An Empirical Evaluation of Sketched SVD and its Application to Leverage Score Ordering

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Two Motivating Questions

1. Is there a better way to generate mini-batch than random shuffle?
2. Do matrix sketching methods for leverage scores work in real life?
3. (We were taking a class by the David Woodruff of matrix sketching.)
Outline

1. Motivation
2. Empirical Evaluation of Sketched Leverage Scores
3. Sketched Leverage Score Ordering
4. Experiments of SLSO
5. Conclusion
Statistical Leverage Score

The statistical leverage score measures how much an outlier the data point is from other points in data matrix $A$, assuming a linear data model.

Naïve computation requires a costly matrix inversion. Fortunately, the techniques of matrix sketching can be used to approximate the leverage scores [Drineas, 2011].

<table>
<thead>
<tr>
<th>Algorithm 1</th>
<th>Exact Leverage Score by SVD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Input</strong></td>
<td>Given $N \times D$ matrix $A$</td>
</tr>
<tr>
<td><strong>Output</strong></td>
<td>Leverage score of $i$th row as $l_i$</td>
</tr>
<tr>
<td></td>
<td>Compute SVD, $A = U\Sigma V^T$</td>
</tr>
<tr>
<td></td>
<td>Compute $l_i$ from the first $D$ columns of the $i$th row $l_i =</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Algorithm 2</th>
<th>Approximation Leverage Score by Sketching</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Input</strong></td>
<td>Given $N \times D$ matrix $A$</td>
</tr>
<tr>
<td><strong>Output</strong></td>
<td>Approximate Leverage score of $i$th row as $l_i$</td>
</tr>
<tr>
<td></td>
<td>Compute Sketch of $A$, $SA$</td>
</tr>
<tr>
<td></td>
<td>Compute SVD, $SA = U\Sigma V^T$</td>
</tr>
<tr>
<td></td>
<td>Compute $U_{approx} = AV^T\Sigma^{-1}$</td>
</tr>
<tr>
<td></td>
<td>Compute $l_i$ from the first $D$ columns of the $i$th row $l_i =</td>
</tr>
</tbody>
</table>
Sketching Techniques Evaluated

1. Random Gaussian [Woodruff, 2014]
2. Fast Johnson Lindenstrauss transform (FJLT) [Sarlos, 2006]
3. Subsampled Randomized Hadamard Transform (SRHT) [Boutsidis and Gittens, 2012]
4. CountSketch [Meng and Mahoney, 2013]
5. OSNAP [Nelson and Nguyen, 2013]

These sketching techniques have the ”subspace embedding property”
Empirical Evaluation of Sketched Leverage Score

Number of Instances vs Number of Features for Image Datasets

Most of the real world data sets do not meet the row-dim requirements for even CountSketch!
Empirical Evaluation of Sketched Leverage Score

- Quality of the sketch is highly dependent on the column rank of the data.
- Approximation error can be unbounded if the column rank < column dimension
- Requiring a full column rank matrix for sketching is a **hard requirement** [Clarkson and Woodruff, 2017].
Empirical Evaluation of Sketched Leverage Score

Most real data, like MNIST, have a high rank, low power noise. => The data “appears” full column rank
Insights from Sketched Leverage Scores

1. SRHT requirement of having rows as a power of 2 was impractical. The memory and timing overheads required to satisfy that requirement is prohibitive.

2. OSNAP is more useful for real world data as it requires less rows $O(d)$.

3. Most curated real world datasets do not have enough rows ($n$) to satisfy the CountSketch (most analyzed) requirements.

4. Quality of the sketch is highly dependent on the column rank of the data. If the column rank of the data is less than the column dimension, then the approximation error can be unbounded.

5. Small singular values in real data can corrupt the approximate leverage scores returned by sketching. This happens often as real data have high rank noise.
Approximate Leverage Score with Truncation

**Input**: Given an $n \times d$ matrix $A$, threshold $\epsilon$.

**Output**: Approximate Leverage score of $i$th row as $l_i$.

2. Compute SVD, $SA = U\Sigma V^T$.
3. Truncate $V$, $\Sigma$ at small singular values less than $\epsilon$. Let this truncated matrices be $V'$, $\Sigma'$.
4. Compute $U_{approx} = AV'^T\Sigma'^{-1}$.
5. Compute $l_i$ from the first $d$ columns of the $i$th row $l_i = |U_{i,approx}|_2^2$. 

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**References**

- ACML 2018
Sketched Leverage Score Ordering

Using ideas from curriculum learning, the sketched leverage score is used to generate sampling policies to order the training data based on their leverage scores:

1. **Decreasing (dec):**
   Ordered based on strictly decreasing leverage scores. The most important and diverse training points are seen first.

2. **decreasing, sampling with replacement** (dec, swr):
   The most important and diverse training points are seen first, but with randomness introduced.

3. **decreasing, sampling without replacement** (dec, swor):
   Same but sampling without replacement.

