A Generalized Fast Subset Sums Framework for Bayesian Event Detection

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Given all of this nationwide health data on a daily basis, we want to obtain a complete situational awareness by integrating information from the multiple data streams.

More precisely, we have three main goals: to detect any emerging events (i.e. outbreaks of disease), characterize the type of event, and pinpoint the affected areas.
We must provide the prior probability $\Pr(H_1(S, E_k))$ of each event type $E_k$ in each region $S$, as well as the prior probability of no event, $\Pr(H_0)$.

MBSS uses Bayes’ Theorem to combine the data likelihood given each hypothesis with the prior probability of that hypothesis: $\Pr(H | D) = \Pr(D | H) \Pr(H) / \Pr(D)$.

Given a set of event types $E_k$, a set of space-time regions $S$, and the multivariate dataset $D$, MBSS outputs the posterior probability $\Pr(H_1(S, E_k) | D)$ of each type of event in each region, as well as the probability of no event, $\Pr(H_0 | D)$. 

Overview of the MBSS method

The Bayesian hierarchical model

Space-time region affected

Type of event

Effects on each data stream

Expected counts

Effects of event

Parameter priors

Observed counts

Relative risks

\[ c_{i,m}^t \sim \text{Poisson}(q_{i,m}^t b_{i,m}^t) \]

\[ x_m = 1 + \theta (x_{km,\text{avg}} - 1) \]

\[ q_{i,m}^t \sim \text{Gamma}(x_m \alpha_m, \beta_m) \text{ inside } S, \]

\[ q_{i,m}^t \sim \text{Gamma}(\alpha_m, \beta_m) \text{ elsewhere} \]
Interpretation and visualization

MBSS gives the total posterior probability of each event type $E_k$, and the distribution of this probability over space-time regions $S$.

**Visualization**: $\Pr(H_1(s_i, E_k)) = \sum \Pr(H_1(S, E_k))$ for all regions $S$ containing location $s_i$.

**Posterior probability map**
Total posterior probability of a respiratory outbreak in each Allegheny County zip code.
Darker shading = higher probability.
MBSS: advantages and limitations

MBSS can detect faster and more accurately by integrating multiple data streams.

MBSS can model and differentiate between multiple potential causes of an event.

MBSS assumes a uniform prior for circular regions and zero prior for non-circular regions, resulting in low power for elongated or irregular clusters.

There are too many subsets of the data (2^N) to compute likelihoods for all of them!

How can we extend MBSS to efficiently detect irregular clusters?
We define a non-uniform prior $\Pr(H_1(S, E_k))$ over all $2^N$ subsets of the data.

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2. Choose **neighborhood size** $n$ from \{1…n_{max}\}, given multinomial $\Pr(n)$. 
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3. For each $s_i \in S_{cn}$, include $s_i$ in $S$ with probability $p$, for a fixed $0 < p \leq 1$.

This prior distribution has non-zero prior probabilities for any given subset $S$, but more compact clusters have larger priors.

Parameter $p$ controls the sparsity of detected clusters. Large $p$ = compact clusters. Small $p$ = dispersed clusters.
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3. For each \( s_i \in S_{cn} \), include \( s_i \) in \( S \) with probability \( p \), for a fixed \( 0 < p \leq 1 \).

\[ p = 0.5 \] corresponds to the original Fast Subset Sums approach described in (Neill, *Stat. Med.*, 2011), assuming that all subsets are equally likely given the neighborhood.

\[ p = 1 \] corresponds to MBSS, searching circular regions only.
Generalized Fast Subset Sums

Naïve computation of posterior probabilities using this prior requires summing over an exponential number of regions, which is infeasible.

However, the total posterior probability of an outbreak, $\Pr(H_1(E_k) \mid D)$, and the posterior probability map, $\Pr(H_1(s_i, E_k) \mid D)$, can be calculated efficiently **without** computing the probability of each region $S$.

In the original MBSS method, the **likelihood ratio** of spatial region $S$ for a given event type $E_k$ and event severity $\theta$ can be found by multiplying the likelihood ratios $LR(s_i \mid E_k, \theta)$ for all locations $s_i$ in $S$.

