Homework Assignment 8

Due via Canvas, **Friday** April 7th by midnight

SDS 321 Intro to Probability and Statistics

1. (2+2+3+1) Suppose that the height, in inches, of a 25-year-old man is a normal random variable with parameters $\mu = 71$ and $\sigma^2 = 6.25$.

   (a) What percentage of 25- year-old men are over 6 feet, 2 inches tall? $P(X > 74) = P((X - 71)/2.5 > 3/2.5) = P(Z > 1.2) = 1 - \Phi(1.2) = .115$

   (b) What percentage of men in the 6-footer club are over 6 feet, 5 inches?

   $$P(X > 77|X > 72) = \frac{P(X > 77)}{P(X > 72)} = (1 - \Phi(2.4))/(1 - \Phi(.4)) = .024$$

   (c) What is the probability that more than half of the group of 5 25-year old men are over 6 feet, 2 inches tall? $X \sim Binomial(5, .115)$. So $P(X \geq 3) = .0127$

   (d) For the last question, what assumptions are you making? We are assuming that the height of individual individuals are independent and identically distributed random variables.

2. (2+2) The time (in hours) required to repair a machine is an exponentially distributed random variable with parameter $\lambda = 1/2$. What is

   (a) the probability that a repair time exceeds 2 hours? $P(X \geq 2) = 1 - P(X < 2) = 1 - (1 - \exp(-1/2 \times 2)) = \exp(-1)$.

   (b) the conditional probability that a repair takes at least 10 hours, given that its duration exceeds 9 hours? Use the memoryless property. $P(X \geq 10|X \geq 9) = P(X \geq 10, X \geq 9)/P(X \geq 9) = P(X \geq 10|X \geq 9) = P(X \geq 1) = \exp(-1/2)$. If anyone uses $\lambda = 12$ give full score.

3. (1+1+3+1+2 pts) The joint probability density function of X and Y is given by Let X be a random variable with PDF

   $$f_{X,Y}(x, y) = \begin{cases} \frac{6}{7}\left(x^2 + \frac{xy}{2}\right) & 0 < x < 1, 0 < y < 2 \\ 0 & \text{Otherwise} \end{cases}$$

   (a) Verify this is a joint density function?

   $$\begin{aligned} \int_{x=0}^{1}\int_{y=0}^{2} f_{X,Y}(x, y)dxdy &= 6/7 \int_{x=0}^{1}\int_{y=0}^{2} (x^2 + xy/2)dxdy \\ &= 6/7(x^3y/3 + x^2y^2/8)|_{x=0,y=0}^{x=1,y=2} \\ &= 6/7(2/3 + 1/2) = 1 \end{aligned}$$

   So f is non-negative and integrates to one. It is a valid joint PDF.

(b) Compute $f_X(x)$.

$$\begin{aligned}\int_{y=0}^{2} f_{X,Y}(x,y)dy &= 6/7\int_{y=0}^{2}(x^2+xy/2)dy \\ &= 6/7(x^2y+xy^2/4)|_{y=0}^{y=2} \\ &= 6/7(2x^2+x)\end{aligned}$$

So

$$f_X(x) = \begin{cases} 6/7(2x^2+x) & x \in [0,1] \\ 0 & \text{Otherwise} \end{cases}$$

(c) Compute $P(X > Y)$.

$$\begin{aligned}\int_{x=0}^{1}\int_{y=0}^{x} f_{X,Y}(x,y)dydx &= 6/7\int_{x=0}^{1}\int_{y=0}^{x}(x^2+xy/2)dydx \\ &= 6/7\int_{x=0}^{1}\int_{y=0}^{x}(x^3+x^3/4)dx \\ &= 15/56\end{aligned}$$

(d) Compute $P(Y > X)$. $P(Y > X) = 1 - P(X > Y) = 41/56$.

(e) What is the CDF (Cumulative Distribution Function) of X?

$$F_X(t) = \begin{cases} 0 & t < 0 \\ 6/7\int_0^t(2x^2+x)dx = 6/7(2t^3/3+t^2/2) & t \in [0,1] \\ 1 & t > 1 \end{cases}$$

.

	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990