

In Search of Musicality: Can Fractals Show the Way?

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Abstract—Research on characterization and classification of musical compositions have focused on several mathematical and statistical techniques. In this paper, we take a different approach based on music visualization and fractals to investigate the nature of musicality. A musical composition is defined as a sequence of notes and self-similarity plots (SSPs) are constructed on the basis of certain similarity metric. We observe that SSPs are fractals by nature and can distinguish musical compositions from randomly generated sequence of notes. We define the *signature* of a composition based on fractal dimensions of SSPs and observe that signature of a tune can help classify and characterize a composition. Thus, we conclude that fractals might provide us with the answer to the age-old question of musicality.

I. IN SEARCH OF MUSICALITY

What makes a sequence of sounds musical? It is a question that has raised much more philosophical arguments than possible explanations - both the so called “scientific” and “non-scientific” ones. We now have sufficiently satisfactory explanations for the basic set of notes used in musical compositions - thanks to Pythagoras and Bach [1]. Even then, not any combination of notes are musical. Some compositions are aesthetically more *musical* than others. What are the properties of a sequence of notes which make it musical or soothing to the ears? Why do we like some type of music more than some other types? Why do some compositions arouse a feeling of melancholy while some other compositions are embodiment of serenity or divinity? How can we associate a composition to Beethoven and another piece to Bach just by listening? In this paper, we take a look at a new approach towards musicality and try to answer some of these questions computationally.

A. Components of a Composition

A Musical composition is multidimensional piece of work consisting of the basic composition as a sequence of notes, the rhythmic beats, the lyrics and its vocal rendition. Any research on the musicality of a piece should consider the piece as a whole (i.e. as an audio signal) rather than the individual parts. Nevertheless, in this work, we focus only on the sequence of notes, because our aim here is to define musicality on the basis of this parameter. Some of the other components of a composition, that have been completely ignored in this work are

- **Lyrics:** The lyrics pertaining to a composition has its own pragmatics, and hence biases the listener towards a

specific mood. Since we do not want to use such explicit linguistic information, we will avoid using this.

- **Rendition:** Vocal quality of the renderer and the specific instruments used in a piece are often sufficient to produce some bias, which might be based on cultural conventions. In the Indian context, *Shehnai* is associated with festivities and *Jaltarang* or *Santoor* have a lively tonal effect. Such prejudices should be avoided.
- **Rhythm:** Rythm or the *beat pattern* is an integral part of any composition. It is a measure of the pace and the periodicity of the composition, and hence is an important parameter of a composition that governs its musical nature to a large extent.

The sequence of notes in a composition plays an important role in imparting the characteristics of that music. Mood, genre, temporal and spatial settings are largely governed by the note sequence. This concept has been the heart of both Western and Indian Classical Music¹. Therefore, it seems worthwhile to search for the basic nature of musicality of a composition in its note sequences.

B. Brief Sketch of the Work

In this paper, we describe an approach to investigate the nature of musicality based on fractal dimensions of *self-similarity plots*. Section 2 describes the construction of the self-similarity plots and through examples illustrates the patterns of music as compared to random strings. Section 3 begins with a discussion on the fractal nature of the self-similarity plots followed by the description of the method used to measure the fractal dimension of the plots. This section also talks about the *signature* of a tune based on the fractal dimension, that can be used for clustering and categorization of compositions. The concluding section summarizes our basic findings and outlines future research.

II. VISUALIZING MUSIC

Acoustic signals such as speech and music are stretched over a period of time. The temporal dimension makes the comprehension of the overall structure of the signal difficult for the human brain. Contrary to this, the human brain can

¹Indian Classical Music is based on the *Raga* paradigm, which are considered as the basic structure of a music [2].

identify patterns in a visual image much faster as the whole structure can be perceived at a single point of time. Therefore, a visual representation of a composition can help us understand its features quite fast and provide us with useful insights about musicality. The idea is not new and considerable effort has gone behind music visualization. The ancient Indian endeavour for anthropomorphic descriptions of the *Ragas* and modern artistic realizations of images elicited by sounds, for example the Walt Disney film *Fantasia*, are just some of the attempts towards this direction. In this section, we describe a more formal approach towards music visualization based on the works of Foote and Cooper [3], [4].

A. Related Works

Visualization of acoustic signals for scientific or quantitative analysis is normally done through oscillograph and sound spectrograph, which quantitatively render the time and/or frequency content of the audio signal [5], [6]. There has been some work on visualization from note-based or score-like representation of music as well [7]. Foote [3] suggested a *similarity matrix* based visualization technique for similarity analysis in and among musical pieces.

