Endogenously Formed Communities

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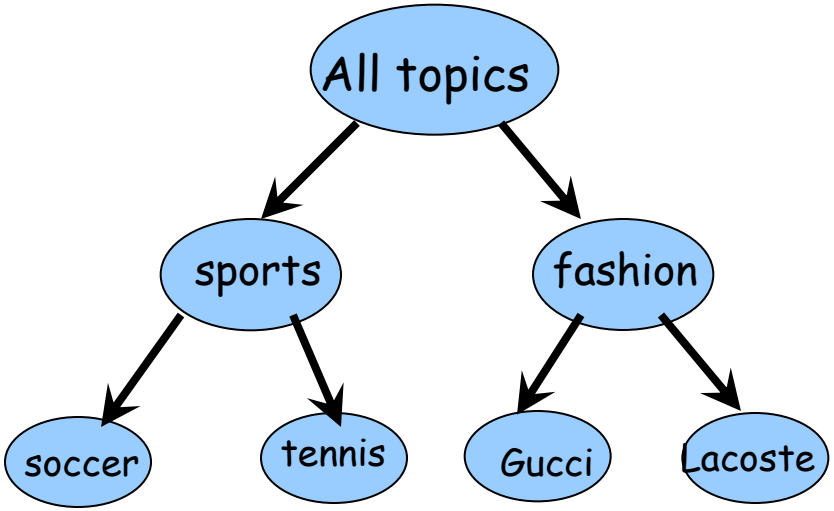
Unsupervised Learning/Clustering

- Extensively studied in many fields.
- Classic goals: output a partition of the data.

