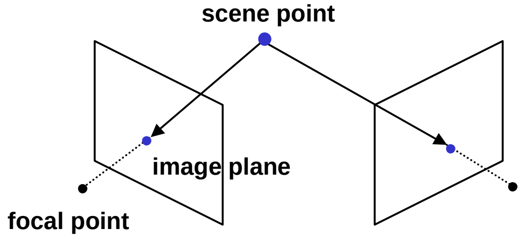
Stereo and Motion

The Stereo Problem

- Reconstruct scene geometry from two or more *calibrated* images



Stereo

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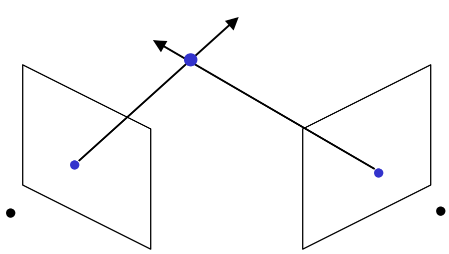


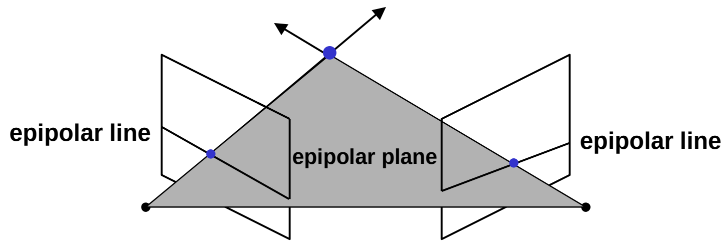
Basic Principle: Triangulation

- Gives reconstruction as intersection of two rays
- Requires *point correspondence*
  - This is the hard part

Stereo Correspondence

Determine Pixel Correspondence

- Pairs of points that correspond to same scene point



Epipolar Constraint

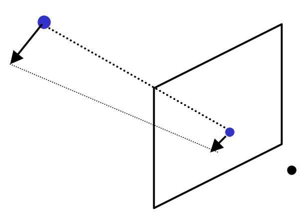
- Reduces correspondence problem to 1D search along *conjugate epipolar lines*
- Stereo rectification: make epipolar lines horizontal
  - this is what the prewarp did in *view morphing*

Correspondence and Optical Flow

Stereo requires just 1D motion estimation

But in general the motion field is 2D

- Epipolar lines not known in advance
- Non-rigid motion (no epipolar lines)



True motion field: projected point displacements

Optical flow is *apparent* motion in the image

- Generally these will not be the same

The Aperture Problem

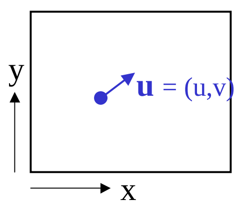
We can't measure the true 2D motion field from local image measurements

Example: *Barber Pole Illusion*

http://www.sandlotscience.com

Optical Flow Equation

Several of the following slides adapted from P. Anandan, 1999



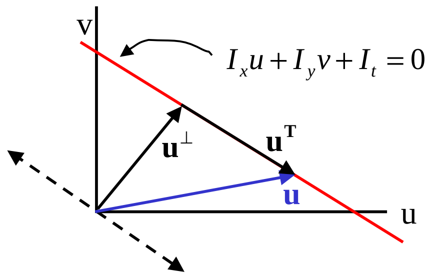
Assumptions

- Brightness Constancy: intensity *I* of a moving point is constant over time
- Pixel intensity is linear in t (for small time steps)

$$I(x, y, t) = I(x + u\delta t, y + v\delta t, t + \delta t)$$

$$\begin{aligned} 0 &= \frac{dI}{dt} \\ &= \frac{\partial I}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial I}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial I}{\partial t} \\ &= I_x u + I_y v + I_t \\ &= \nabla I \bullet \mathbf{u} + I_t \end{aligned}$$

