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An Evolutionary Game-theoretic Model for Ethno-religious Conflicts between Two Groups

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Abstract Ethno-religious conflict in multi-cultural societies has been one of the major causes of loss of life and property in recent history. In this paper, we present and analyze a multi-agent game theoretic model for computational study of ethno-religious conflicts in multi-cultural societies. Empirical fact-based research in sociology and conflict resolution literature have identified (a) ethno-religious identity of the population, (b) spatial structure (distribution) of the population, (c) existing history of animosity, and (d) influence of leaders as some of the salient factors causing ethno-religious violence. It has also been experimentally shown by Lumsden that multi-cultural conflict can be viewed as a Prisoner's Dilemma (PD) game. Using the above observations, we model the multi-cultural conflict problem as a variant of the repeated PD game in graphs. The graph consists of labeled nodes corresponding to the different ethno-religious types and the topology of the graph encodes the spatial distribution and interaction of the population. We assume the structure of the graph to have the statistical properties of a social network with the high degree nodes representing the leaders of the society. The agents play the game with neighbors of their opponent type and they update their strategies based on neighbors of their same type. This strategy update dynamics, where the update neighborhood is different from the game playing neighborhood, distinguishes our model from conventional models of PD games in graphs. We

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present simulation results showing the effect of various parameters of our model to the propensity of conflict in a population consisting of two ethno-religious groups. We also compare our simulation results to real data of occurrence of ethno-religious violence in Yugoslavia.

Keywords Conflict behavior · Game theory · Multi-cultural society · Multi-agent systems · Oscillation · Prisoner's dilemma · Social network · Social simulation · Steady state

1 Introduction

Ethno-religious conflict in multi-cultural societies has been one of the major causes of loss of life and property in recent history (e.g., violence in Yugoslavia [Ball et al(2002)], Sudan [Prunier(2005)]). There has been substantial empirical research in sociology and conflict resolution literature on analyzing the causes of such violence (see [Toft(2006)] and references therein). Apart from the ethno-religious identity, other social and economic factors identified in the literature in violence prone areas are economic grievances, competition for scarce natural resources, historical precedents, territorial claims, influence of political/religious elites, and international influences. The spatial structure of the different population groups is also correlated to the occurrence of violence [Toft(2006),Lim et al(2007)].

More recently, there has been focus on computational study of civil conflict in societies (see [Epstein(2002),Goh et al(2006),Srbljinovic et al(2003)] and references therein). Some works have also studied societies with multiple ethno-religious groups [Lim et al(2007),Goh et al(2006)]. Although the proposed computational models vary in detail, the common features of existing agent-based computational models are: (a) the agents are assumed to be distributed on a grid and the interactions between agents are restricted to a neighborhood around their position (usually their Moore-neighborhood, i.e., nearest 8 neighbors) and (b) the interactions between agents are non-adaptive (except in [Goh et al(2006)]), e.g., agents will die with a certain probability if they are in the neighborhood of opponent agents or they will migrate towards agents of only their type. Since it is well established that the structure of a social network of interacting population is not like a grid [Barabasi and Albert(1999)], we model the interaction topology among the agents as a graph. Moreover, we model the interaction between agents as a repeated prisoner's dilemma (PD) game where the agents update their strategies. Thus, in this paper, we present a simple multi-agent game-theoretic model for studying conflict situations in multi-cultural societies.

The primary cause of conflict in multi-cultural societies is the fear of the minority population about loss (or suppression) of their ethno-religious identity by the majority group. Lumsden demonstrated this through a series of experiments in the context of the Cyprus conflict between Greek Cypriots and Turkish Cypriots [Lumsden(1973)]. Both the Greek Cypriots and the Turkish Cypriots gave higher value to maintaining their position on *right to self-*

determination (which is directly related to the importance the groups have for their own ethno-religious identity) than to compromise even if the former meant war in the long run (that is detrimental to both) and the latter meant peace (that is beneficial to both). In other words, the payoff matrix of the conflict viewed as a 2×2 matrix game had the structure of the prisoner's dilemma (PD) game. In this paper, we use this insight of Lumsden combined with the ethno-religious identity and the spatial structure of the population to form a multi-agent game-theoretic model for studying conflict in multi-cultural societies.

We model the problem as a PD game in graphs where the nodes of the graph are the agents and the edges between the agents represent interaction between them. Each agent represents an individual or a group of people (e.g., a household or a family). The graph topology encodes the spatial distribution of the population and the interaction between the different groups of the population is abstracted as the PD game. The nodes in the graph have different labels denoting their ethno-religious identity. The agents *play the game* with members of their *opponent groups* and they *update their strategies* based on the members of *their own group*. We note that the different neighborhoods we use for game playing and strategy updating is a departure from the standard model of PD games on graphs (where the game playing and strategy updating neighborhoods are assumed to be the same). The degree of an agent node within its own group is the measure of the agent's influence within its group. Consequently, the nodes with high in-group degree are designated as the *leader nodes*. Assuming that there are two different ethno-religious types in the population, we present simulation studies (on synthetic data) showing the effect of our model parameter variations.

Contributions The main contributions of our paper are: (1) we introduce a simple and novel game theoretic model for studying ethno-religious conflict, (2) we present simulation studies showing the effect of various parameters of our model to a measure of the potential for conflict (that is defined by Equation 3), (3) we incorporate and evaluate the effect of leaders (influential agents) in our model, (4) we demonstrate the predictive ability of our model in terms of potential conflict locations by experimental evaluation on the real data of former Yugoslavia. Since the model of the PD game in graphs that we introduced is new, we also present theoretical results that (partially) characterize the long term strategy update dynamics of our model. In particular, we prove that the strategy of an agent may not reach a fixed point by showing counter-examples of graphs where the strategies of the agents oscillate.

This paper is organized as follows: In Section 2, we present a brief overview of the literature related to conflict modeling and PD games in graphs. In Section 3, we present the formal definition of PD games in graphs and other definitions used in the paper. In Section 4, we present in detail our model for multi-cultural conflict modeling based on PD game in graphs. Thereafter, in Section 5, we present some theoretical aspects of the steady state solutions of the models and in Section 6, we present our simulation results. In Section 7 we include the effect of leaders in our basic model and present simulation results showing the effects of leaders' strategies on the overall tendency of conflict. In Section 8, we do a case study of former Yugoslavia based on the demographic data to evaluate our model. In Section 9 we present a discussion of some limitations of our model and indicate ways to overcome them. Finally, in Section 10, we provide our conclusions and outline problems to be addressed in the future. A preliminary version of this work appeared in [Luo et al(2009), Luo et al(2010)].

2 Related Work

There has been a variety of social and economic causes put forward for ethnic conflicts based on empirical fact-based research [Toft(2006), Gurr(2000), Horowitz(2005)]. These causes can be divided into three generic categories [Toft(2006), Ackelson(2006)] (a) non-material causes like ethno-religious identity, culture, history of violence, mutual fear (b) material causes like uneven distribution of natural resources, uneven economic development and (c) use of ethno-religious identity based nationalism by political and religious elites. These factors are not independent of each other and can be thought of as factors that enhance the importance of ethno-religious identity in the population. Geographical factors like territorial indivisibility and the spatial distribution of the population has also been proposed as another reason for ethnic violence [Toft(2006)].