4. **Shuffle** (Shuffle):
   Baseline order where models are trained on shuffled training data.
Experiments Setup

3 different datasets for Deep Learning. Models and hyper parameters kept constant across runs.

**MNIST**: image classification on digits handwriting.

**SST**: sentiment classification.

**CMU-MOSI**: multimodal sentiment analysis over 2 different runs using an early fusion method.
## Experiments Results

### MNIST 10 Class Image Classification Accuracy (%)

<table>
<thead>
<tr>
<th>Method</th>
<th>LR</th>
<th>NN Small</th>
<th>NN Large</th>
<th>CNN Small</th>
<th>CNN Large</th>
</tr>
</thead>
<tbody>
<tr>
<td>shuffle</td>
<td>92.84</td>
<td>98.50</td>
<td>98.28</td>
<td>98.98</td>
<td>99.34</td>
</tr>
<tr>
<td>dec</td>
<td>89.09</td>
<td>98.43</td>
<td>98.34</td>
<td>99.01</td>
<td>98.99</td>
</tr>
<tr>
<td>dec, swr</td>
<td>92.70</td>
<td>98.42</td>
<td>98.46</td>
<td>99.01</td>
<td>99.35</td>
</tr>
<tr>
<td>dec, swor</td>
<td><strong>92.88</strong></td>
<td><strong>98.55</strong></td>
<td><strong>98.57</strong></td>
<td><strong>99.01</strong></td>
<td><strong>99.39</strong></td>
</tr>
</tbody>
</table>

MNIST, “decreasing order without sampling” improves performance.

### SST 5 Class Sentiment Classification Accuracy (%)

<table>
<thead>
<tr>
<th>Method</th>
<th>LR</th>
<th>DAN Small</th>
<th>DAN Large</th>
<th>LSTM Small</th>
<th>LSTM Large</th>
</tr>
</thead>
<tbody>
<tr>
<td>shuffle</td>
<td>42.22</td>
<td>39.68</td>
<td>40.27</td>
<td>42.35</td>
<td>41.67</td>
</tr>
<tr>
<td>dec</td>
<td><strong>42.94</strong></td>
<td>39.86</td>
<td><strong>42.76</strong></td>
<td>42.35</td>
<td>41.31</td>
</tr>
<tr>
<td>dec, swr</td>
<td>41.22</td>
<td>40.72</td>
<td>40.72</td>
<td><strong>42.44</strong></td>
<td><strong>43.71</strong></td>
</tr>
<tr>
<td>dec, swor</td>
<td>42.26</td>
<td>39.41</td>
<td>40.14</td>
<td>41.27</td>
<td>41.36</td>
</tr>
</tbody>
</table>

SST, “decreasing order with replacement” improves performance.

### CMU-MOSI Sentiment Analysis

<table>
<thead>
<tr>
<th>Metric</th>
<th>DAN Large</th>
<th>LSTM Small</th>
<th>LSTM Large</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A²</td>
<td>F1</td>
<td>MAE</td>
</tr>
<tr>
<td>shuffle</td>
<td>61.2</td>
<td>59.9</td>
<td>1.314</td>
</tr>
<tr>
<td>dec</td>
<td>59.0</td>
<td>56.0</td>
<td>1.365</td>
</tr>
<tr>
<td>dec, swr</td>
<td>60.1</td>
<td>58.0</td>
<td>1.336</td>
</tr>
<tr>
<td>dec, swor</td>
<td><strong>64.3</strong></td>
<td><strong>64.3</strong></td>
<td><strong>1.271</strong></td>
</tr>
</tbody>
</table>

CMU-MOSI, “sampling in a decreasing order” improves accuracies.

Decreasing SLSO improves performance compared to random shuffle!
Why Should this Even Work?

**Curriculum Learning**
manually determining a training order (Bengio et al., 2009; Spitkovsky et al., 2010; Khan et al., 2011)
Using auxiliary networks to jointly optimize for the optimal training order (Kumar et al., 2010; Jiang et al., 2015).

**Solving Linear System**
Randomized Coordinate Descent, Analysis of the Kaczmarz Method [Gower et al, 2015]
Why Should this Even Work?

Comparison of Leverage Scores for Neural Net Features (Training Data)

MNIST data
Mid layer vs Last Layer
Two Motivating Questions

1. Is there a better way to generate mini-batch than random shuffle?
   Yes! SLSO is one approach.

2. Do matrix sketching methods for leverage scores work in real life?
   Only OSNAP is feasible for DL.
Conclusion

Empirical evaluation of Sketched Leverage Scores, identifying the limitations of matrix sketching to real world problems and guidelines for real world implementation.

Sketched Leverage Score Ordering, a technique for determining the ordering of data in the training of neural networks by computing leverage scores efficiently via truncated sketching.

Our method shows improvements in convergence and results.
Thanks for your attention!

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