The idea of *self-similarity plots* (SSP) is as follows. The composition is divided into n overlapping windows each of length t seconds. The (i, j) entry of the *similarity matrix* $S[1..n][1..n]$ is a measure of the similarity between the parts of the composition in the i th and j th windows. The SSP is an $n \times n$ square image constructed by coloring each point (i, j) with a shade of gray proportionate to the value of $S[i][j]$. Thus, in an SSP if a point (u, v) is darker than a point (u, w) , then one can infer that the part of the composition around the window u (at $u.t$ seconds from the beginning) is acoustically more similar to the part at $v.t$ seconds than the part at $w.t$ seconds.

Foote used the average autocorrelation between the MFCC feature vectors over the audio windows i and j as the similarity metric and got interesting checkerboard-like patterns for musical compositions. This visualization technique was later used for music summarization and similarity analysis with considerable success [4]. However, in order to capture the overall structure of the composition, statistical clustering techniques were used to filter out the finer details and only the broader structure was considered.

In this work, we import Foote's idea of music visualization, but with two major differences - (1) SSPs are constructed for the note sequences rather than the audio signals and (2) both the broader structure and the finer details are captured through the notion of fractals.

B. The Formal Model

We define the *notes*, *compositions* etc. following the system of Indian classical music. [2], [8] provide a good introduction to the subject. However, the model described is capable of capturing any musical composition and the choice of Indian classical music is only for notational convenience.

1) *Basic Definitions*: Let Σ be the set of notes. Then, Σ can be expressed as

$$\Sigma = \Sigma_{lower} \cup \Sigma_{middle} \cup \Sigma_{upper}$$

Where, Σ_{lower} , Σ_{middle} and Σ_{upper} refers to the notes in the lower, middle and upper octaves respectively.

$$\Sigma_{lower} = \{S, r, R, g, G, m, M, P, d, D, n, N\}$$

$$\Sigma_{middle} = \{\dot{S}, \dot{r}, \dot{R}, \dot{g}, \dot{G}, \dot{m}, \dot{M}, \dot{P}, \dot{d}, \dot{D}, \dot{n}, \dot{N}\}$$

$$\Sigma_{upper} = \{\ddot{S}, \ddot{r}, \ddot{R}, \ddot{g}, \ddot{G}, \ddot{m}, \ddot{M}, \ddot{P}, \ddot{d}, \ddot{D}, \ddot{n}, \ddot{N}\}$$

S, R, G, m, P, D and N stand for *sa, re, ga, ma, pa, dha* and *ni* respectively. r, g, d and n stand for the corresponding flat notes (*komal swar*) and M stands for *kadi ma*. Although the Σ (and Σ_{upper} , Σ_{middle} and Σ_{lower}) has been described here as a set, it is actually more than a simple set - it is an ordered set. We can define a *total ordering* on Σ on the basis of the frequency of the notes. The *notes* have been listed above in the ascending order of frequency. However, the frequencies of the notes are relative, rather than any fixed absolute value. In general,

$$\frac{fr(\dot{S})}{fr(S)} = \frac{fr(\ddot{S})}{fr(\dot{S})} = 2$$

where, $fr(S)$ stands for the frequency of the note S . Also, the ratio of frequencies of two adjacent notes, i.e.

$$\frac{fr(r)}{fr(S)} = \frac{fr(R)}{fr(r)} = \dots = 2^{1/12} = 1.059$$

In order to represent a note more succinctly and formally, we define a function $id : \Sigma \rightarrow \mathbf{Z}$, such that

$$id(\dot{S}) = 0,$$

$$\frac{fr(x)}{fr(\dot{S})} = 2^{id(x)/12}, \forall x \in \Sigma$$

$$i.e., fr(x) = fr(\dot{S}) \times 2^{id(x)/12}, \forall x \in \Sigma$$

Thus, the function id explicitly defines the *total ordering* on Σ and provides a way to calculate the frequency of a note relative to the base note \dot{S} . However, the motivation behind this definition lies in the fact that human ear can perceive frequencies in the logarithmic scale. Therefore, if x and y are two notes, then the human ear perceives the difference between the frequencies of the two as

$$\begin{aligned} & \log(fr(x)) - \log(fr(y)) \\ &= \log(fr(\dot{S}) \times 2^{id(x)/12}) - \log(fr(\dot{S}) \times 2^{id(y)/12}) \\ &= (id(x) - id(y)) \times k \end{aligned}$$

where, k is a constant ($\log 2/12$). Thus, it is the difference between $id(x)$ and $id(y)$ that is relevant to the human ears.

A *musical composition* of length n is a string $\sigma_1 \sigma_2 \dots \sigma_{n-1} \sigma_n$, where $x_i \in \Sigma \forall i \in [1, n]$. The set of all *musical compositions* defines a language \mathcal{M} over Σ . Our aim precisely is to characterize \mathcal{M} .