Normal Flow

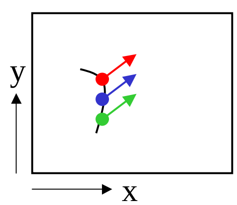


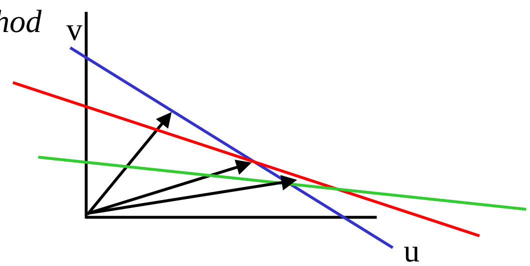
Optical flow equation is a line constraint

- Normal component $\mathbf{u}^{\perp}$ can be computed
- Tangent component $\mathbf{u}^{\mathbf{T}}$ is undefined

Integrating Neighborhood Information

Lucas and Kanade Method





We want to minimize: $\sum_i (I_x^i u + I_y^i v + I_t^i)^2$

This corresponds to solving:

$$\begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} -\sum I_x I_t \\ -\sum I_y I_t \end{bmatrix}$$

Matrix on the left is singular if all gradients point in same direction

(i.e., if points are on a line--just get normal flow)

Limits of the Gradient Method

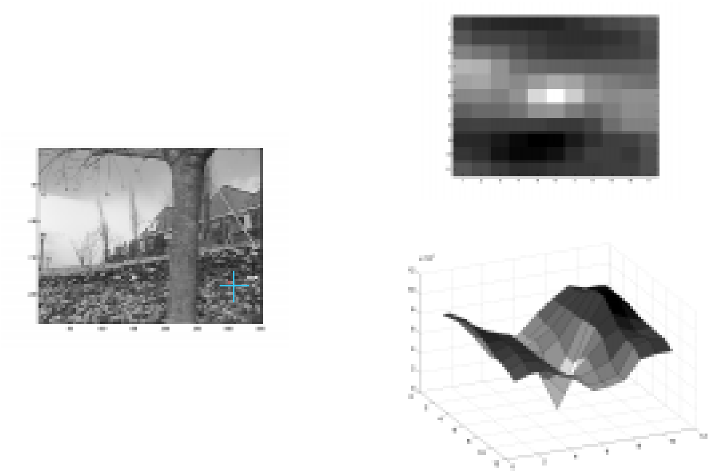
Fails When

- Not enough variation in local neighborhood
- Motion is large (much greater than a pixel)
  - Linear brightness assumption is not met

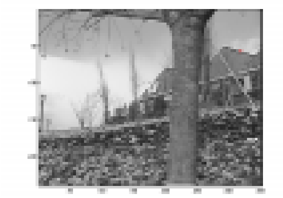
For larger displacements, match templates instead

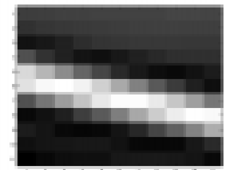
- Define a small area around a pixel as the template
- Search locally for template in next image
- Use a match measure such as correlation, normalized correlation, or sum-of-squares difference (SSD)
- Choose the maximum (or minimum) as the match
- Window size is important
  - small windows lead to false matches
  - big windows lead to over-smoothing

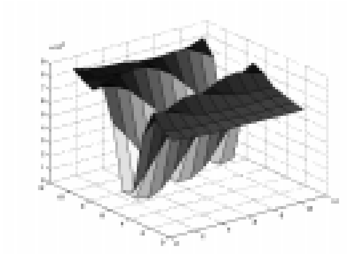
SSD Surface – Textured area



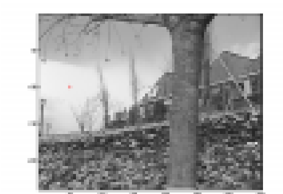
SSD Surface -- Edge

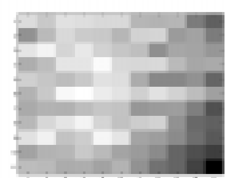


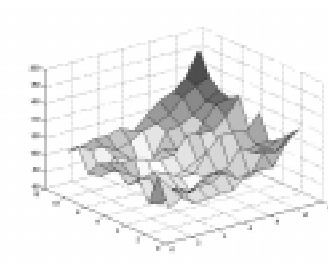




SSD Surface – homogeneous area





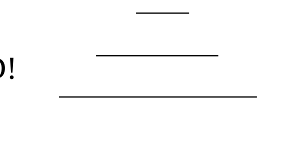


Coarse to Fine Estimation

First use large windows and search over large displacement range
Refine these estimates using smaller windows

Can do this more efficiently by using:

A PYRAMID!



