18-452/18-750 Wireless Networks and Applications
Lecture 5: Physical Layer Modulation and Diversity

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Announcement

• Please form P1 teams on campus
  » 2 students per team

Outline

• RF introduction
• Modulation and multiplexing
• Channel capacity
• Antennas and signal propagation
• Modulation
• Coding and diversity
• OFDM

(Limited) Goals

• Non-goal: turn you into electrical engineers
• Basic understanding of how modulation can be done
• Understand the tradeoffs involved in speeding up the bit rate
From Signals to Packets

Packet Transmission

Sender → Receiver

Packet

000101110001

Header/Body

Bit Stream

0 0 1 0 1 1 1 0 0 0 1

“Digital” Signal

Analog Signal

Basic Modulation Techniques

• Encode digital data in an analog signal
  • Amplitude-shift keying (ASK)
    » Amplitude difference of carrier frequency
  • Frequency-shift keying (FSK)
    » Frequency difference near carrier frequency
  • Phase-shift keying (PSK)
    » Phase of carrier signal shifted

Amplitude-Shift Keying

• One binary digit represented by presence of carrier, at constant amplitude
• Other binary digit represented by absence of carrier

\[ s(t) = \begin{cases} \frac{A \cos(2\pi f_c t)}{t} & \text{binary 1} \\ 0 & \text{binary 0} \end{cases} \]

– where the carrier signal is \( A \cos(2\pi f_c t) \)

• Inefficient because of sudden gain changes
  » Only used when bandwidth is not a concern, e.g. on voice lines (< 1200 bps) or on digital fiber
• A can be a multi-bit symbol

How Can We Go Faster?

• Increase the rate at which we modulate the signal, or …
  » I.e., a higher frequency base signal
  » Signal time becomes short
• Modulate the signal with “symbols” that send multiple bits
  » I.e., each symbol represents more information
  » Longer signal time but more sensitive to distortion
• Which solution is the best depends on the many factors
  » We will not worry about that in this course
Binary Frequency-Shift Keying (BFSK)

- Two binary digits represented by two different frequencies near the carrier frequency
  \[ s(t) = \begin{cases} 
  A \cos(2\pi f_1 t) & \text{binary 1} \\
  A \cos(2\pi f_2 t) & \text{binary 0} 
  \end{cases} \]
  - where \( f_1 \) and \( f_2 \) are offset from carrier frequency \( f_c \) by equal but opposite amounts
- Less susceptible to error than ASK
- Sometimes used for radio or on coax
- Demodulator looks for power around \( f_1 \) and \( f_2 \)

Multiple Frequency-Shift Keying (MFSK)

- More than two frequencies are used
- Each symbol represents \( L \) bits
  \[ s_i(t) = A \cos 2\pi f_i t \quad 1 \leq i \leq M \]
  - \( f_i = f_c + (2i - 1 - M)f_d \)
  - \( L \) = number of bits per signal element
  - \( M \) = number of different signal elements = \( 2^L \)
  - \( f_c \) = the carrier frequency
  - \( f_d \) = the difference frequency
- More bandwidth efficient but more susceptible to error
  » Symbol length is \( T_s = LT \) seconds, where \( T \) is bit period

Phase-Shift Keying (PSK)

- Two-level PSK (BPSK)
  » Uses two phases to represent binary digits
  \[ s(t) = \begin{cases} 
  A \cos(2\pi f_c t) & \text{binary 1} \\
  A \cos(2\pi f_c t + \pi) & \text{binary 0} 
  \end{cases} \]
  - \( f_c = f_c + (2i - 1 - M)f_d \)
- Differential PSK (DPSK)
  » Phase shift with reference to previous bit
  - Binary 0 – signal of same phase as previous signal burst
  - Binary 1 – signal of opposite phase to previous signal burst
Phase-Shift Keying
Four Level PSK

- Each element represents 2 (or more) bits

\[ s(t) = \begin{cases} 
A \cos \left( 2\pi f_c t + \frac{\pi}{4} \right) & 11 \\
A \cos \left( 2\pi f_c t + \frac{3\pi}{4} \right) & 01 \\
A \cos \left( 2\pi f_c t - \frac{3\pi}{4} \right) & 00 \\
A \cos \left( 2\pi f_c t - \frac{\pi}{4} \right) & 10 
\end{cases} \]

Time and Point View of Signal

- Remember: communication is based on the transmission of a modulated carrier signal
  » Focus on amplitude-phase modulation – very common!

