Midterm

SDS321

Spring 2017

You may use a two (2 sided) pages of notes, and you may use a calculator.

This exam consists of five questions, containing multiple sub-questions. The assigned points are noted next to each question; the total number of points is 25. You have 75 minutes to answer the questions.

Please answer all problems in the space provided on the exam. Use extra pages if needed. Of course, please put your name on extra pages.

Read each question carefully, show your work and clearly present your answers. Note, the exam is printed two-sided - please don't forget the problems on the even pages!

Good Luck!

Name: ____

UTeid: ____

1 Short questions (10 points)

1. (3 pts) I am waiting for bus number 1. The probability that the next bus is bus number 1 is 0.2. If the first three buses are not the number 1 bus, what is the probability that the number 1 bus will come **after** the first 5 buses?

   Solution: *This is the same as asking* $P(X > 5|X > 3)$ *where* $X \sim$ *Geometric*$(.2)$. *We know that this number is* $(1-.2)^5/(1-.2)^3 = (1-.2)^2$

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   Grading: 1/2 point for understanding its geometric. One point for setting up conditional probability. 1 point for evaluating $P(X > t)$***. 1/2 point for correct answer. If anyone writes*** $P(X > t) = (1-p)^{t-1}$***, take half a point off. All algebraic mistakes, take half point off.***

2. (3 pts) The PMF of a random variable X, which takes values in $\{0, 1, 2\}$ is given as $P(X = 0) = 1/2$, $P(X = 1) = b$, $P(X = 2) = b$.

   (a) (2 pts) $b =?$

   Solution: $P(X = 0) + P(X = 1) + P(X = 2) = 1$. *So* $1/2 + 2b = 1$ *and* $b = 1/4$

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   Grading: 1 point for setting problem up. 1 point for correct answer. If they did everything right, but made silly algebraic mistake, then take 1/2 pt off.

(b) (1 pt) What is $E[X]$? *This should not be in terms of b, it should be a real number.*

Solution: $E[X] = 1 \times b + 2 \times b = 3/4$

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Grading: 1/2 point for correct formula. 1/2 point for correct answer.

3. (4 pts) I pick one letter at random from the word BUBBLE and independently one letter at random from the word BURST.

   (a) (2 pts) What is the probability that both letters are 'B'?

   Solution: *W_1 is the letter picked from word 1 and W_2 is letter picked from word 2.* $P(W_1 =' B', W_2 =' B') = P(W_1 =' B')P(W_2 =' B') = 3/6 \times 1/5 = 1/10.$

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   Grading: 1/2 pt calculating $P(W_1 =' B')$***, 1/2 pt for*** $P(W_2 =' B')$***, 1/2 pt for independence, 1/2 pt for correct answer.***

   (b) (2 pts) What is the probability that the two letters are the same?

   Solution: $P(W_1 = W_2) = P(W_1 =' B', W_2 =' B') + P(W_1 =' U', W_2 =' U') = 1/10 + 1/6 \times 1/5 = 1/10 + 1/30 = 4/30.$

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   Grading: 1 pt for setting the problem up. One point for calculating $P(W_1 = W_2 =' U')$***.***

2 Long questions (15 points)

1. (5 pts) 60% of the students at a certain school take at least one of the two available Math classes (Math I and Math II). 20% percent takes Math I but do not take Math II.

   (a) (2 pts) If one of the students is chosen randomly, what is the probability that this student takes Math II?

   Solution: $P(M_1 \cup M_2) = .6$ *and* $P(M_1 \cap M_2^c) = .2$. $P(M_2) = .6 - .2 = .4$.

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   Grading: 1/2 pt for set-up. 1 pt for understanding how to calculate M_2***. 1/2 pt for correct answer.***

   (b) (2 pts) If a randomly picked student does not take Math II, what is the probability that he/she does not take Math I either?

   Solution: $P(M_1^c|M_2^c) = P(M_1^c \cap M_2^c)/P(M_2^c) = (1 - P(M_1 \cap M_2))/(1 - P(M_2)) = .4/.6 = 2/3$.