Computational modeling for understanding/analyzing ethno-religious violence has received extensive attention recently [Epstein(2002), Lim et al(2007), Goh et al(2006), Srbljinovic et al(2003), Bhavnani et al(2008)]. Although [Epstein(2002), Goh et al(2006), Srbljinovic et al(2003), Bhavnani et al(2008)] view civil violence as a result of pent-up grievances, they vary significantly in their details. [Epstein(2002), Goh et al(2006), Srbljinovic et al(2003)] look at the conflict as a function of mass mobilization whereas [Bhavnani et al(2008)] considers it as a function of economic causes. However, all the models have the common feature that they consider the agents to be distributed on a grid and interact with their neighbors in the grid graph. In [Goh et al(2006)], the authors consider the interaction between the agents as an iterated game which they call a PD game. However, their definition does not correspond to the standard definition of a PD game [Nowak and Sigmund(1989)].

To demonstrate the importance of spatial structure of the population in ethnic conflicts, Lim et al. [Lim et al(2007)] proposed a mathematical model based on the dynamics of type separation. Type separation models were originally proposed to explain pattern formation in physical or chemical phase separation processes and has been used in the social segregation context in [Schelling(1971)]. Lim at al. [Lim et al(2007)] assumed that the population consists of different types and occupy the nodes of a $2D$ grid with some empty nodes. Like agents move towards each other whereas unlike agents move away from each other. This leads to formation of patches of different types and a patch of one type surrounded by agents of another type with certain patch size is predicted as a site of violence. Thus, they demonstrate the importance of the spatial structure of the population for ethno-religious violence modeling. However, it is not apparent whether it is possible to extend this model to take into consideration other factors like effect of leaders or uneven distribution of natural resources.

Lumsden conducted experiments to elicit the structure of the Cyprus conflict in game-theoretic terms [Lumsden(1973)]. He concluded that the essential factors of the conflict can be captured as a two-party, two-choice game and showed that the payoff matrix of the game has the structure of the Prisoner's Dilemma game. The Prisoner's Dilemma game is a well known model for many social choice situations. In this two-player game, each player has two actions, cooperate (C) or defect (D). The payoff matrix for the game has the following characteristics: (a) the highest payoff is obtained by the defector against the cooperator, (b) the total payoff for mutual cooperation is the highest and (c) the defector's extra income (relative to its income for mutual cooperation) is less than the loss of the cooperator (please see Section 3 for a formal statement). In a single shot PD game, each player should choose to defect and this is the Nash equilibrium. However, the total payoff for both players in this case is lesser than the case when both play cooperate. In many social situations, cooperation emerges among self-interested people. The iterated PD game [Axelrod(1984),Nowak and Sigmund(1989)] was proposed as a model to capture this and it was shown that cooperation indeed emerges in iterated PD games where the number of iterations are possibly infinite (for finite iterated PD games with the number of iterations known to the players, defect should be the best strategy; this can be shown by backward induction using the Nash equilibrium solution of the single-shot PD game at the last step).

The PD game in graphs has also received attention in the recent past. We will give a very brief discussion about the literature that is directly relevant to this work (for a more extensive review and discussion on evolutionary games on graphs in general, see [Szab and Fth(2007)]). In this literature also, emergence of cooperation has been demonstrated in social networks by simulation studies [Ebel and Bornholdt(2002),Santos et al(2006),Zimmermann and Eguíluz(2005)]. Santos et al. [Santos et al(2006)] presented simulation studies showing the emergence of cooperation in graphs of fixed topology for a range of values of the parameters in the payoff matrix. Their main goal was to study the effect of the variation of degree of the nodes on the evolution of cooperation. Zimmermann at al. [Zimmermann and Eguíluz(2005)] presented simulation results showing the evolution of cooperation in graphs with variable topology, where the dynamics of the network was much slower than the dynamics of the strategies. In this paper, the specific game model of PD games that we study is very similar to that in [Santos et al(2006),Zimmermann and Eguíluz(2005)]. One difference of our model from [Zimmermann and Eguíluz(2005)] is that we assume the network topology to be fixed. The main distinction of our model is that we consider two different types of agents that form the nodes of the graph and thus the game-playing and strategy update neighborhood for the agents can be different.

3 Preliminaries

In this section we introduce the notations and definitions that will be used in the remainder of the paper.

Undirected graph An undirected graph G is an ordered pair, $G = (V, E)$, where $V = \{v_1, v_2, \ldots, v_n\}$ is a set of n nodes, and $E \subseteq V \times V$ is a set of edges. Two nodes v_i and v_j are called *neighbors* of each other if $(v_i, v_j) \in E$. The set $\mathcal{N}_i = \{v_j | (v_i, v_j) \in E\}$ is the set of v_i's neighbors, and $|\mathcal{N}_i|$ is defined as the *degree* of node v_i. Denote $\mathcal{N}_i^+ = \mathcal{N}_i \cup \{v_i\}$.

Scale-free network A scale-free network is a graph where the degree distribution of nodes follow a power law, i.e., $N_d \propto d^{-\gamma}$, where N_d is the number of nodes of degree d and γ is a constant called the power law degree exponent (typically $\gamma \in [2, 3]$).

Prisoner's Dilemma Game In its simplest form, the prisoner's dilemma game is a single-shot two-player game where the players have two available strategies – cooperate (C) and defect (D). The payoffs of the players are given by the following table

	C	D
C	σ_1, σ_2	a_1, b_2
D	b_1, a_2	δ_1, δ_2

where the index 1 corresponds to the row player and the index 2 to the column player. The entries in the payoff matrix should satisfy $b_i > \sigma_i > \delta_i > a_i, i = 1, 2$. From the constraints of parameters, we can see that no matter whether one player cooperates or defects, the better strategy for the other player is always defection (since b_i is the largest among all payoff values); however, if both players defect, their payoff can be much less than the case when both cooperate ($\delta_i < \sigma_i$). Hence the dilemma arises. The payoffs of both players are usually assumed to be identical. In this paper, we assume $\sigma_1 = \sigma_2 = \sigma$, $a_1 = a_2 = a$, $\delta_1 = \delta_2 = \delta$. b represents the extent of an agent's playing defection, i.e., an agent with higher b has more incentive to defect than an agent with lower b. For repeated PD games an additional constraint is $2\sigma > a + b_i, i = 1, 2$. We further follow the convention in Nowak [Nowak and Sigmund(1989)] and set $a = 0$, to reduce the number of parameters.

PD Game in Graphs A PD game in a graph is a repeated game where the n-players form the nodes of the graph and the game proceeds in two phases: (i) game playing phase (ii) strategy update phase. The parameters that define different versions of PD games in graphs are: (a) topology of the graph (fixed or variable) (b) game playing and strategy update neighborhood (c) strategy update rule (d) assumptions on synchronous or asynchronous strategy update. The version of the PD game on graphs that is most relevant to this paper is defined below.

PD game in fixed graphs with synchronized strategy update is a repeated game where each iteration of the game proceeds in the following two phases: (a) In the game playing phase the players play the PD game with all their neighbors with a fixed strategy and compute their total payoff. (b) In the strategy update phase, each player compares the payoffs of all its neighbors (including itself) and chooses the strategy of its neighbor with the highest payoff for the next iteration. In other words, our strategy update rule is: *imitate your best/wealthiest neighbor*.

In this version of the game, the agents are not distinguished by group labels, and the neighborhoods for game playing and strategy update are assumed to be same. As we shall discuss in the next section, in our model, we assigned agents to different groups (so each agent will have a new property of group label) and distinguish the game playing and strategy update neighborhood of each agent based on its group label.

4 Model Definition

In this section, we present our agent-based model for conflict among different groups in multi-cultural societies. We first present the basic model without considering the effect of leaders. In Section 7, we include the effect of leaders in our model. We model the whole multi-cultural population in a geographical region as a collection of agents. An agent represents an individual. Since an individual interacts with few other individuals in a society, we model the collection of interacting population as a graph where the nodes are the agents and the edges denote interaction between the agents. We assume that the population consists of two different ethno-religious groups (i.e., there are two different types of nodes in the graph). The interaction between agents in two different groups is modeled as the prisoner's dilemma game. In this context, the strategy cooperate (C) implies the willingness of the agent to compromise with the other group whereas the strategy defect (D) implies unwillingness to compromise with the other group. Thus, the fraction of links between the two groups where both agents play D can be used as a measure of tension between the two groups.