Steps:

- Convolve image with a small kernel
  - Typically 5x5 Gaussian or Laplacian filter
- Subsample to get lower resolution image
- Repeat for more levels

Result:

- A sequence of low-pass or band-pass filtered images

Pyramids

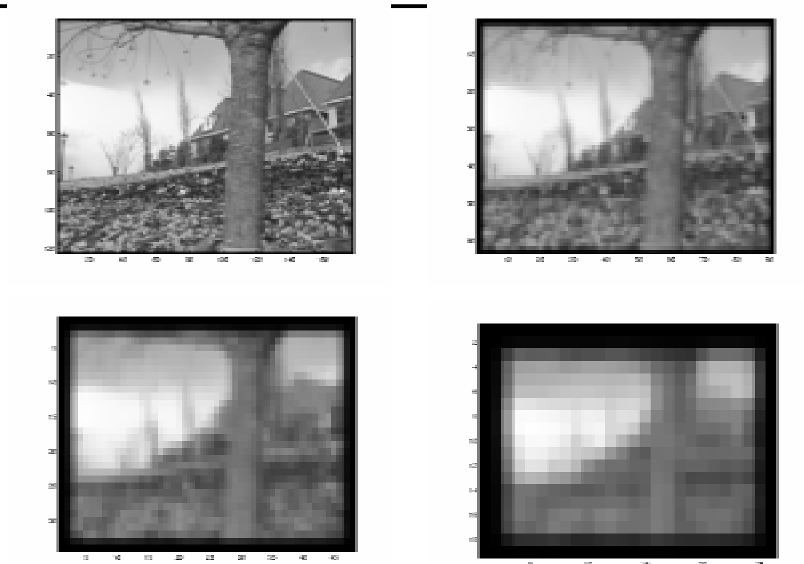
Pyramids were introduced as a multi-resolution image computation paradigm in the early 80s.

The most popular pyramid is the Burt pyramid, which foreshadows wavelets

Two kinds of pyramids:

Low pass or “Gaussian pyramid”

Band-pass or “Laplacian pyramid”



Coarse-to-Fine Flow Estimation (Anandan)

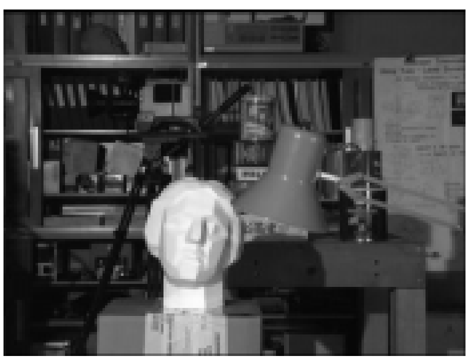
Construct pyramids from each image (Gaussian)

Start at coarsest level, initialize flow to 0

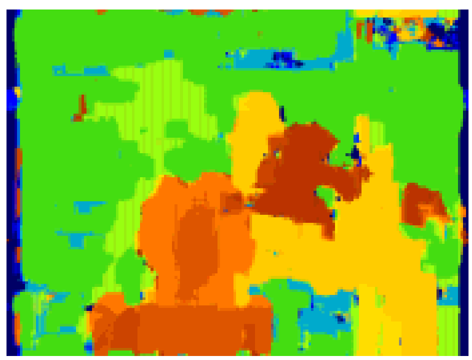
1. Do local search (3x3 or 5x5 area) using small (5x5) templates
2. Around the peak perform subpixel refinement
   1. Either analytically, using the Lucas-Kanade formulation or
   2. Numerically by fitting quadratic surface to the peak and interpolating to find the sub-pixel peak
3. Warp one image toward the other using the flow field
4. Repeat steps 1,2, and 3 a few times (usually 5-10)
5. Project the flow field to next finer level
6. Move to the next finer level and repeat 1-5.