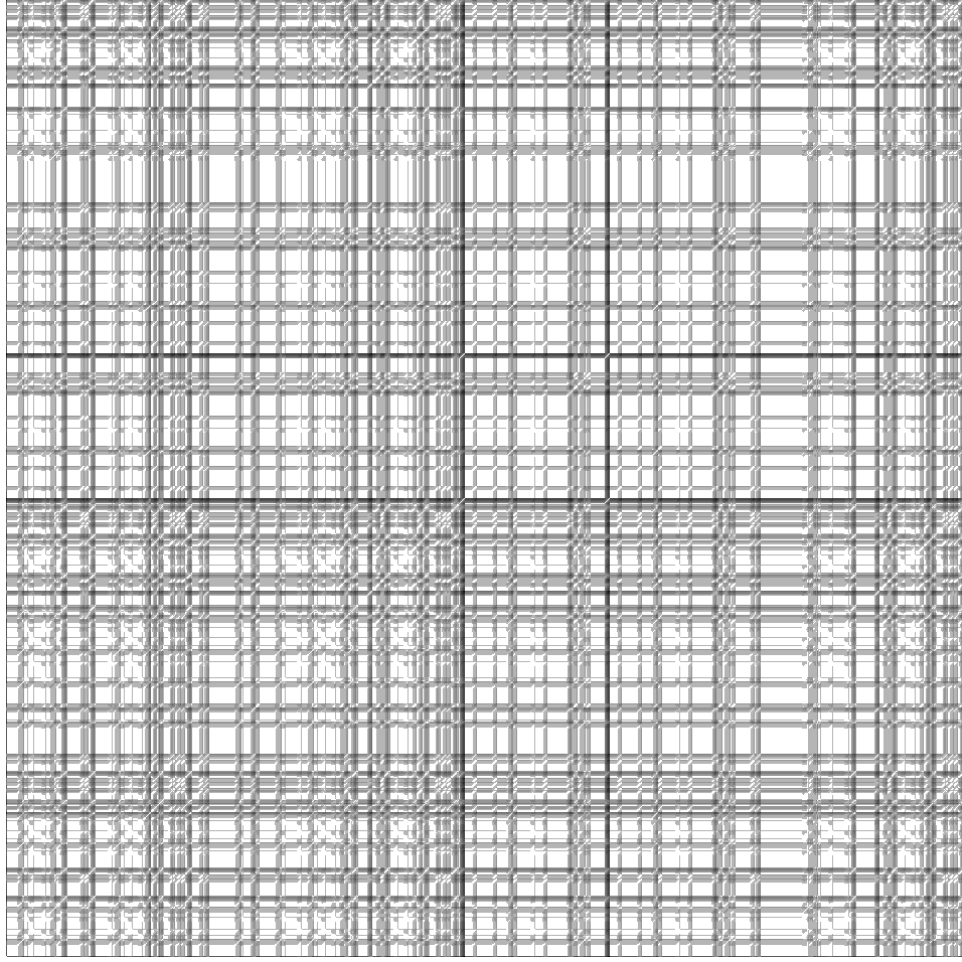


Fig. 1. Self-similarity plot of a Tagore Song, $w = 5$

2) *Similarity Metric*: We define a transformation Θ from a string Σ^* to a string over \mathbf{Z}^* as follows. then

$$\Theta(\sigma_1\sigma_2 \dots \sigma_{n-1}\sigma_n) = id(\sigma_1)id(\sigma_2) \dots id(\sigma_{n-1})id(\sigma_n)$$

where, $\sigma_i \in \Sigma \forall i \in [1, n]$. Since, it is the relative frequency changes that characterize the musicality of a sequence of notes, rather than the frequency of the notes themselves, we define a transformation Ψ from \mathbf{Z}^* to \mathbf{Z}^* for *normalization* of a sequence of note after the Θ transformation.

$$\Psi(x_1x_2 \dots x_{n-1}x_n) = (x_2 - x_1) \cdot (x_3 - x_1) \dots (x_{n-1} - x_1) \cdot (x_n - x_1)$$

where $x_i \in \mathbf{Z} \forall i \in [1, n]$ and ‘ \cdot ’ stands for the concatenation operator.