- What about a mathematical representation of the received signal?
  » We can then reason about the impact of channel impairments on the signal and error rates

Channel State

- The channel state \( c \) is a complex number that captures attenuation, fading, ... effects
  » Represents instantaneous phase and amplitude

- \( c \) changes over time, e.g., fading
  » Change is continuous, but represented as a sequence of values \( c_i \)
  » The sampling rate depends on how fast \( c \) changes – must sample twice the frequency (Nyquist rate)

- \( c \) typically depends on carrier frequency: \( c(f) \)
  » Frequency selective fading or attenuation, e.g., \( f \) impacts loss and phase caused by multi-path and obstacles
  » The dependency on \( f \) is much more of a concern for wideband signals

Channel Model

1. Transmits signal \( x \): modulated carrier at frequency \( f \)
2. Signal is attenuated
3. Multi-path + mobility cause fading
4. Noise is added
5. Doppler effects distorts signal
6. Receives distorted Signal \( y \)

\[ x \times c + n = y \]
Tradeoff: Bit Rate versus Error Rate - Informal

- Amplitude and phase modulation places transmitted symbols into 2D space
  » Represented by a complex number
- Channel distortion “moves” the symbol
  » Large shift can map it onto another symbol
- Large symbols means denser packing of symbols in the plane
  » Results in high bit rate but distortions are more likely to result in errors
- Smaller symbols are more conservative
  » Lower bit rate but more resistant to errors

Good channels

Bad channels

How Does Distortion Impact a Constellation Diagram?

- Changes in amplitude, phase or frequency move the points in the diagram
- Large shifts can create uncertainty on what symbol was transmitted
- Larger symbols are more susceptible
- Can Adapt symbol size to channel conditions to optimize throughput

Signal Constellations

- Each pair \((A_k, B_k)\) defines a point in the plane
- Signal constellation set of signaling points

\[
\begin{align*}
(A, B) & \quad & (A, -B) \\
(-A, B) & \quad & (-A, -B)
\end{align*}
\]

Good channels

Bad channels

Adapting to Channel Conditions

- Channel conditions can be very diverse
  » Affected by the physical environment of the channel
  » Changes over time as a result of slow and fast fading
- Fixed coding/modulation scheme will often be inefficient
  » Too conservative for good channels, i.e. lost opportunity
  » Too aggressive for bad channels, i.e. lots of packet loss
- Adjust coding/modulation based on channel conditions – “rate” adaptation
  » Controlled by the MAC protocol
  » E.g. 802.11a: BPSK – QPSK – 16-QAM – 64 QAM

Bad

Good Channel
Summary

- Key properties for channels are:
  - Channel state that concisely captures many of the factors degrading the channel
  - The power budget expresses the power at the receiver
  - Channel reciprocity
- Modulation changes the signal based on the data to be transmitted
  - Can change amplitude, phase or frequency
  - The transmission rate can be increased by using symbols that represent multiple bits
    - Can use hybrid modulation, e.g., phase and amplitude
  - The symbol size can be adapted based on the channel conditions – results in a variable bit rate transmission
    - Details do not matter!

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- RF introduction
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- Channel capacity
- Antennas and signal propagation
- Modulation
- Diversity and coding
  - Space, time and frequency diversity
- OFDM

Diversity Techniques

- The quality of the channel depends on time, space, and frequency
- Space diversity: use multiple nearby antennas and combine signals
  - Both at the sender and the receiver
- Time diversity: spread data out over time
  - Useful for burst errors, i.e., errors are clustered in time
- Frequency diversity: spread signal over multiple frequencies
  - For example, spread spectrum
- Distribute data over multiple “channels”
  - “Channels” experience different frequency selective fading, so only part of the data is affected

Space Diversity

- Use multiple antennas that pick up/transmit the signal in slightly different locations
- If antennas are sufficiently separated, instantaneous channel conditions are independent
  - Antennas should be separated by ¼ wavelength or more
- If one antenna experiences deep fading, the other antenna has a strong signal
- Represents a wide class of techniques
  - Use on transmit and receive side - channels are symmetric
  - Level of sophistication of the algorithms used
  - Can use more than two antennas!
Selection Diversity

- Receiver diversity: receiver picks the antenna with the best SNR
  - Very easy
- Transmit diversity: sender picks the antenna that offers the best channel to the receiver
  - Transmitter can learn the channel conditions based on signals sent by the receiver
  - Leverages channel reciprocity

\[ x_1 \rightarrow h_1 \rightarrow y_1 \]
\[ x_2 \rightarrow h_2 \rightarrow y_2 \]