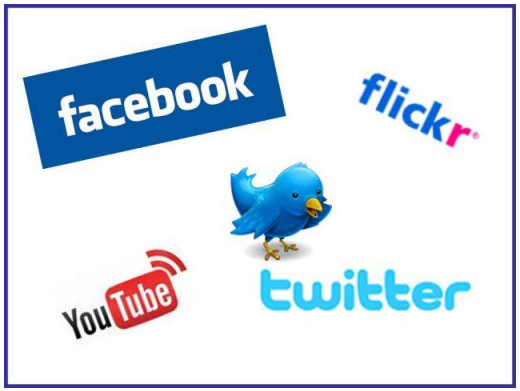
Unsupervised Learning/Clustering

- Extensively studied in many fields.
- Classic goals: hierarchical clustering



Overlapping communities

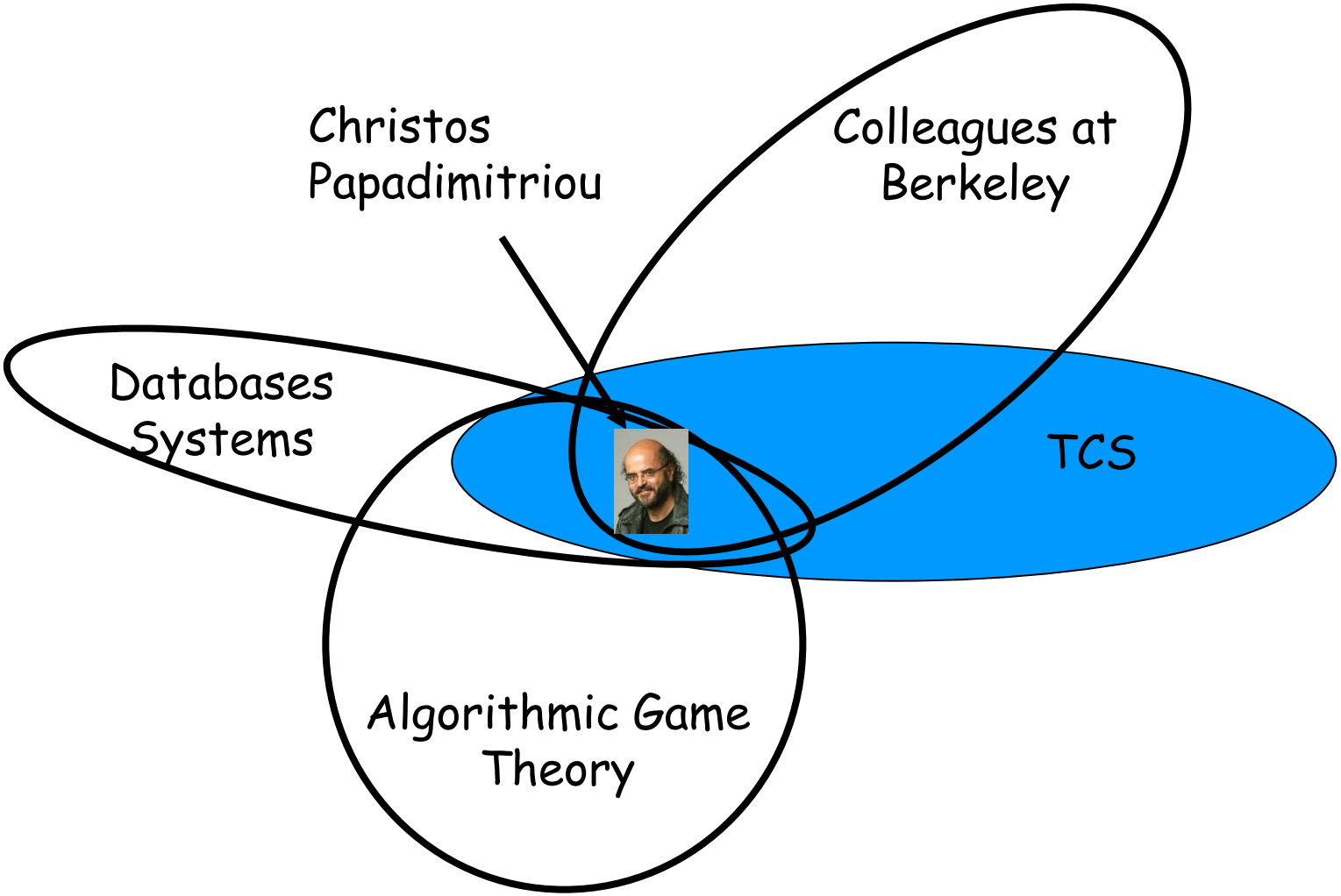
- Social networks



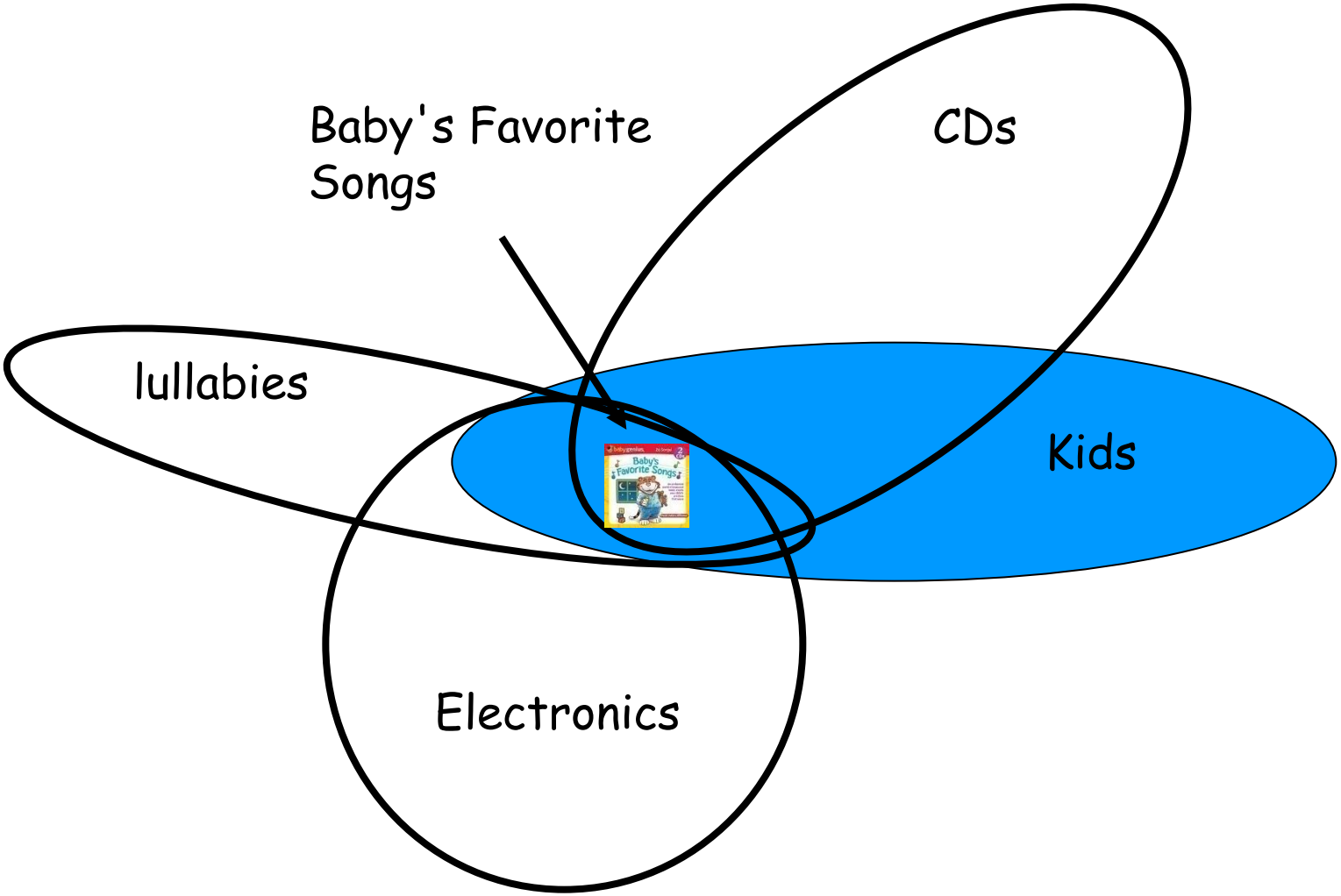
- Professional networks

- Product Purchasing Networks, Citation Networks, Biological Networks, etc

Overlapping communities



Overlapping communities



Overlapping communities

- Used usually as preprocessing step for data analysis or decision making.

Open Question: rigorous & natural notions; algorithms for finding all of them.

Prior Work:

- Various heuristics and optimization criteria [N'06, K11]
- No general guarantees on # and time needed to find communities meeting natural criteria [MSST07]

Self-Determined Communities in General Affinity Systems

Affinity systems

Basic model (ordinal): (V=[n], π_1, π_2, …, π_n)

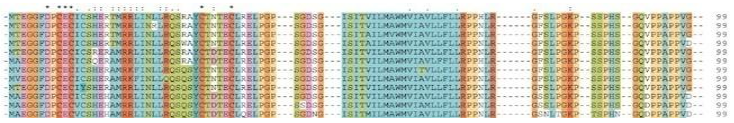
Weighted affinity systems: (V=[n], a_1, a_2, …, a_n)

$a_{i,j} \in [0,1]$ – affinity of member i for member j

Arise in different areas:

- social sciences
- social networks

- Data mining (e.g., documents, DNA sequences, etc.)



Self Determined Communities in Affinity Systems

Basic model (ordinal): $(V=[n], \pi_1, \pi_2, \ldots, \pi_n)$

$S \subseteq V$ self-determined community if members of S collectively prefer each other to anyone else outside the community

- # votes i in S receives from members in $S \geq$ # of votes j not in S receives from S
- each i in S casts a vote for his $|S|$ most preferred members

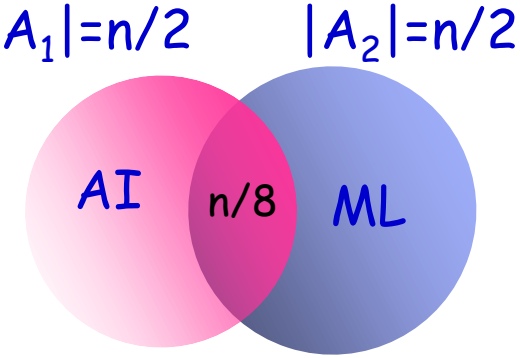