Network Construction As stated in Section 3, for defining the PD game on graphs we need to first define the graph on which the game should be played. We use an undirected graph $G = (V, E)$ to represent the agents of two groups and their connections. $V = \{v_i | i = 1, \ldots, n\}$. Let n_1 and n_2 be the number of agents in the two groups, respectively with $n = n_1 + n_2$. We construct G in two steps:

1. Construct the graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ for each group separately, where $|V_1| = n_1$ and $|V_2| = n_2$.
2. Construct the set of edges $E_3 \subseteq V_1 \times V_2$ such that each agent in one group is connected to at least one agent in the other group. The edges are added by picking two nodes from the two different groups uniformly at random. Let

the average number of edges connecting an agent in one group to agents in the other group be k (a parameter that captures the degree of connectivity between the two groups).

Thus, after the two steps of construction, we get the graph $G = (V, E)$, with $V = V_1 \cup V_2$ and $E = E_1 \cup E_2 \cup E_3$. Figure 1 illustrates the network structure of G. By construction, in the graph G, each agent i has two types of neighborhood: (a) neighborhood of agents of its own type $\mathcal{NS}_i = \{v_j | (v_i, v_j) \in E_1 \cup E_2\}$ and (b) neighborhood of agents of the other group's type $\mathcal{ND}_i = \{v_j | (v_i, v_j) \in E_3\}$.

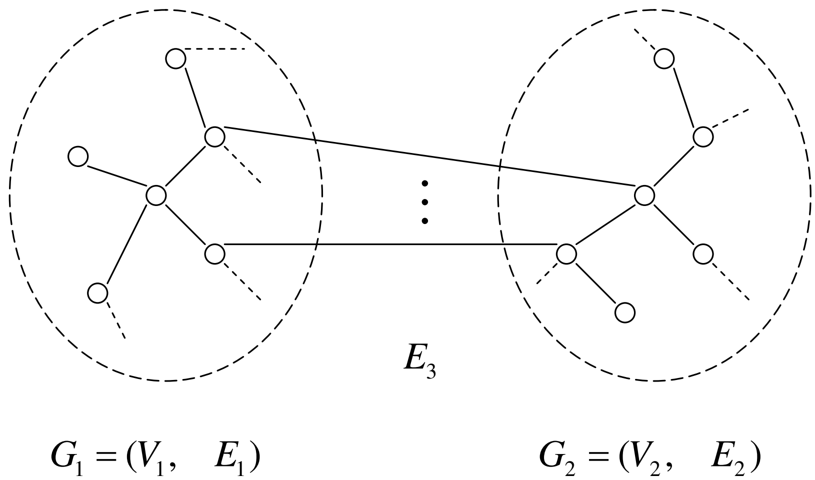


Fig. 1 The network structure G of the model. $G = (V_1 \cup V_2, E_1 \cup E_2 \cup E_3)$. On the left (or right) oval is a sub-graph of network G_1 (or G_2). Edges connecting nodes in the two ovals represent E_3, the set of edges between two groups.

Phases in each round of the game As stated before, the PD game on graphs proceeds in rounds where each round (iteration) consists of the game playing phase and the strategy update phase. We assume that the network structure, G, is fixed and the strategy update is synchronous. In the *game playing phase*, each agent plays the PD game with its neighbors *of the other type*, i.e., $\mathcal{ND}_i$ is the game playing neighborhood of each agent. Let $s_i(t)$ denote the strategy of agent i at round t, where $s_i(t) = 0$ implies cooperation (C) and $s_i(t) = 1$ implies defection (D). We assume that at each iteration, each agent plays the same strategy with all agents in its game playing neighborhood. The aggregate payoff, $p_i(t)$, of agent i in iteration t can be computed by summing up the individual payoffs obtained from playing with agents in $\mathcal{ND}_i$.

$$p_i(t) = \sum_{j: v_j \in \mathcal{ND}_i} \sigma(1 - s_i(t))(1 - s_j(t)) + b s_i(t)(1 - s_j(t)) + \delta s_i(t) s_j(t) \quad (1)$$

In the *strategy update phase*, each agent i imitates the strategy of the agent with highest payoff at previous iteration from a set $\mathcal{C}_i^+ = \mathcal{C}_i \cup \{v_i\}$ where $\mathcal{C}_i$ is the strategy update neighborhood. In this paper we assume that each agent updates its strategy based on the payoffs of neighboring agents *of its own type*, i.e., $\mathcal{C}_i = \mathcal{NS}_i$ (we will discuss another choice for this below). If there is more than one agent with the highest payoff, an agent randomly selects one of the agents and imitate its strategy. Thus the strategy update for agent i can be written as:

$$s_i(t) = s_j(t-1)$$
$$\text{where } j = \arg\max_{k \in \mathcal{C}_i^+} (p_k(t-1)) \quad (2)$$

Let $S(t) = [s_1(t), \ldots, s_n(t)]$ and $P(t) = [p_1(t), \ldots, p_n(t)]$ the strategy vector and the aggregate payoff vector for all n agents at iteration t, respectively. The initial strategy vector $S(1)$ is randomly chosen so that the probability that an agent's initial strategy is cooperation is f_c (which represents the initial fraction of cooperators). Figure 2 illustrates the evolution of strategy vector $S(t)$ and payoff vector $P(t)$.

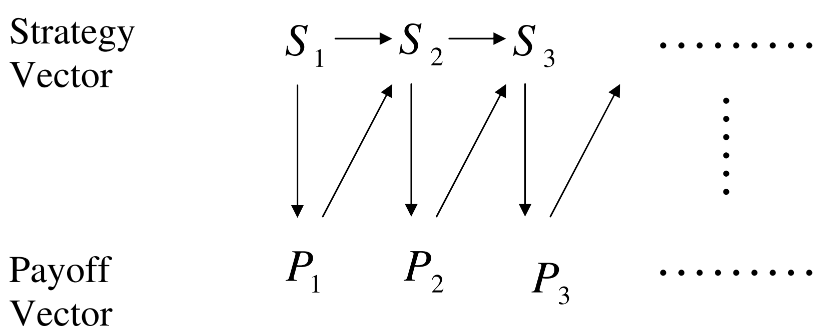


Fig. 2 The evolution of $S(t)$ and $P(t)$. The arrows represent the dependence relationship of the variables. The value of the arrows' destination is given by the values of the arrows' origins.

Game playing and strategy update neighborhood In our model, we chose our game playing neighborhood for an agent as $\mathcal{ND}_i$ (i.e., agents in its neighborhood that are of different type). Intuitively, this encodes the fact that we are interested in inter-cultural disputes and not in disputes within the culture. For the strategy update neighborhood we have chosen $\mathcal{NS}_i$. This encodes the assumption that an agent gives more importance to the opinions of neighbors of its own type regarding the opposite type than the opinion of the other type for its own type. Mathematically, there is another possibility that the strategy update neighborhood for an agent can be all agents within its neighborhood, i.e., $\mathcal{NS}_i \cup \mathcal{ND}_i$. However, we have done some simulations showing that the results obtained with the two choices are qualitatively very similar. Therefore we do not consider the second case any further in the paper.