Stop when you finish the iterations at the finest level

Stereo Matching



Stereo Pair

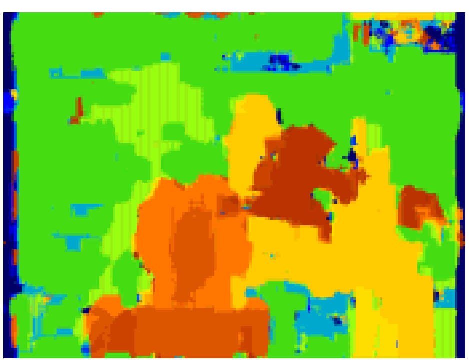


Quantized Depth Map

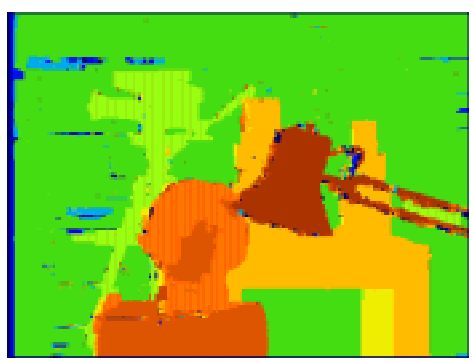
normalized cross-correlation search

Stereo Matching Algorithms

- Pitfalls
  - specularities (non-Lambertian surfaces)
  - ambiguity (aperture problem, low-contrast regions)
  - missing data (occlusions)
  - intensity error (quantization, sensor error)
  - position error (camera calibration)
- Numerous approaches
  - course-to-fine [Anandan 89]
  - edge-based [Marr-Poggio]
  - dynamic programming [Baker-Binford 81]
  - MRF’s, graph cuts [Zabih]
  - adaptive windows [Kanade 91]
  - multi-baseline [Okutomi 93]
  - many more...



Normalized cross-correlation

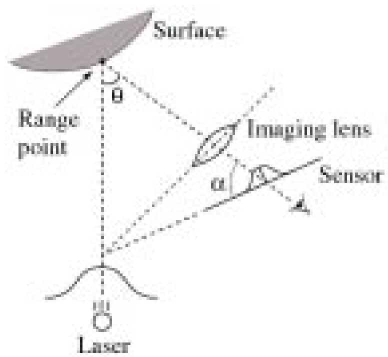


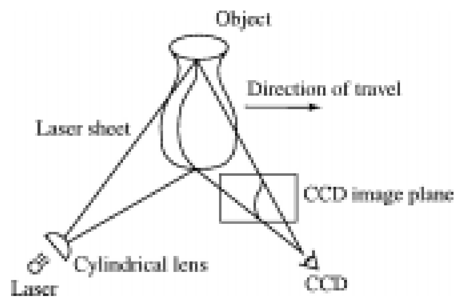
Graph cuts [Zabih 99]

Active Stereo (Laser Scanning)

One way to solve the aperture problem

- Create your own texture by projecting light patterns onto the object
- Most precise way is to use a laser
- Triangulate as before, but between laser and sensor

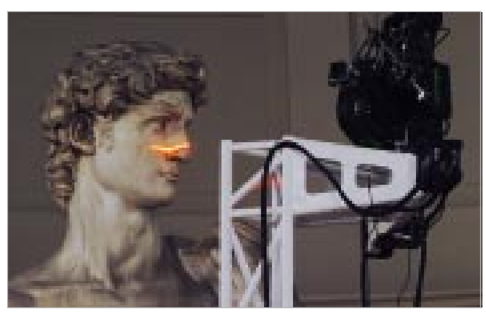




Figures by Brian Curless, 1999

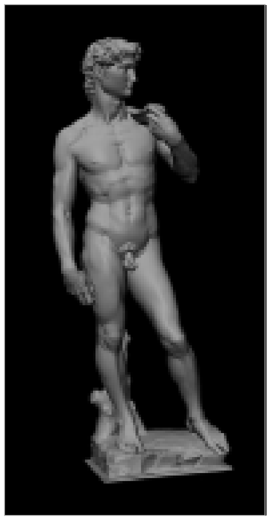
Stanford's Digital Michelangelo Project

http://graphics.stanford.edu/projects/mich/



maximum height of gantry: 7.5 meters
weight including subbase: 800 kilograms

Statistics about the scan



480 individually aimed scans
2 billion polygons
7,000 color images
32 gigabytes
30 nights of scanning
1,080 man-hours
22 people