Lecture 5.5:
Multiplying By A Global Phase Doesn’t Matter
Discriminating Two Quantum States at Angle $\theta$

Two-sided error: $\frac{1}{2} - \frac{1}{2} \sin \theta$

One-sided error: $1 - (\sin \theta)^2$

Zero-sided error: $1 - \frac{(\sin \theta)^2}{2}$
Discriminating Two Quantum States at Angle $\theta$

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Error prob.
The state $|u\rangle$ and the state $-|u\rangle$ are **indistinguishable**.

Given an unknown qubit $|\psi\rangle$, promised to be either $|u\rangle$ or $-|u\rangle$, there is **no physical experiment** you can do to distinguish them.
The state $|0\rangle$ and the state $-|0\rangle$ are indistinguishable.
You’ll read out “\( |0\rangle \)” with probability **100%**, and nothing will change.

Could measure \( |\psi\rangle \).
The state $|u\rangle$ and the state $-|u\rangle$ are **indistinguishable**.

*Also:* The state $|u\rangle$ and the state $i|u\rangle$ are **indistinguishable**.

*Also:* The state $|u\rangle$ and the state $c|u\rangle$ are **indistinguishable** whenever $c$ is a complex number of magnitude 1.

Such a $c$ is called a “global phase”.
The state $|u\rangle$ and the state $-|u\rangle$ are indistinguishable.

Means our notation for quantum states is slightly clunky. Would be better if indistinguishable states had identical notations. In fact, when we later study “mixed quantum states”, this clunkiness will be fixed!
The following “mixed quantum states” are also indistinguishable...!

**Scenario ρ₁**
A fair coin is flipped.
If Heads: $|ψ⟩$ set to $|0⟩$
If Tails: $|ψ⟩$ set to $|1⟩$

**Scenario ρ₂**
A fair coin is flipped.
If Heads: $|ψ⟩$ set to $|+⟩$
If Tails: $|ψ⟩$ set to $|−⟩$