In our model of the PD game in graphs we have a separate neighborhood for game playing and strategy update. This is different from the conventional case where the game playing and strategy update neighborhood are the same. We make the distinction in the game playing and strategy update neighborhood to encode the fact that there are two different types of agents in the graph. We emphasize this difference because the literature on PD games in graphs mostly talks about the emergence of cooperation, whereas we will show that considering two different types of agents leads to the emergence of defection between the two groups.

Model parameters The PD game in graphs that we defined has a number of parameters: (a) The parameters defining the payoff matrix σ, δ, a, b_1 and b_2. We have already mentioned that we choose $a = 0$. We can also choose $\sigma = 1$ without any loss of generality. The main parameters of interest in the payoff matrix are b_1 and b_2. They represent the payoff to an agent of group 1 (group 2) when an agent in group 2 (group 1) compromises on its stand. According to the results in [Lumsden(1973)], agents in smaller groups have more incentive to defect than those in larger groups. That is, b for agents in a smaller group is larger than for agents in a larger group. So we distinguish b values for the two groups: b_1, b_2 and if $n_1 > n_2$ we have $b_1 < b_2$. We choose the value of the parameter δ, the payoff for $D-D$ pair, to be 0.1. δ does not change the results significantly as long as it is much smaller than σ (increasing the value of δ will increase the fraction of $D-D$ links); (b) The ratio of population in the two groups n_1/n_2 is another parameter; (c) The initial fraction of cooperators in the PD game that roughly encodes the prevalent level of animosity between the two groups (e.g., due to historical reasons or due to occurrence of an external event); (d) The topology of the graphs and the average number of edges from one group to another. This last parameter is an assumption that we make since we cannot know the exact interaction patterns between individuals in the population. In Section 6, we will present simulation results obtained by systematically varying these parameters.

Measure of Conflict Potential The measure of conflict potential between two different groups should consider the interactions between agents in different groups, which can be expressed as the strategy pairs for two neighboring agents belonging to different groups (in the steady state). So we use the fraction of $D-D$ links (two neighboring agents both play defection) between two groups as a measure of the potential of conflict between the two groups, which can be computed as follows:

$$f_{dd} = \frac{\sum_{(v_i,v_j)\in E_3} s_i \cdot s_j}{|E_3|} \tag{3}$$

5 Analysis

As discussed in Section 4, the strategy vector $S(t)$ of all the agents will evolve according to equation 2. We define the state of each agent as its strategy s_i and therefore the vector $S(t)$ is the state vector of the whole graph. Thus equation 2 gives the state evolution equations of the whole dynamical system. We will call the set of all possible states as the state space of the system. Since each component of the state vector can take on only two possible values (i.e., $s_i(t) = 0$ or 1), the state space is discrete with cardinality 2^n. In this section, we will give partial characterization of the long term behavior of the state vector $S(t)$ for the model as the iteration progresses.

The first question one might ask for this dynamical system is whether the state vector will converge to a fixed state (or fixed point in the state space). Our main result here is that the state vector $S(t)$ whose components evolve according to equation 2 may not converge to a fixed state, which means the system's conflict measure in this model may fluctuate depending on the network structure.

Lemma 1 *For the model of PD game in graphs defined in Section 4, as $t \to \infty$, the state vector $S(t)$, may not converge to a fixed point.*

Proof: We will prove this lemma by constructing a counter-example, namely an example of oscillation for the model in Section 4. At iteration t, the fact that the system has reached the oscillatory state means that the sequence of strategy vectors $\{S(t), \ldots, S(t+k-1)\}, k > 1$ will repeat forever (where k is the period of the oscillation). Figure 3 shows an example of oscillation with two agents in the middle switching their strategies forever (oscillation with period 4). Agents in the first row belong to one group and agents in the second row belong to the other group. The payoff values of the four agents on the corners are indicated by the adjacent labels for the 4 phases, e.g., with x, y, w and z for Phase 1. We can construct the four agents' other neighbors (not shown to simplify the graph) so that the following two conditions are satisfied: (1)the four agents have higher payoff values than the two middle agents at all times; (2) $y + a - \sigma > x > y$ and $z + b - \delta > w > z$. Please note a,b,σ and δ are four parameters of PD game payoff matrix. Then the two agents in the middle column will alternate between C and D and their strategies will traverse all alternatives: (C,D), (D,D), (D, C) and (C, C). In Figure 3, the notation $\underline{C}$ or $\underline{D}$ means that the agent will switch strategy at next iteration; the italic dot-underlined C or D means that the agent just switched strategy at previous iteration; the bold **C** or **D** means that the agent's payoff has changed due to the fact that its neighboring agent from the other group has switched strategy. The four states will repeat forever. ■

Although we have shown above that there are cases where the strategy vector $S(t)$ does not reach a fixed point, there are also cases when the strategy vector will reach a fixed point. In fact, in our simulations we have observed that the strategy vector reaches a fixed point in most cases. In general, whether we will reach a fixed state depends on the initial strategy distribution and the

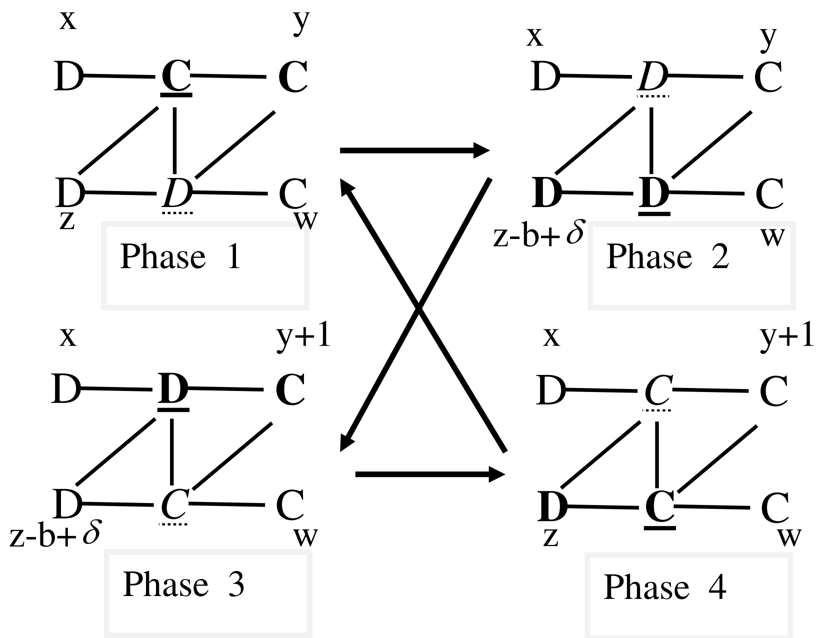


Fig. 3 Oscillation in our model with period 4.

structure of the graph. We can construct trivial examples of fixed states for the model by setting the identical strategies for agents in the group where the strategy update neighborhood is defined. For example, one group of agents play C while the other group of agents play D or all agents play C (or D). It is easy to conclude that on a complete graph, the steady state can only be trivial examples of fixed states above. However, there can be many other non-trivial fixed states on a general graph. Below we try to capture some of the common features of these states.

Given a node i, define $l(i)$ as i's neighboring node from its own group whose strategy i imitates. $l(i)$ can be computed according to Equation 2, and is time-varied until the system reaches a fixed state. At a fixed state, if we keep each node i and each edge $(i, l(i))$ (*imitation edges*), and remove other edges in G, we will get a new graph

$$G' = (V, E') \text{ where } E' = \{(i, l(i))|i \in V\}$$

that represents imitation relation of agents in fixed states. Since we removed non-imitation edges in G to form G', G' may not be a connected graph as G. According to the strategy update rule, the imitation edges can not form any cycles. So G' can be represented as a set of trees:

$$G' = \cup_{i:i=l(i)} T_i$$

where T_i is a tree (*imitation trees*) composed of root node i imitating itself, all other nodes connecting to i through imitation edges in E', and the imitation edges between them. In each imitation tree, each node j will have one parent node $l(j)$. All agents in each imitation tree will play the same strategy and the payoff of any ancestor node must be greater than or equal to that of offspring nodes. So the imitation trees can be classified as cooperative imitation tree or defective imitation tree.