Let u and v be two strings of length $n + 1$ over Σ . Then, $u' = \Psi(\Theta(u))$ and $v' = \Psi(\Theta(v))$ are strings of length n over \mathbf{Z}^* . We define the *Euclidean distance* between the strings u' and v' averaged over the string length n as the *distance* between the two note sequences u and v . Thus, if δ denotes the distance function and $u' = u'_1u'_2 \dots u'_n$ and $v' = v'_1v'_2 \dots v'_n$

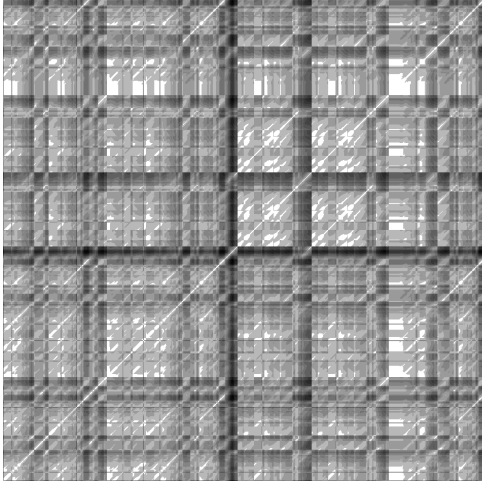
$$\delta(u, v) = \left[\frac{1}{n} \sum_{i=1}^n (u'_i - v'_i)^2 \right]^{1/2}$$

For example, if $n = 4$, $u = \dot{S}\dot{R}\dot{G}\dot{m}\dot{P}$ and $v = \dot{R}\dot{S}n\dot{R}\dot{S}$, we have $\Theta(u) = 0, 2, 4, 5, 7$ and $\Theta(v) = 2, 0, -2, 2, 0$. Therefore, $u' = 2, 4, 5, 7$ (subtracting 0 from each element) and $v' = -2, -4, 0, -2$ (subtracting 2 from each element). Hence, $\delta(u, v) = [((2 - (-2))^2 + (4 - (-4))^2 + (5 - 0)^2 + (7 - (-2))^2)/4]^{1/2} = [(16 + 64 + 25 + 81)/4]^{1/2} = 6.82$.

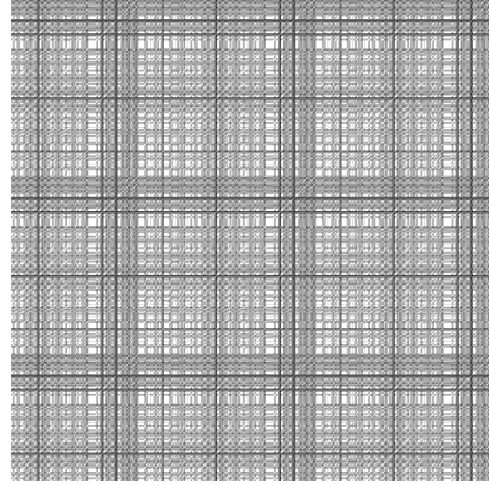
The higher the value of $\delta(u, v)$, the more different they (u and v) are from each other. As explained above, the applied transformations Θ and Ψ capture the human perceptual mechanism. However, the choice of Euclidean distance as the distance metric is arbitrary. It is a commonly used distance function and computationally less expensive. In future, other distance metrics can be explored as well.

3) *Self-similarity Matrix*: For a musical composition $c = c_1c_2 \dots c_{l-1}c_l$ of length l , the *self-similarity matrix* of window w is an $n \times n$ square matrix S , where

$$n = l - w$$



(a) Tagore song of figure 1, $w=25$



(b) Song "Hotel California" by Eagles, $w=5$

Fig. 2. Some more self-similarity plots for different window sizes

$$S[i][j] = \delta(c[i, i + w], c[j, j + w]), \quad 1 \leq i, j \leq n$$

Here, $c[i, i + w]$ stands for the substring of c beginning at position i and of length $w + 1$. The size of the window w can vary from 1 to $l - 1$. For very large w (comparable to l) we get very small self-similarity matrices. On the other hand, for very small w (≤ 2) the substrings being compared are too small to entail any notion of musicality.

C. Self-similarity Plots

Given a musical composition and a window size w , we can easily construct the self-similarity matrix S of size $n \times n$. The *self-similarity plot* corresponding to S can be constructed as follows. Each entry of $S[i][j]$ is linearly scaled up or down so that the largest entry of the matrix is mapped to 255. This is because, we use a 256-bit colour map and want most dissimilar parts of the composition to be plotted in black (R=255, G=255, B=255). Every point (i, j) in the plot gets a color (R=G=B= $s[i][j]$) equivalent to $s[i][j]$ the scaled value of $S[i][j]$. Thus, we get a black-and-white square plot.

Figure 1 shows the self-similarity plot for a Tagore song². The window size was 5. The white diagonal line from the bottom left corner to top right corner divides the image into two symmetric triangles. This line is white because $S[i][i] = 0$ for all i , or in other words, every piece is identical to itself. Due to the symmetric nature of the distance metric δ , $S[i][j] = S[j][i]$ for all i, j . Therefore, the two triangular halves are symmetric. Figure 2 shows self-similarity plots for some other compositions.