Different communities have different degrees of robustness

Self Determined Communities in Affinity Systems

Definition $S \subseteq V$ (θ, α, β) self-determined community if

- if each i in S receives $\geq \alpha$ |S| votes from members of S
- if each j not in S receives $\leq \beta$ |S| votes from members of S
- each i in S casts a vote for its θ |S| most preferred members

Allows for overlapping communities



- Each s in $A_i \setminus A_j$ ranks elements in A_i first
- A_1, A_2 are (1, 3/4, 1/4) self-determined comm.

Given a set S, easy to efficiently determine whether or not S is SD.

Self Determined Communities, Main Results

- A multi-stage approach that leads to a poly time algorithm for finding all communities if θ, α, β are constant.

- Local procedure: for $\alpha \geq \frac{1}{2}$, given a random v in community S, with prob. $\Omega(2\alpha - 1)$ recovers S in time $O(|S| \log|S|)$.

- Weighted affinity systems, Multi-facet affinity systems.

- Connections to (α, β) clusters [Mishra, Schreiber, Staton, Tarjan]
  - We prove there exists network with superpoly # of (α, β) clusters and even finding one as hard as hidden clique

Self Determined Communities in Affinity Systems

Theorem

Given t=|S| find, output a list L that whp contains S in time:

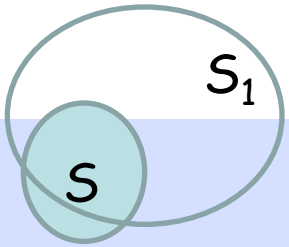
$$n^{O(\log(1/\gamma)/\alpha)} \left(\frac{\theta \log(1/\gamma)}{\alpha}\right)^{O\left(\frac{1}{\gamma^2}\log\left(\frac{\theta(1/\gamma)}{\alpha\gamma}\right)\right)}$$

Leads to a poly time algorithm for finding all communities, when all parameters are constant.