Starting from an agent i in the imitation tree T_j, we can trace the parent node until we reach the root node j to form a chain of agents: $L(i) = \{i = l^0(i), l(i), l^2(i), \ldots, l^p(i) = j\}$, where p is the number of hops from i to j along the tree T_j. Then we can conclude that there will not be edges crossing each other between two chains derived from different imitation trees, as shown in Fig. 4.

Proposition 1 *Consider two chains from different imitation trees: $L(i)$ with length p_1 and $L(j)$ with length p_2. Then $\forall a-1, a-2, b_1, b_2 \in \mathbb{N}, s.t., 0 \leq a_1 < a_2 \leq p_1$, $0 \leq b_1 < b_2 \leq p_2$, if $\{l^{a_1}(i), l^{b_2}(j)\} \in E$, $\{l^{a_2}(i), l^{b_1}(j)\} \notin E$.*

Proof: Proof by contradiction. Suppose $\{l^{a_1}(i), l^{b_2}(j)\} \in E$, $\{l^{a_2}(i), l^{b_1}(j)\} \in E$.

Since $\{l^{a_1}(i), l^{b_2}(j)\} \in E$, we have $p_{l^{a_1+1}(i)} \geq p_{l^{b_2}(j)}$. Also according to the imitation rule, the payoff of any ancestor node must be no less than the payoffs of offspring nodes, so

$$p_{l^{a_2}(i)} \geq p_{l^{a_1+1}(i)}, p_{l^{b_2}(j)} \geq p_{l^{b_1+1}(j)}$$

From the equations above, we get $p_{l^{a_2}(i)} \geq p_{l^{b_1+1}(j)}$. Since $\{l^{a_2}(i)), l^{b_1}(j)\} \in E$, $l^{b_1}(j)$ will imitate $l^{a_2}(i)$ instead of $l^{b_1+1}(j)$, which contradicts the fact. (Even when $p_{l^{a_2}(i)} = p_{l^{b_1+1}(j)}$, according to the imitation rule, $l^{a_2}(i)$ has higher priority to be imitated than $l^{b_1+1}(j)$.)■

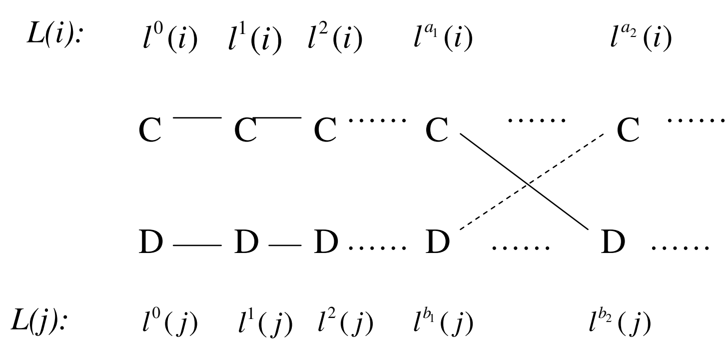


Fig. 4 When there is an edge $(l^{a_1}(i), l^{b_2}(j)) \in E$ between the cooperative chain and the defective chain above, there cannot be an edge between $l^{a_2}(i)$ and $l^{b_1}(j)$, indicated as a dashed line above. Visually, there can not be any two edges crossing each other between different imitation trees.

The characteristic of the steady state captured in Proposition 1 means that as iteration progresses, the network will evolve to form some kind of tree structure of "organization" based on the imitation relationship and the connectivity among different trees should maintain some kind of social hierarchy (tree levels) according to Proposition 1.

6 Simulation Results

In this section, we present simulation results showing the effect of changing the various parameters that define our model. As stated before, our measure of conflict is the fraction of D-D links between the two groups. The parameters we consider include: (i) the initial fraction of cooperators, f_c, (ii) the number of nodes in the two groups, n_1, n_2, (iii) the average number of edges from an agent in one group to the other group, k, and (iv) the payoffs (b_1 or b_2) to the agent playing D when its opponent plays C.

The default values for the parameters in our simulation are: $f_c = 0.5$, $n_1 = n_2 = 300$, $k = 5$, $b_1 = b_2 = b = 1.5$. When we change the value of one parameter, other parameters have their default values unless otherwise stated. The graphs G_1 and G_2 used for the simulations are scale-free networks generated by using the Barabasi-Albert algorithm [Barabasi and Albert(1999)]. The set of edges E_3 between nodes in V_1 and V_2 are generated randomly. However, we ensure that each node in V_1 is connected to at least one node in V_2 and the average number of edges between the two groups is k. Each data point in the figures for fraction of final D-D links were generated using an average of 500 runs on randomly generated graphs as described in Section 4. In the simulation, we observed that after a transient time of 30 iterations, the strategy vector either converges to a fixed state or occasionally comes to an oscillation state with small magnitude and period. So we compute the fraction of final D-D links by averaging over 5 iterations after a transient time of 30 iterations.

Figure 5 shows the effect of the fraction of initial cooperators (f_c) on the final fraction of $D - D$ links for various values of b with the other parameters remaining constant. For a given b, as f_c is increased, the final fraction of $D - D$ links between the two groups decreases (as expected). However, as the value of b increases, even when the initial fraction of cooperators is as high as 0.9, our model still predicts a high fraction of final $D - D$ links (e.g., more than 0.6, for $b = 1.5$), implying that the model captures the potential of conflict between two different groups. To verify that this is due to the fact that we are considering two different groups in our model, we ran simulations on the same graphs where we do not distinguish between the two types of agents (i.e., we simulate a conventional PD game). In those cases, we found (although we do not show it here) that the fraction of final $D - D$ links is quite small when f_c is high (as expected and reported in the literature), which means cooperation will emerge there.

To investigate the influence of the number of nodes in the network on the evolution of conflict, we ran two sets of simulations by increasing n (the number of nodes in each group) from 100 to 1000, and increasing k (the number of

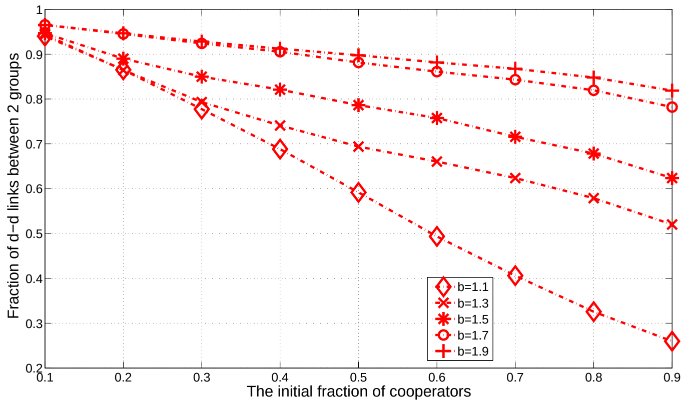


Fig. 5 The plot of fraction of $D-D$ links between two groups as function of initial fraction of cooperators of all agents for different b from 1.1 to 1.9. $n = 300, k = 5$.

average edges from an agent in one group to agents in the other group) from 1 to 20, respectively. Figure 6 shows that different values of n do not influence the fraction of final $D - D$ links for our model (it stays within 0.78 to 0.79 as n varies from 300 to 1000). However, in Figure 7, we find that with k increasing, the fraction of final $D-D$ links does increase when k is sufficiently large ($k \geq 5$). Since k represents the connectivity of the two groups, we can see that in the conflict model, the more interactions between two groups, the more conflicts will emerge.