D. Music Does Have a Pattern

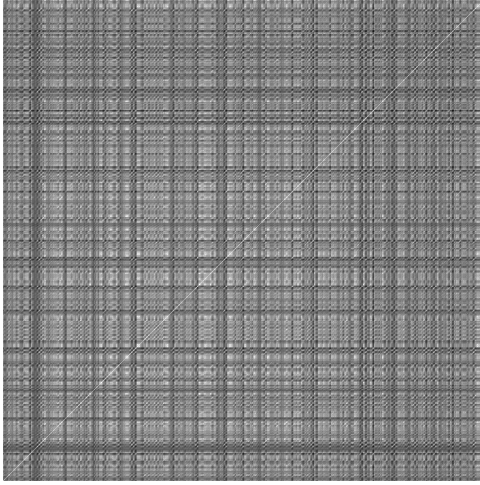
The self-similarity plots as illustrated in figures 1 and 2 do have a pattern. The patterns are quite similar to the ones shown

in [3], [4], though those were based on audio signals and not note sequences. However, can we claim from these evidences that music has a pattern? It might well be the case that the patterns are inherent to the construction procedure of the self-similarity matrix and the self-similarity plot. For example, the self-similarity plots are by definition symmetric with a white diagonal line. Therefore, we need more evidence to claim that the visual patterns in the self-similarity plots emerge from musicality and not the construction algorithm.

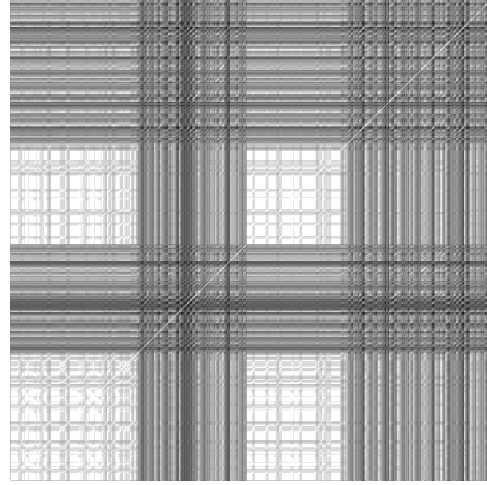
One way to settle this question is to artificially generate "non-musical" compositions and compare their self-similarity plots with that of musical compositions. We tried this by generating random sequences of strings and constructed their similarity plots. However, for a pure random sequence, where every note in the sequence is selected independently of the other notes, we get a uniformly dark-colored similarity plot with a bright diagonal line. This is expected, because correlation between different parts of the composition is zero by construction. On the other hand, we do have non-musical compositions, where this may not be true. Therefore, we chose a more realistic model of random music.

A common observation about musical sequences is that there are very few abrupt jumps of frequencies. For example, it is very unlikely that in a composition there will be a jump from the note S to the note \hat{S} (the frequency of the latter is twice that of the former). We incorporated this property in the random music model as described below. First a note σ_1 is chosen randomly from Σ . A *normal distribution* is considered with $id(\sigma_1)$ as the *mean* and *standard deviation* 3. The next note σ_2 is chosen according to this distribution. If the next note is out of the bounds of Σ (that is higher than \bar{N} or lower than S), then the procedure is repeated until a valid note is generated. Again a normal distribution is considered for the generation of the next note, but this time with mean

²Songs composed by Rabindranath Tagore (1861-1941) are known as *Rabindra Sangeet* or Tagore songs.



(a) Randomly generated composition of length 1000, $w=5$



(b) A mixed composition consisting of alternate sequences of a musical piece and random sequence, $w=5$

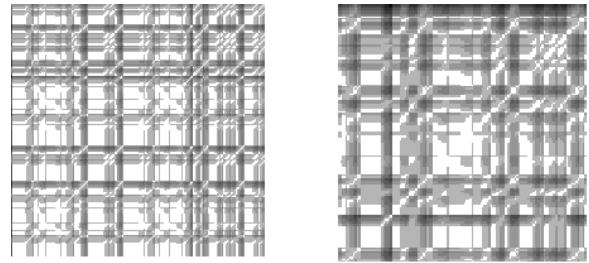
Fig. 3. Self-similarity plots of musical pieces versus random sequences

$id(\sigma_2)$. In this way the whole sequence is constructed.

Thus, from the properties of normal distribution, we can expect that for two adjacent notes σ_i and σ_{i+1} , $id(\sigma_i) - id(\sigma_{i+1})$ will be within ± 3 for 68% cases, ± 6 for 95% cases and almost always (99.7%) within ± 9 . Random compositions of length 1000 were constructed based on this model. Fig. 3(a) shows the self-similarity plot of such a composition. Although the checker-board pattern is present, the pattern is uniform and the image is much darker than the plots of musical composition for same window size (such as fig. 1) implying less self-similarity compared to musical compositions.

