Pseudo homework Assignment

SDS 321 Intro to Probability and Statistics

1. Evaluate the following integrals. For (c)-(f), you will have revisit/learn integration by parts. These will come in handy for continuous random variables. Show your calculations.

   (a) (1 pt) $\int_0^\infty \exp(-2x-3)dx$

   $$e^{-3}\int_0^\infty \exp(-2x)dx = .5e^{-3}\int_0^\infty 2\exp(-2x)dx = .5e^{-3} \tag{1}$$

   I am using the fact that $\int_0^\infty 2\exp(-2x)dx$ is basically the integral of the pdf of an Exp(2) random variable and should be 1.

   (b) (1 pt) $\int_0^\infty \exp(-x/2)dx$

   $$\int_0^\infty \exp(-x/2)dx = 2\int_0^\infty 1/2\exp(-x/2)dx = 2 \tag{2}$$

   (c) (2 pts) $\int_0^\infty x\exp(-2x)dx$

   $$\int_0^\infty x\exp(-2x)dx = 0.5\int_0^\infty x\times 2\exp(-2x)dx = 0.5\times 1/2 = 1/4 \tag{3}$$

   I am using the fact that $\int_0^\infty x\times 2\exp(-2x)dx$ is basically $E[X]$ where $X\sim Exp(2)$ random variable and should be $1/2$.

   (d) (2 pts) $\int_0^\infty x\exp(-x/2)dx$

   $$\int_0^\infty x\exp(-x/2)dx = 2\int_0^\infty x\times 1/2\exp(-x/2)dx = 2\times 2 = 4 \tag{4}$$

   I am using the fact that $\int_0^\infty x\times 1/2\exp(-x/2)dx$ is basically $E[X]$ where $X\sim Exp(1/2)$ random variable and should be 2.

   (e) (2 pts) $\int_0^\infty x^2\exp(-2x)dx$

   $$\int_0^\infty x^2\exp(-2x)dx = 0.5\int_0^\infty x^2\times 2\exp(-2x)dx = 0.5\times 2/2^2 = 1/4 \tag{5}$$

   I am using the fact that $\int_0^\infty x^2\times 2\exp(-2x)dx$ is basically $E[X^2]$ where $X\sim Exp(2)$ random variable and should be $2/2^2 = 0.5$.

   (f) (2 pts) $\int_0^\infty x^2\exp(-x/2)dx$

   $$\int_0^\infty x^2\exp(-x/2)dx = 2\int_0^\infty x^2\times 1/2\exp(-x/2)dx = 2\times 2/(1/2)^2 = 16 \tag{6}$$

   I am using the fact that $\int_0^\infty x^2\times 1/2\exp(-x/2)dx$ is basically $E[X^2]$ where $X\sim Exp(1/2)$ random variable and should be $2/0.5^2 = 8$.