Multi-Stage Approach

Input: Info I about unknown community S (e.g., |S|).

Output: List L of subsets of V.



Generate Rough Approximations Step

- Generate a list L_1 of sets S_1 s.t. at least one of them is a rough approximation to S.

Purification Step

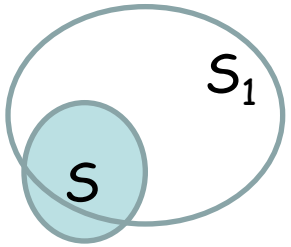
- Run a purification procedure to generate a list L s.t. at least one of the elements in L is identical to S.

Eliminate from L sets that are not self-determined.

Self Determined Communities in Affinity Systems

Key Fact $k_1 = \log(1/\gamma)/\alpha$

$\exists\ i_1, \ldots, i_{k\_1}$ in S s. t. the union of their votes contains $\geq 1-\gamma/16$ fraction of S.



$|S_1| \leq k_1\ \theta\ t$, $|S|=t$

Proof

Any $\tilde{S} \subseteq S$ the receives at least $\alpha|S|\ |\tilde{S}|$ votes from S, so $\exists\ i_{\tilde{S}} \in S$ that votes for at least $\alpha\ |\tilde{S}|$ members of $\tilde{S}$.

The existence of $i_1, \ldots, i_M$ proven in a greedy fashion.

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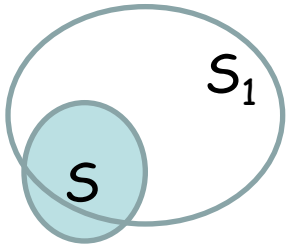


$|S_1| \leq k_1\, \theta\, t$, $|S|=t$

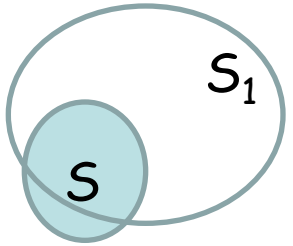
Generate Rough Approximations Step

- Search over all sets U, $|U| = k_1$. For each U, let S_1 be the set of elements voted by U; add S_1 to L_1 .

Self Determined Communities in Affinity Systems

Key Fact $k_2 = O(\log(16\theta k_1/\delta\gamma)/\gamma^2)$

$|S_1| \leq k_1\,\theta\,t$, $|S|=t$



If draw U_2 a set of k_2 pts at random from $S \cap S_1$, consider S_2 set voted by $(\alpha - \gamma/2)$ fraction of U_2 , then whp

$$|\Delta(S_2,S)| \leq \gamma\, t/8.$$

Purification Step $N_2 = O((\theta k_1)^{k_2}\log(1/\delta))$

For each S_1 to L_1, repeat N_2 times

- Pick k_2 points at random from S_1 (get U_2) and let S_2 be the set voted by $(\alpha - \gamma/2)$ fraction of U_2 .
- Let S_3 be the set voted by $(\alpha - \gamma/2)$ fraction of S_2.

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Leads to a poly time algorithm for finding all communities all parameters are constant.

Local Procedure

Theorem: For $\alpha \geq \frac{1}{2}$, given a random v in community S, with prob. $\Omega(2\alpha - 1)$ recovers S in time $O(|S| \log|S|)$.

- Similar multi-stage approach; main challenge, providing a local version for rough approxs.

Note: not possible to start from any seed vertex v in S.

- e.g., if v is voted first by everyone in V 

- We show that a constant fraction of the nodes in S are sufficiently "representative" of S to enable recovering S

Local Procedure

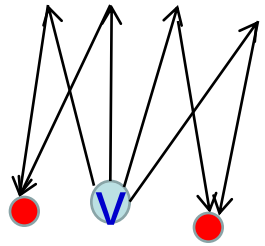
Key Fact $\eta = 2\alpha - 1$, $\exists\ T \subseteq S$, $|T| \geq \eta\, t$ s.t. for $v \in T$ and $u \in S$,

$$\Pr[R(R(v)) = u] \geq \frac{(\alpha - 1/2)/\theta^2}{t}$$

p

Generate Rough Approximations Given $v \in T$:

- Compute $R(R(v))$ for $O((1/p)\log t)$ times.
- S_1 = all u hit at least $a \log t$ times.



- Whp S_1 includes all of S, and in total of $O(t)$ points.
- This together with an analysis for the purification step, leads the local algo.

Conclusions

- Natural notion of self-determined community.
- A poly time algorithm for finding all communities if θ, α, β are constant; stronger guarantees for $\alpha \geq \frac{1}{2}$.

Open Questions

- Input affinity system is typically only a projection of the true underlying affinity system..
- Interactive community detection.