Fig. 3(b) is even more interesting. It is the self-similarity plot of a mixed composition. Two sequences from a musical piece (Tagore song of Fig. 1) were interleaved with randomly generated sequences of comparable length. The portions are clearly demarcated in the image with alternate brighter and darker blocks corresponding to the musical piece and random sequences respectively.

It is tempting to conclude from the above findings that self-similarity plots can indeed distinguish between musical and non-musical compositions; or in other words “music does have a pattern”. However, we should be a little more conservative in claiming so. The findings only show that we can distinguish pieces generated by the random model described here from musical composition. We do not have enough reasons to believe that any sequence of notes which has a sufficiently “interesting” self-similarity plot is musical. Non-musical pieces can yield quite beautiful self-similarity pattern and moreover the concepts of “musicality” and “interesting” or “beautiful” patterns are very subjective.



(a) Size = 1/25

(b) Size = 1/100

Fig. 4. Self-similarity plot of fig. 1 zoomed in

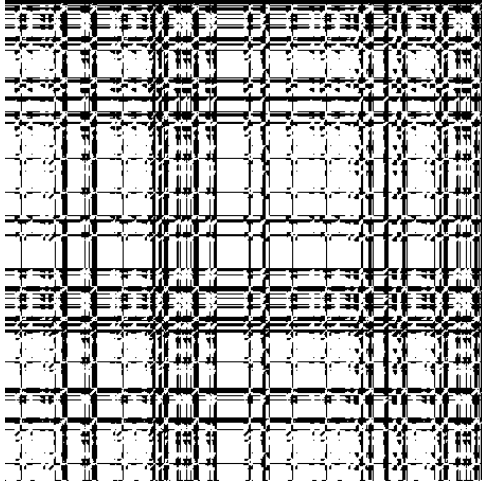
III. CAN FRACTALS SHOW THE WAY?

A. Patterns within Patterns

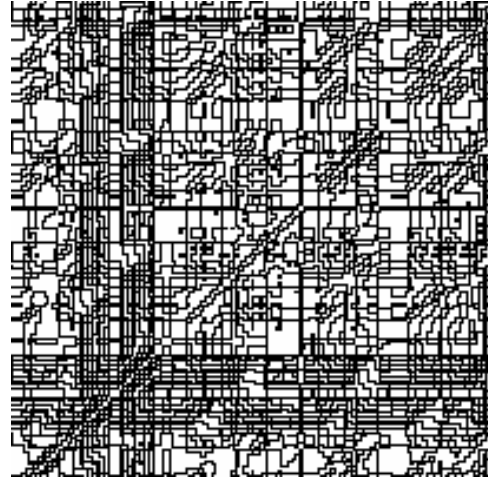
The self-similarity plots have an intriguingly complex pattern. The patterns are repeated at different levels of detail. Fig. 4 shows some part of the self-similarity plot of Fig. 1 zoomed by several times. The repetition of the same pattern at the different levels is apparent. Therefore, it is quite possible that the SSPs are fractals.

In order to verify the fractal nature of SSP, their *fractal dimensions* must be measured, for which the gray images should be mapped to some appropriate binary image. Two different approaches were tried.

- **Thresholding:** The mean value of the self-similarity matrix $S[i][j]$ was set as the threshold value. The entries above the threshold value were mapped to 1 and correspondingly colored black in the SSP and the entries below the threshold were mapped to 0 and colored white. Figure 5(a) shows the SSP of Figure 1 after thresholding.



(a) SSP of a fig. 1 after thresholding. The image is only 1/4 part of the SSP.



(b) SSP of a Tagore song at $w=9$, after skeletonization. The image is only 1/2500 part of the SSP.

Fig. 5. Approaches for converting the gray SSP to a binary image for measuring fractal dimension

- **Skeletonization:** The idea behind skeletonization³ is to extract all the edges from the SSP and include them in the binary version. The algorithm for skeletonization is as follows. If the four neighbouring points $(i, j+1)$, $(i+1, j)$, $(i-1, j)$ and $(i, j-1)$ of a point (i, j) and the point itself have the same color, then the point is colored white; otherwise it is colored black. Fig. 5(b) shows the skeletonized version of a musical composition. It is only 1/2500 part of the original SSP.

It was found that *thresholding* resulted in clustering of points and hence, loss of details. The *box counting method* for measuring the fractal dimension of such plots ended up in measuring the area covered by black points. Thus, the fractal nature of the patterns were lost due to thresholding. On the other hand, *skeletonization* preserved the details and consequently the fractal nature of the SSP. Therefore, skeletonization was chosen for further experimentation on the fractal nature of SSPs.

