Homework Assignment 1

Due by 4pm at GDC 7.504, February 2nd

SDS 321 Intro to Probability and Statistics

1. Let A, B and C be three events. Find expressions for:

   (a) (1 pts) Both A and B, but not C occur.
   Solution: $A \cap B \cap C^c$

   (b) (1 pts) None of the three events occur.
   Solution: $A^c \cap B^c \cap C^c$ or $(A \cup B \cup C)^c$

   (c) (1 pts) At least one of the three events occur.
   Solution: $A \cup B \cup C$

   (d) (2 pts) At least two of the three events occur.
   Solution: $(A \cap B) \cup (B \cap C) \cup (A \cap C)$

   (e) (2 pts) Exactly one of the three events occur.
   Solution: $(A^c \cap B^c \cap C) \cup (A \cap B^c \cap C^c) \cup (A^c \cap B \cap C^c)$

   For the last two questions, if the answer is incomplete, i.e. they have included 2 of the cases but not all, then take a point off.

2. In a student community, 30% of the students own a car and 50% of the students who own a car also own a bicycle. Also, 60% of the student community own a bicycle. Furthermore 25% of students who own a bicycle, also own a two-wheeler. Car owners do not own two-wheelers. Finally, 30% of the students own a two-wheeler.

   (a) (2 pts) What is the probability that a randomly selected student owns a car and a bicycle? $B = \{\text{Owns a bicyle}\}$, $C = \{\text{Owns a car}\}$, $T = \{\text{Owns a two-wheeler}\}$. $P(C) = .3$, $P(B|C) = .5$. So $P(B \cap C) = .15$.

   (b) (2 pts) What is the probability that a randomly selected student owns a bicycle,but does not own a car? $P(B \cap C^c) = P(B) - P(B \cap C) = .6 - .15 = .45$.

   (c) (2 pts) What is the probability that a randomly selected student does not own any of the three? $1 - P(T \cup B \cup C) = 1 - (P(T) + P(B) + P(C) - P(B \cap C) - P(T \cap B)) = 1 - (.3 + .6 + .3 - .15 - .15) = .1$

   (d) (2 pts) What is the probability that a randomly selected student owns a bicycle, but no car or two-wheeler? $P(B \cap T^c \cap C^c) = P(B) - P(B \cap C) - P(B \cap T) = .6 - .15 - .15 = .3$.

   One point for setting it up and one point for correct answer.

3. (5 pts) The admissions committee codes the applications for graduate school according to their publication record and GPA. The coding for publications is (a) for more than

3 papers, (b) for 1-3 publications and (c) for no publications. The coding for GPA is (1) for GPA$\geq$ 4.0, (2) for $3.5 \leq GPA < 4$ and (3) for any other GPA. Consider an experiment of the coding of such an incoming application. For example if an applicant has more than 3 papers and GPA 3.7, then he/she will be coded as $a2$.

(a) (1 pt) Give the sample space of this experiment. $a1, a2, a3, b1, b2, b3, c1, c2, c3$

(b) (1 pt) Let A be the event that the applicant has less than 3.0 GPA. What would be the outcomes in A? $A = \{a3, b3, c3\}$

(c) (2 pts) Let B be the event that the applicant has at least one publication. Specify the outcomes in B. $B = \{a1, a2, a3, b1, b2, b3\}$

(d) (1 pt) Give the outcomes in $B \cap A^c$. $B \cap A^c = \{a1, a2, b1, b2\}$