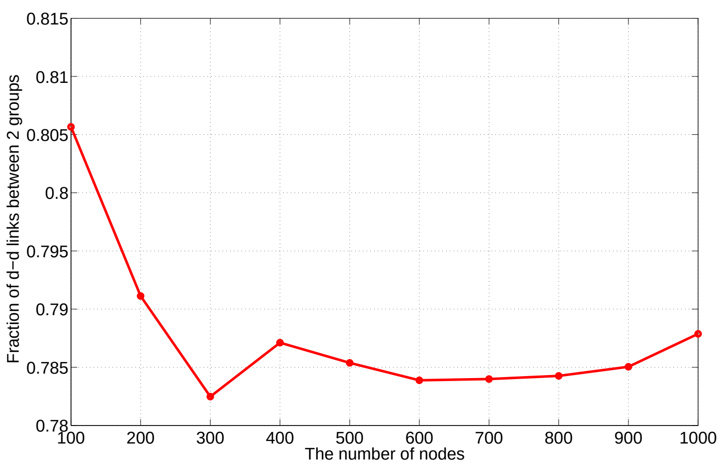


Fig. 6 The plot of fraction of D-D links between two groups as function of the number of nodes in each group. $f_c = 0.5, k = 5, b = 1.5$.

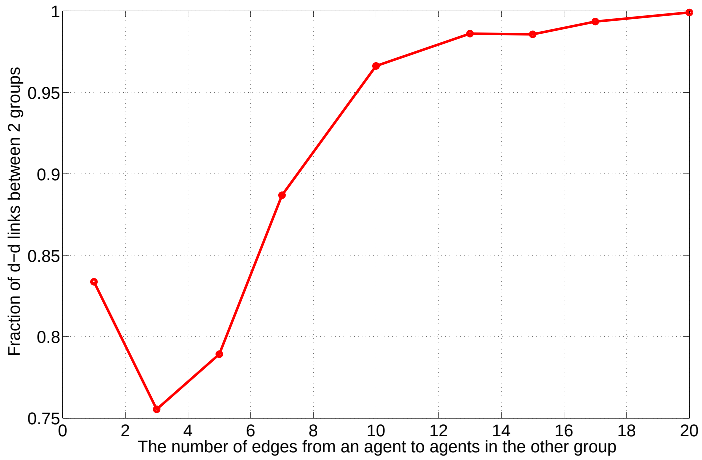


Fig. 7 The plot of fraction of $D-D$ links between two groups as function of k. $n = 300$, $f_c = 0.5$, $b = 1.5$.

Figure 8 shows that when b increases from 1.1 to 1.9, the fraction of final $D - D$ links will increase. This happens because b represents the incentive of agents to play defection. With higher b, the higher payoff values of defection versus cooperation encouraged agents to play defection instead of cooperation.

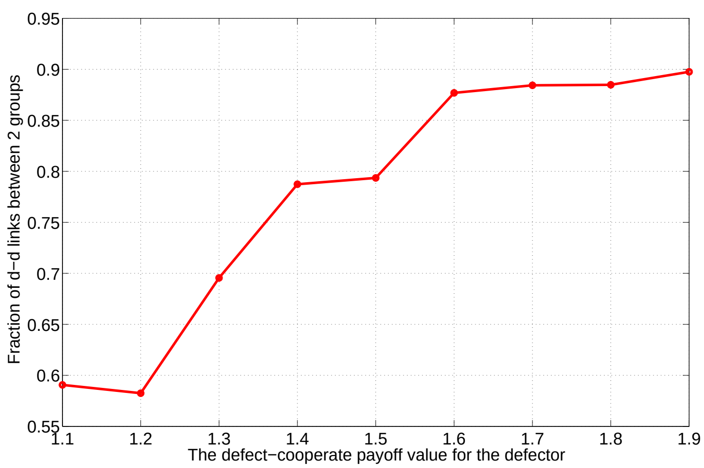


Fig. 8 The plot of fraction of $D-D$ links between two groups as function of b. $n = 300$, $f_c = 0.5$, $k = 5$.

In the simulation above, we assume that both groups have the same size $n_1 = n_2$. Next we remove the assumption to consider the effect of having two groups with different sizes $n_1 \neq n_2$. We fixed the size of the larger group $n_1 = 500$, and increase $\frac{n_2}{n_1}$, the relative size of the smaller group to the larger one, from 0.1 to 0.9. Meanwhile according to the results in [Lumsden(1973)], smaller groups have higher tendency to defect than larger groups, so we distinguish b values for the two groups: $b_1 < b_2$. Figure 9 shows the result when we change the relative size of the two groups and b values for them. From the figure, we can find that the conflict will increase when the ratio of the smaller group to the larger group decreases, e.g., the conflict is more drastic (the fraction of $D-D$ links is almost 1) between a larger group and a smaller one with $\frac{1}{10}$ size of the former.

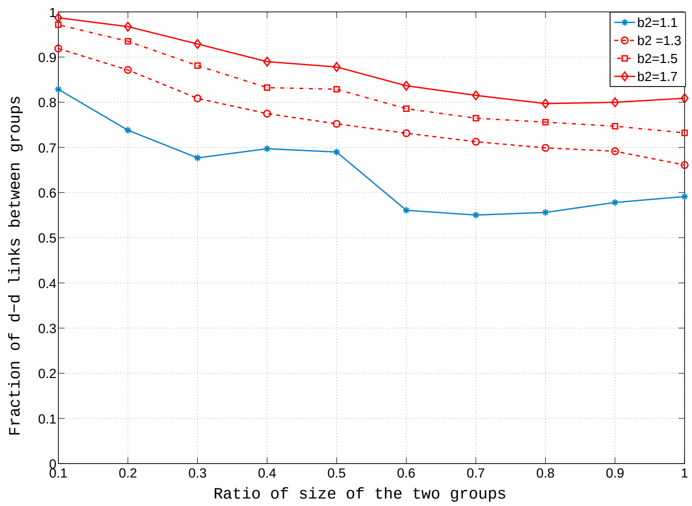


Fig. 9 The plot of fraction of $D-D$ links between two groups as function of $\frac{n_2}{n_1}$ corresponding to $b_1 = 1.1$ and b_2 increased from 1.1 to 1.7. $f_c = 0.5, k = 5$.

7 Effect of leaders

In the basic model that we presented above, we did not incorporate the fact that different agents in a society may have different levels of influence. In any population group, the strategies of some agents (leaders of the group) may have more effect on the society than the strategies of others. In this section, we incorporate the effect of leaders in our model. Since we model the population of each ethnic group as a scale-free network, we can use the degree of each node in the graph as a measure of its social influence. Consequently, high degree nodes can be thought of as the agents with high influence on the population. As in Section 4, agents play against agents of the other group. According to our construction of edges between the two ethnic groups, each node in one group will have almost the same number of neighbors in the other group. By multiplying the payoffs of the agents at each round by a factor proportional to the degree of their nodes, we can ensure that the strategies of the leader nodes have high payoffs. In other words, we calculate the payoff of each agent, $p'_i(t)$, using

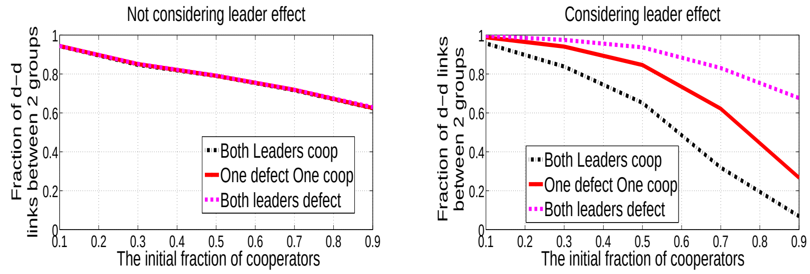
$$p'_i(t) = p_i(t) * |\mathcal{N}_i| \tag{4}$$

where $|\mathcal{N}_i|$ is the degree of node i and $p_i(t)$ is the payoff for node i at round t without considering leaders' effect and is given by Equation 1. The strategy update rule is the same as in the basic model, i.e., *imitate your neighbor within your own group with highest payoff*, and is given by Equation 2.