B. Measuring Fractal Dimensions

The *box-counting method* was used to calculate the fractal dimension (also called the *box-counting dimension* due to the method used) of the skeletonized SSP. This is a standard method⁴ and therefore, the basic details will be skipped here. One problem with the box-counting method when applied on a real image consisting of only edges (as in the case of skeletonized SSP) is that the dimension abruptly drops to one when the box-size becomes comparable to a single pixel. Therefore, the log-log plot was constructed using the

³Skeletonization is a technical term in image processing and here it refers to a similar but not the same idea.

⁴[9] provides a good introduction to fractals and the box-counting dimension (pp 177-179).

values near the knee point (just before abrupt drop in the box-counting dimension); the plot was almost a straight line. The slope of the best fitting straight line was considered to be the fractal dimension of the plot.

The fractal dimension of SSPs were found to depend on the window size. For small (around 5) window sizes, musical compositions have fractal dimensions between 1.7 to 1.4, whereas randomly generated sequences have dimensions greater than 1.95. For large (around 50) window sizes, the values are between 1.2 to 1.6 for musical compositions and around 1.8 for random sequences.

C. Signature of a Tune

The fractal dimension changes with the window size of the SSP. The SSPs of a Tagore song for window sizes 5 and 25 are shown in figures 1 and 2(a) respectively. It can be seen that as the window size increases, the image becomes darker and the density of the edges decreases. Therefore, in general, the fractal dimension decreases as the window size increases. Figure 6 shows the variation of fractal dimension with window size for the composition of figure 1.

Window size versus fractal dimension curves were plotted for seven compositions from different genres - 3 compositions by Tagore, 2 compositions based on Hindustani classical music (Raga-based), 1 western pop and 1 Hindi movie melody composed by R. D. Burman with a fusional touch. Such curves were also plotted for two randomly generated note sequences. Figure 7 shows the curves for different compositions. The distinction between the random note sequences and musical compositions is clear. It seems that this curve can also discriminate between different genres. For example, the curves for fusional and pop music is largely separated from the two compositions based on Indian classical music, which have

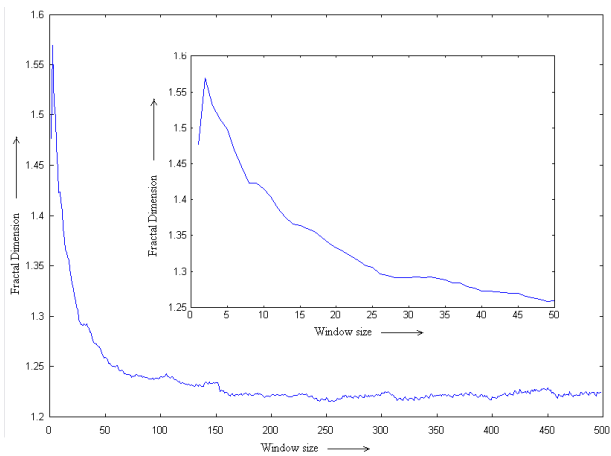


Fig. 6. Fractal dimension of SSP versus window size for the composition of figure 1. In inset: the curve from window size 1 to 50 zoomed in.

almost overlapping curves. Surprisingly, the curve for one of the compositions by Tagore (Tagore1) is very close to the two curves for *raga*-based compositions. On further investigation, it was discovered that the composition "Tagore 1" is also a *raga*-based composition.

Thus, we have enough evidence to believe that the window size versus fractal dimension curve for SSPs is quite informative and representative of a particular composition. We define this curve as the *signature* of a composition. On the basis of our findings, we can conclude that the *signature* of a tune can be used for classification of compositions, however, it is not clear on what basis (mood, genre, tempo etc.) the compositions will get classified. Therefore, further investigation is required to answer this question. One can take a large number of compositions subscribing to different genres, moods etc., automatically cluster them on the basis of their *signatures* and manually investigate each of the clusters to identify the basis of clustering.

IV. CONCLUDING NOTES

In this paper, we described some of the experiments carried out to investigate the nature of musicality. We defined a musical composition as a sequence of notes, constructed the SSP on the basis of a similarity metric and observed the fractal nature of SSPs. We also found that the SSP of a musical composition significantly differs from the SSP of a randomly generated sequence of notes.

In order to quantify the properties of SSPs, we measured their fractal dimensions and defined the *signature* of a tune based on the window-size versus fractal dimension plots of SSPs. Experiments on a small set of compositions show that the *signature* of a composition is helpful in its characterization and classification.

