \to \infty$.

$$\begin{aligned}
\ln(365!) &\approx \frac{1}{2}\ln(2\pi) + \left(365 + \frac{1}{2}\right)\ln(365) - 365 = 1792.3 \\
\ln(335!) &\approx \frac{1}{2}\ln(2\pi) + \left(335 + \frac{1}{2}\right)\ln(335) - 335 = 1616.6 \\
\ln\left(\frac{365!}{335!}\right) &= \ln(365!) - \ln(335!) \approx 1792.3 - 1616.6 = 175.55 \\
\frac{365!}{335!} &\approx e^{175.55} = 2.1711 \times 10^{76}
\end{aligned}$$

- The actual value is 2.1710×10^{76} – not bad!

Birthdays

- So, we with 30 people, we have 7.392×10^{76} possible combinations of birthdays.
- 2.171×10^{76} of these possible combinations of birthdays have no repeats.
- So, the probability of no one having the same birthday is:

$$\frac{2.171 \times 10^{76}}{7.392 \times 10^{76}} \approx 0.296$$

- Odds are, there's a shared birthday!