

Continuous r.v practice problems

SDS 321 Intro to Probability and Statistics

- (2+2+1+1 = 6 pts) The annual rainfall (in inches) in a certain region is normally distributed with mean 40 and standard deviation 4.
 - What is the probability that, starting with this year, it will take over 10 years before a year occurs having a rainfall of over 50 inches?
 - What is the probability that at least 4 out of the next 50 years will have a rainfall of over 50 inches?
 - What is the expected number of years with over 50 inches of rainfall in the next 50 years?
 - What assumptions are you making?

- (2+1+3+1+3 = 10pts) The joint pdf of two random variables X and Y are given by:

$$f_{X,Y}(x,y) = \begin{cases} 24xy & x,y \in [0,1], 0 \leq x+y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- Show that $f_{X,Y}(x,y)$ is a valid joint probability density function.
 - Find $f_X(x)$.
 - Find $E[X]$ and $\text{var}(X)$.
 - Find $f_Y(y)$.
 - Find $E(Y)$ and $\text{var}(Y)$.
- (2+1+1+2+2=8pts) The random variables X and Y have joint density function

$$f_{X,Y}(x,y) = \begin{cases} cxy(1-x) & 0 < x < 1, 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

- Find c .
 - Find $E[X]$.
 - Find $E[Y]$.
 - Find $\text{Var}(X)$.
 - Find $\text{Var}(Y)$.
- The random variables X and Y have a joint density function given by:

$$f(x,y) = \begin{cases} 2e^{-2x}/x & 0 \leq x < \infty, 0 \leq y \leq x \\ 0 & \text{otherwise} \end{cases}$$

- What are $f_X(x)$ and $f_Y(y)$?
- What are $E[X]$ and $E[Y]$?

5.

$$f_X(x) = \begin{cases} C(2x - x^3) & 0 < x < 5/2 \\ 0 & x \leq 0 \end{cases}$$

Could f be a probability density function? If so find C .

6. Consider the density function

$$f_X(x) = \begin{cases} C(2 - x) & 0 < x < 2 \\ 0 & x \leq 0 \end{cases}$$

Could f be a probability density function? If so find C .

- (a) We know that $\int_0^2 C(2 - x)dx = 1$ and so $C(4 - (2)^2/2) = 2C = 1$ and so $C = 1/2$.
- (b) Also $(2 - x)/2 \geq 0$ for $0 < x < 2$.
- (c) So this is a valid pdf.

7. Let U be an uniform $[0, 1]$ r.v and let $a < b$ be constants. Show that:

- (a) If $b > 0$ then $bU \sim Uniform([0, b])$.
- (b) $a + U \sim Uniform([a, a + 1])$
- (c) What function of U is distributed as $Uniform([a, b])$
- (d) Show that $\min(U, 1 - U) \sim Uniform(0, 1/2)$

8. The joint density of X and Y are given by:

$$f_{X,Y}(x, y) = \begin{cases} xe^{-(x+y)} & x > 0, y > 0 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Are X and Y independent?
- (b) What are $f_X(x)$ and $f_Y(y)$?
- (c) What is $P(X + Y \leq 2)$?

9. The joint density function of X and Y is:

$$f_{X,Y}(x, y) = \begin{cases} x + y & 0 < x < 1, 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Are X and Y independent?
- (b) Find $f_X(x)$
- (c) Find $P(X + Y < 1)$

10. The running time in seconds of an algorithm on a medium sized data set is approximately normally distributed with mean 30 and variance 25.

- (a) What is the probability that the running time of a run selected at random will exceed 25 seconds?
- (b) What is the probability that the running time of at least one of four randomly selected runs will exceed 25 seconds?

- (c) What is the probability that the running time of all runs will exceed 25 seconds given at least one of four randomly selected runs will exceed 25 seconds?

11. (2+2+1+1 + 1+1= 8pts) The joint density of X and Y is given by

$$f_{X,Y}(x,y) = \begin{cases} c & 0 < x < y, 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

- (a) What is c ?
- (b) What is $f_Y(y)$?
- (c) What is $E[Y]$?
- (d) What is the conditional pdf $f_{X|Y}(x|y)$?
- (e) Are X and Y independent?

	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990