Nevertheless, many questions remain unanswered. We do not know on what basis the signature can classify a composition. A composition and its *reverse* (if we go up by two notes in a composition, we go down by two in the reverse composition and so on) will have identical SSPs due to the *even* nature of the Euclidean distance function. Does that imply the reverse of a composition will sound similar to the composition? At the least, can we say that the reverse of a composition will be musical? Most importantly, there is no absolute notion of musicality; ones socio-cultural background and aesthetic senses play an important role in determining the musicality of a composition to him/her. How do we capture this subjectivity in this approach?

All these issues should be systematically investigated. Moreover, in this work, only the sequence of notes has been considered for defining a composition. A composition is much more than the sequence of notes. Therefore, the experiments described in this paper can be repeated with audio-signals as input. The findings from such experiments can be used to develop smart music retrieval systems.

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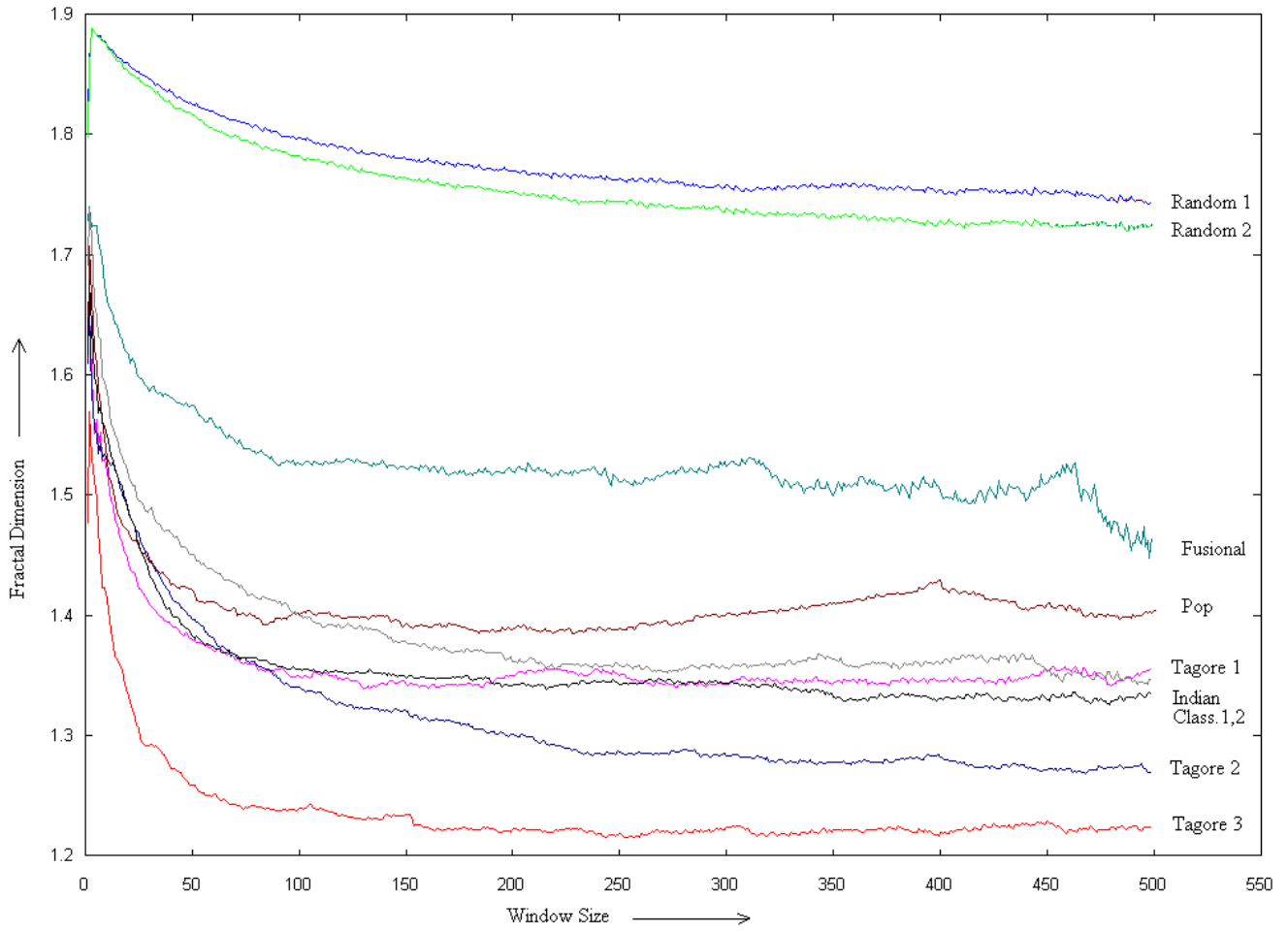


Fig. 7. Window size versus fractal dimension plots (signature) of different types of compositions.