According to Equation 4, leader nodes with high degrees in their own group (i.e., high in-group degree) will have greater probability to get higher payoffs. Consequently, during the strategy update phase, the leaders will have more effect on the strategies of the whole population compared to low degree nodes. In other words, the willingness of the leaders to compromise or not will play a significant role in the potential of conflict between the two groups.

To study the effect of leaders' strategies on the tendency of conflict between the two groups, we ran simulations with the leader as the highest degree node in each group. For each set of testing parameters, we randomly generated 500 graphs, and compared the average final fraction of *D*-*D* links for three different combinations of leaders' initial strategies: both leaders cooperate, one cooperates while the other defects, and both defect. In each graph randomly generated for the simulations, G_1 and G_2 are scale-free networks generated using the Barabasi-Albert algorithm [Barabasi and Albert(1999)]. The set of edges E_3 between nodes in V_1 and V_2 are generated randomly and ensure that each node in V_1 is connected to at least one other node in V_2 and the average number of edges between the two groups is k. The total number of iterations for each run was set at 30 and we verified that the system converged to a steady state.

Figure 10 shows the variation of the final fraction of *D*-*D* links f_{dd} with the initial fraction of cooperators f_c for different leader strategies (both leaders cooperating, both leaders defecting, and only one of them cooperating). The results in Figures 10 and 12 were generated with $f_c = 0.1, 0.3, 0.5, 0.7, 0.9$, and each data point is an average over 500 runs. Figure 10(a) shows that when we do not use Equation 4 for obtaining the payoffs of the agents, the strategy of the leaders does not affect the conflict measure (all the curves are almost identical), while 10(b) shows when we use Equation 4 for computing the payoffs, the conflict measure is dependent on the strategies of the leaders. A similar effect on the conflict measure for variation of b is shown in Figure 11. From Figure 10(b), we see that by changing the initial strategies of the leader nodes in each group from cooperate to defect, f_{dd} always increases. This is especially important when f_c is high, because this shows that although most



(a) (b)

Fig. 10 The plot of f_{dd} as function of f_c using (a)the basic model without Equation 4, (b)the model incorporating leader's effect with Equation 4. The three curves in the fig corresponds to three different combinations of initial strategies of the two leader nodes with the highest degree in each group. $n = 300, b = 1.5, k = 5$.

of the population may be willing to cooperate, if the leaders are willing to defect then the conflict potential is high. For example, when $f_c = 0.9$, if both leaders cooperate, the average (over the 500 different runs) f_{dd} is about 0.1 while if both defect, f_{dd} is about 0.7. When both f_c is high and the leaders cooperate, the conflict measure is low. Apparently, it seems that when the majority of the population is willing to defect, the initial willingness of the leaders to cooperate does not have any significant effect (look at Figure 10(b) for $f_c = 0.1$). However, we note that apart from the leader, there are other high degree nodes present in each group. These influential agents may have less influence than the leader node, individually, but as a collective they may have more influence and may force the leaders to change their strategies.

To test this intuition, we also checked the fraction of leaders that changed their initial strategies. From Figure 12 it is apparent that when f_c is low, the leaders initially cooperating almost always change their strategies to defect. For example, when $f_c = 0.1$, leaders initially cooperating have a fraction of strategy switching as high as almost 1.0. On the other hand, for leaders initially defecting, the fraction of leaders switching strategy is uniformly lower than 0.03. Although the results in Figure 12 are for a constant $b = 1.5$, the nature of the curves remain same for other values of b, and there is no qualitative difference.

In Figures 10 and 12, we have kept the value of the incentive to defect as a constant, i.e., $b = 1.5$. Now, we study the effect of varying b while keeping f_c constant. The results in Figures 11 and 13 were generated with $b = 1.1, 1.3, 1.5, 1.7, 1.9$, and each data point is an average over 500 runs. From Figure 11(b), we can see that irrespective of the value of b, there is more conflict potential if the leaders defect instead of cooperate. Moreover, even when b is low ($b = 1.1$), if the leaders initially defect, the potential of conflict is quite high ($f_{dd} = 0.8$). For high incentive to defect ($b = 1.9$), the potential of conflict is high even if the leaders are initially willing to cooperate ($f_{dd} = 0.8$). Different from before, this is not only due to the fact that the leaders switch strategies from cooperate to defect, since other influential leaders in the soci-

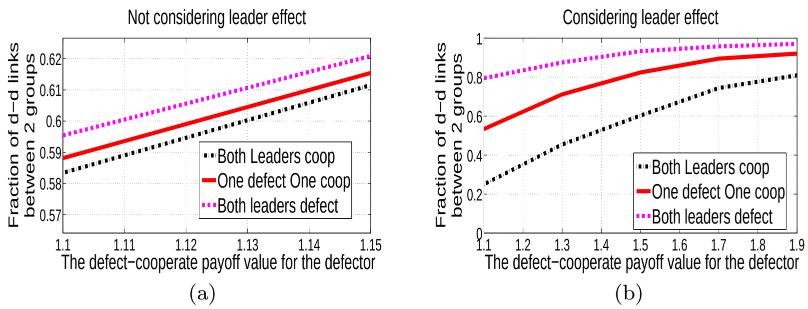


Fig. 11 The plot of f_{dd} as function of b using (a)the basic model without Equation 4, (b)the model incorporating leader's effect with Equation 4. $n = 300, f_c = 0.5, k = 5$.

ety are defecting (see Figure 13), but also due to the fact that the incentive of agents in both groups to defect is high.

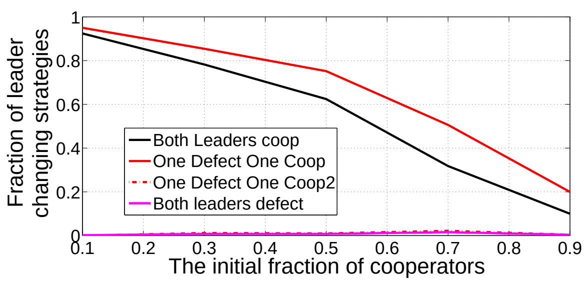


Fig. 12 Variation of fraction of cases where leaders switch their initial strategies to different final ones as a function of f_c; The four curves in the figure corresponds to three different combinations of initial strategies of the two leader nodes, where we distinguish the case when one leader cooperates and the other defects. $n = 300, b = 1.5, k = 5$.

8 Case Study: Former Yugoslavia

In this section, we present preliminary results of using our basic model (i.e., the model without leader effects) to obtain a set of potential conflict locations. As a test case, we use the demographic data for the different ethnic groups in former Yugoslavia before the ethno-religious conflict in Yugoslavia erupted. The spatial population distribution of the different ethnic groups according to the 1991 census is shown in Figure 14 [UTexasLib(1992)]. In Figure 14, the different colors represent the areas where each ethnic group is a majority and the white regions are areas where there is no group that is a clear majority. The different population groups that we consider in our simulation are Serbs, Croats, Muslims, and Albanians.

