Homework Assignment 4

Due by 5pm via Canvas, Feb 17

SDS 321 Intro to Probability and Statistics

1. (1+3 pts) A committee of 6 is to be selected from 10 men and 10 women. If every configuration is equally likely, what is the probability that,

   (a) the committee contains equal number of men and women?

   Total number of ways $\binom{20}{6}$. 3 men and 3 women $\binom{10}{3}\binom{10}{3}$. So $\frac{\binom{10}{3}\binom{10}{3}}{\binom{20}{6}}$

   (b) more than half of the committee are women? $\sum_{i=4}^{6} P(\text{Committee has i women}) = \sum_{i=4}^{6} P(\text{Committee has i women and 6-i men}) = \sum_{i=4}^{6} \frac{\binom{10}{i}\binom{10}{6-i}}{\binom{20}{6}}$

2. (1+1) An urn contains 4 red, 8 blue, and 10 green balls. I pick a set of 3 balls without replacement. What is the probability that each of the balls will be,

   (a) of the same color? Total number of ways $\binom{22}{3}$. Same color: P(all balls are red or all are green or all a $\frac{\binom{4}{3}}{\binom{22}{3}} + \frac{\binom{8}{3}}{\binom{22}{3}} + \frac{\binom{10}{3}}{\binom{22}{3}}$.

   (b) of different colors? Total number of ways $\binom{22}{3}$. Different colors: 4 ways to pick a red, 8 ways to pick a blue and 10 ways to pick a green. So $\frac{4\times 8\times 22}{\binom{22}{3}}$

3. (2+2) Alice and Bob are sitting with eight of their friends. What is the probability that Alice and Bob will sit together if they

   (a) sit in a line? Total number of ways 10. Take Alice and Bob as a unit, so total number ways to sit 9 units is 9!. But Alice and Bob can switch places, so the total number ways becomes $2 \times 9!$. Answer is $2 \times 9!/10! = 2/10$.

   (b) sit in a circle? Total number of ways to sit in a circle 9because you fix one person and permute the others. Now the same logic as before gives $2 \times 8!/9! = 2/9$.

4. (2+4+4) I am building a 5 digit number (without repetitions) by **arranging** the digits $\{1, 2, 3, 4, 5\}$. Consider that each configuration is equally likely. What is the probability that:

   (a) There is exactly one digit between '1' and '2'? 1 and 2 with one place in between can be places in 3 places. So 1-2–, -1-2-, –1-2. Now the remaining 3 places can be arranged in 3! ways. And 1 and 2 can be switched too. So $2 \times 3 \times 3!/5!$

   (b) There are exactly two digits between '1' and '2'? 1 and 2 with two places in between can be places in 2 places. So 1–2-, -1–2. Now the remaining 3 places can be arranged in 3! ways. And 1 and 2 can be switched too. So $2 \times 2 \times 3!/5!$

(c) There are exactly three digits between '1' and '2'? 1 and 2 with two places in between can be places in 1 place. So 1—2, -1–2. Now the remaining 3 places can be arranged in 3! ways. And 1 and 2 can be switched too. So $2 \times 3!/5!$