   Grading: 1/2 pt for setting up conditional probability, 1/2 pt for formula, 1/2 pt for numerator, 1/2 for denominator.

   (c) (1 pts) Do you have enough information to calculate the probability that a randomly picked student takes Math I? If yes, please calculate it. If not, say in 2-3 lines why not.

   Solution: *No, because I don't know what* $P(M_1 \cap M_2)$ *is.*

   Grading: 1 pt for correct answer.

2. (5 pts) In the market, there are three varieties of oranges: halos, clementinas and navel. You want to buy 5 oranges. Assume that all oranges of a particular variety are indistinguishable. How many ways can you buy your oranges, such that,

   (a) (1 pt) There are at least one of each variety?

   Solution: $x_1 + x_2 + x_3 = 5,\ x_i > 0.\ \binom{5-1}{3-1} = \binom{4}{2} = 6$

   Grading: They should know this. 1 pt for correct answer.

   (b) (2 pts) There are at least two navel oranges?

   Solution: $x_1+x_2+x_3 = 5,\ x_3 \geq 2.\ y_3 = x_3-2,$ *so* $x_1+x_2+y_3 = 3,$ $\binom{3+3-1}{3-1} = \binom{5}{2} = 10$

   Grading: They should know this. 1 pt setting up, 1 for correct answer.

   (c) (2 pts) There are at least two navel oranges and at least one halo orange?

   Solution: $x_1 + x_2 + x_3 = 5,\ x_1 \geq 0, x_2 \geq 1, x_3 \geq 2.\ y_3 = x_3 - 2,$ $y_2 = x_2 - 1$ *so* $x_1 + y_2 + y_3 = 2,\ \binom{2+3-1}{3-1} = \binom{4}{2} = 6$

   Grading: They should know this. 1 pt setting up, 1 for correct answer.

3. (5 pts) In a bad winter season (too cold), the probability of a person committing a suicide is .006 in a particular city; whereas in a good winter, this probability is .002 in that city. The probability that a winter season will be bad is 0.1. Assume that there are 500 people in that city in a winter season. In your solution, let X be the number of suicides and $B = \{\text{Bad winter}\}$.

   (a) (2 pts) Calculate the probability of the event that there are no suicides.

   Solution: $P(X=0) = P(X=0|B)P(B) + P(X=0|B^c)P(B^c) = e^{-3}.1 + e^{-1}.9.$

   Grading: I think you will see a lot of people doing $\lambda = .006$ ***(or .002). Take one point off for that and don't carry it over. 1/2 pt for total probability rule. 1/2 pt for understanding*** $X|B \sim Poisson(3)$***, 1/2 pt for*** $X|B^c \sim Poisson(1)$***. 1/2 pt for correct answer. Take a point off if they write*** $P(X=0|B) + P(X=0|B^c)$

   (b) (1 pt) If there are no suicides, what is the probability that it was a bad winter season?

   Solution: $P(B|X=0) = P(X=0|B)P(B)/P(X=0) = e^{-3}.1/(e^{-3}.1 + e^{-1}.9).$

   Grading: 1/2 pt for Bayes rule. 1/2 pt for using the information from the last part.

   (c) (2 pts) What is the expected number of suicides in that winter season.

   Solution: $E[X] = E[X|B]P(B) + E[X|B^c]P(B^c) = 3 \times .1 + 1 \times .9 = 1.2$

   Grading: 1 pt for total expectation theorem. 1/2 pt each for expectations of Poisson r.v's.

Useful distributions

All PMFs are zero outside the range specified.

	PDF/PMF	$E[X]$	var(X)
Bernoulli	$p^x(1-p)^{1-x}, x=0,1$	p	$p(1-p)$
Binomial	$\binom{n}{k}p^k(1-p)^{n-k}$, k=0,1,...,n	np	$np(1-p)$
Geometric	$p(1-p)^{k-1}, k=1,2,3,\ldots$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Poisson	$\frac{\lambda^k e^{-\lambda}}{k!}, k=0,1,2,3,\ldots$	λ	λ