Simple Algorithm in (older) 802.11

- Combine transmit + receive selection diversity
  - Assume packets are acknowledged – why?
- How to explore all channels to find the best one ... or at least the best transmit antenna
- Receiver:
  - Uses the antenna with the strongest signal
  - Always use the same antenna to send the acknowledgement – gives feedback to the sender
- Sender:
  - Picks an antenna to transmit and learns about the channel quality based on the ACK
  - Needs to occasionally try the other antenna to explore the channel between all four channel pairs

\[ \text{Transmit} \rightarrow \text{Receiver} \]

Receiver Diversity

Can we Do Better?

- But why not use both signals?
  - 2 Signals contain more information than 1
  - What can go wrong?
- Simply adding the two signals has drawbacks:
  - Signals may be out of phase, e.g. kind of like multi-path; can reduce the signal strength!
  - We want to make sure we do not amplify the noise
- Maximal ratio combining: combine signals with a weight that is based on their SNR
  - Weight will favor the strongest signal (highest SNR)
  - Also: equal gain combining as a quick and dirty alternative

\[ \frac{y_1 + y_2}{h_1 + h_2} \]

Receiver Diversity Optimization

- Multiply \( \vec{y} \) with the complex conjugate \( \vec{h}^* \) of the channel vector \( \vec{h} \)
  - Aligns the phases of the two signals so they amplify each other
  - Scales the signals with their magnitude so the effect of noise is not amplified
- Can learn \( \vec{h} \) based on training data
The Details

- Complex conjugates: same real part but imaginary parts of opposite signs
  \[ \hat{h}^\ast \cdot \hat{y} = \hat{h}^\ast \cdot (\hat{h}^\ast \cdot \hat{x} + \hat{n}) \]
  Where \( \hat{h}^\ast = [h_1^\ast \ h_2^\ast] = [a_1+b_1i \ a_2-b_2i] \)
- Result:
  signal \( x \) is scaled by \( a_1^2 + b_1^2 + a_2^2 + b_2^2 \)
  noise becomes: \( h_1^\ast \cdot n_1 + h_2^\ast \cdot n_2 \)

Transmit Diversity

- Same as receive diversity but the transmitter has multiple antennas
- Maximum ratio combining: sender “precodes” the signal
  » Pre-align the phases at receiver and distribute power over the transmit antennas (total power fixed)
- How does transmitter learn channel?
  » Channel reciprocity: learn from packets received \( Y \)

Adding Redundancy

- Protects digital data by introducing redundancy in the transmitted data.
  » Error detection codes: can identify certain types of errors
  » Error correction codes: can fix certain types of errors
- Block codes provide Forward Error Correction (FEC) for blocks of data.
  » (n, k) code: n bits are transmitted for k information bits
  » Simplest example: parity codes
  » Many different codes exist: Hamming, cyclic, Reed-Solomon,…
- Convolutional codes provide protection for a continuous stream of bits.
  » Coding gain is \( n/k \)
  » Turbo codes: convolutional code with channel estimation

Combine Redundancy with Time Diversity

- Fading can cause burst errors: a relatively long sequence of bits is corrupted
- Spread blocks of bytes out over time so redundancy can help recover from the burst
  » Example: only need 3 out of 4 to recover the data
## Bits, Symbols, and Chips

- Redundancy and time diversity can be added easily at the application layer
- Can we do it lower in the stack?
  - Need to adapt quickly to the channel
- So far: use bits to directly modulate the signal
- Idea: add a coding layer – provides a level of indirection
- Can add redundancy and adjust level of redundancy quickly based on channel conditions

![](diagram.png)

## Discussion

- Error coding increases robustness at the expense of having to send more bits
  - Technically this means that you need more spectrum
- But: since you can tolerate some errors, you may be able to increase the bit rate through more aggressive modulation
- Coding and modulation combined offer a lot of flexibility to optimize transmission
- Next steps:
  - Apply a similar idea to frequency diversity
  - Combine coding with frequency and time diversity in OFDM

## Summary

- Space diversity really helps in overcoming fading
  - Very widely deployed
  - Will build on this when we discuss MIMO
- Coding is also an effective way to improve throughput
  - Widely used in all modern standards
  - Coding, combined with modulation, can be adapt quickly to channel conditions