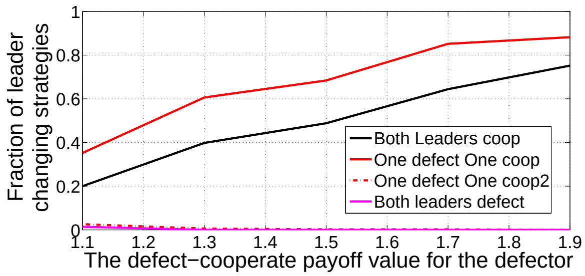


Fig. 13 Variation of fraction of cases where leaders switch their initial strategies to different final ones as a function of b. $n = 300, f_c = 0.5, k = 5$.

To associate the agents with a location, we assign an agent to a pixel. The label of an agent corresponds to the color of the pixel, i.e., the major ethnic group in the region. As stated in Section 4, each ethnic group is organized using a scale-free network. The edges between two different groups are generated based on the spatial distance of agents from other groups so that each agent in one group is connected to the nearest five agents from the other group ($k = 5$). After constructing the graph of agents, we run the game theory model on the network between pairs of ethnic groups. The parameters of initial fraction of cooperators f_c, defect-cooperate payoff for defectors b are manually set for each ethnic groups to be compatible with the ethnic tensions in former Yugoslavia, i.e., the parameters are set so that f_c is relatively low and b is relatively high when running the model between Serb-Croat, Serb-Muslim, and Serb-Albanian. In the simulations, the strategies of the agents converge within 30 iterations. Since our goal is to obtain the set of potential conflict locations, we set the number of D-D between-group edges associated with an agent playing defect as the potential conflict measure at the location of that agent. Note that this is slightly different from the measure of conflict in our simulation for parametric studies in Section 6, where the fraction of D-D links between two groups is used as a global measure of conflict.

The total number of agents we used is approximately 3000. The distribution of the four main groups is 45% Serbs, 20% Croats, 7% Muslims, and 7% Albanians. This is slightly different from the census population composition (36% Serbs, 20% Croats, 10% Muslims, and 9% Albanians) in [Petrovic(1992)]. The error is due to the approximation of considering major ethnic groups composing the population in each region. The results of our simulation is shown in Figure 15. In Figure 15, the light green squares are the potential locations for violence given by our model. The blue cross marks are the reported violence locations during the ethno-religious conflict in Yugoslavia in the nineties obtained from [Lim et al(2007)]. The green squares correspond to the agents (i.e., pixels) playing defect obtained after thresholding of potential conflict measure. As shown in Table 1, we can see that our simple model has a good match with the reported violence locations when they occur at the border of

Table 1 Accuracy analysis of our result of former Yugoslavia.

(a)

	Total Number	Number close to the predicted locations (within 30 pixels/9 miles in the map)	Percentage
All reported violence locations	37	16	43%
Locations along the border	10	9	90%

(b)

Total Number of predicted pixels	131
Number of clustered blocks	19
Number of blocks close to reported locations	14
Percentage	74%

the two major population groups. However, when the blue crosses occur in the middle of an area with a single color, we do not have a good match. This is mainly because of the granularity of the data (in terms of spatial resolution) that we are using. Since we are putting in agents of a type only in the regions where they are a majority, we do not have two different types of agents in the middle of any colored patch. A possible remedy to this is to use a more refined population composition data and using our model with it. As a part of our future work, we are planning to use municipality level population distribution data for all the provinces of former Yugoslavia (as given in [Petrovic(1992)]) and testing our model on it.

9 Discussion

In the very simple model that we developed for studying conflict in multicultural societies, we considered only the effects of ethno-religious identity, spatial distribution of the population, and influential leaders in the society. There are other important factors like (a) migration of population and (b) uneven distribution of natural resources, that should also be considered for making the model more realistic. We state below the possible extensions (or variations) of our model to take these factors into account.

The migration of population can be taken into consideration by using a variable topology graph instead of a fixed topology graph. The migration of a population group would correspond to the breaking of some existing edges and adding new edges corresponding to the new spatial distribution. The effect of migration on an existing state of potential conflict in the society can thus be modeled. The uneven distribution of resources can be taken into account by using a value of b, the incentive to defect as a function of the space. For example, minority agents at places with uneven distribution of natural resources may have a higher value of b than at places where such inequity does not exist. We are currently working on adding these features to our model.

Fig. 14 Population distribution in former Yugoslavia by majority groups according to 1991 census data. [UTexasLib(1992)]

10 Conclusion

In this paper we developed and analyzed a model for studying conflict in multi-cultural societies that is based on the Prisoner's Dilemma game in graphs. Our model captures three essential causes of multi-cultural conflict (a) ethno-religious identity of the different groups, (b) spatial distribution of the population, and (c) effect of influential leaders. The prisoner's dilemma game in graphs usually encourages the evolution of cooperation but we show that dividing the population into two groups leads to an increase in the fraction of $D - D$ links in the two groups (which is our measure of the tendency of conflict).

We analyzed our model by running several sets of simulations showing the effect of different parameters defining our model. We showed that the fraction

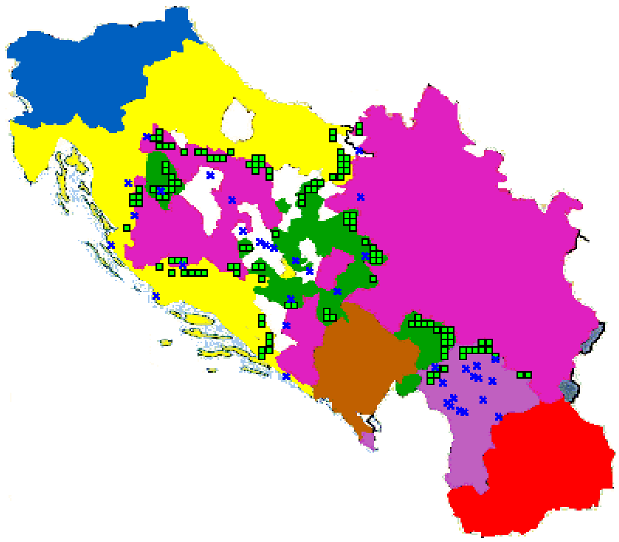


Fig. 15 The Yugoslavia map with reported violence locations and predicted potential violence locations by our model.

of D-D pairs is relatively insensitive to the number of nodes in the two groups, i.e., the number of agents. Thus the exact number of agents present in the society is not very relevant to the tendency of conflict. We found that, as expected, with higher initial fraction of cooperators, there is a lower tendency of conflict. However, the tendency of conflict increases with the number of edges between two groups (which represents the interaction between the groups of agents). We also showed that the tendency of conflict is much higher between two groups of different size than between two groups of similar size. This is mainly because the smaller groups have more incentive to defect than the larger groups [Lumsden(1973)].

We also evaluate the impact of influential agents on the strategy evolution of other agents. Our simulation results show that when the groups have lower inherent tendency of conflict (e.g., high initial distribution of cooperators and low incentive to defect), if the strategy of the leaders is to defect, it can result in a significant fraction of the population defecting. We also see an interesting phenomenon that if the leader's initial strategy is different from the strategy of the majority and the second highest degree agent, the leader's strategy may change to conform with the majority. We experimentally evaluated our model to obtain a set of potential conflict locations using data from the former Yugoslavia.

There are various directions for extending this work. Currently we are working on detailed analysis and further experiments using the municipality level population distribution data of former Yugoslavia. It is also important to design criteria to quantitatively evaluate how well the predictions correlate with the actual occurrence of violent incidents. From the modeling aspect, a natural extension is to extend the model and analysis to more than two groups of different cultures. An additional extension would involve taking into consideration other conflict factors, such as population migration or uneven distribution of resources across population groups.

Acknowledgments

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