In GFSS, the **average likelihood ratio** of the $2^n$ subsets for a given center $s_c$ and neighborhood size $n$ can be found by multiplying the quantities $(p \times LR(s_i \mid E_k, \theta) + (1-p))$ for all locations $s_i$ in $S$.

Since the prior is uniform for a given center and neighborhood, we can compute the posteriors for each $s_c$ and $n$, and marginalize over them.
Preliminary results

We injected simulated disease outbreaks into two streams of Emergency Department data from 97 Allegheny County zip codes.

Results were computed for ten different outbreak shapes, including compact, elongated, and irregularly-shaped (200 injects of each type).

Runtime of GFSS was extremely fast, computing the posterior probability map for each day of data in less than nine seconds.

Smaller values of the sparsity parameter $p$ achieve higher detection performance for elongated clusters, and larger $p$ for compact clusters.
Learning the sparsity parameter

We demonstrate that the sparsity parameter can be learned from a set of labeled training examples $S_1 \ldots S_J$. For each $S_j$, we are given the affected region $S$, but not the values of the latent parameters (center location $s_c$, neighborhood size $n$, and sparsity parameter $p$).

To learn the distribution of $p$, we must marginalize over $s_c$ and $n$:

$$\Pr(S_j \mid p) = \sum_{s_c} \sum_n \Pr(S_j \mid p, s_c, n) \Pr(s_c) \Pr(n)$$

$$\Pr(S_j \mid p, s_c, n) = p^{|S_j|} (1 - p)^{n-|S_j|} 1\{S_j \subseteq S_{cn}\}$$

We assume that the $p_j$ for each $S_j$ is drawn from a multinomial distribution $\theta$ over $\{0.1, 0.2, \ldots, 1.0\}$, assuming a Dirichlet prior on $\theta$.

$$\theta_k = \Pr(p = 0.1 \times k) = \frac{0.1 + \sum_{j=1}^{J} \frac{\Pr(S_j \mid p = 0.1 \times k) \Pr(p = 0.1 \times k)}{\sum_{k=1}^{10} \Pr(S_j \mid p = 0.1 \times k) \Pr(p = 0.1 \times k)}}{1 + J}$$
Evaluation framework

For each $p$: 100 injects for training

1) All injects: $p = 0.2$
2) All injects: $p = 0.4$
3) All injects: $p = 0.6$
4) All injects: $p = 0.8$
5) All injects: $p = 1.0$
6) 50% of injects: $p = 0.2$, 50% of injects: $p = 0.8$. 

For each $p$: 100 injects for testing

Different and mixed values of $p$

Learned $p$ GFSS

MBSS

FSS

Uniform $p$ GFSS
GFSS was able to estimate the true value(s) of $p$. Results were very similar for nausea outbreaks and for as few as 25 injected outbreaks.
Results: detection power

We compared the average time to outbreak detection for the Learned-p GFSS, Uniform-p GFSS, MBSS, and FSS methods, at a fixed false positive rate of 1/month.

When the value of $p$ is small, corresponding to an elongated or irregular outbreak region, GFSS with learned $p$ is able to detect substantially earlier than the other methods.
Results: spatial accuracy

We compared the spatial accuracy (average overlap coefficient between true and detected clusters at day 7 of the outbreak) for Learned-p GFSS, Uniform-p GFSS, MBSS, and FSS.

Comparison of spatial accuracy (cough data)

Comparison of spatial accuracy (nausea data)

GFSS with learned p achieves high spatial accuracy across the entire range of p values.
Results: distinguishing outbreak types

We used the mixture outbreak data to evaluate the ability of GFSS to learn and distinguish between two outbreak types with different values of the sparsity parameter (p = 0.2 and p = 0.8).
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When the two outbreaks also had different effects on the two monitored data streams (cough and nausea), learning both the relative effects and the sparsity further improved detection.

![Graph showing posterior conditional probability of the correct outbreak type](image)
Conclusions

GFSS shares the essential advantages of MBSS: it can integrate information from multiple data streams, and can accurately distinguish between multiple outbreak types.

As compared to the MBSS method, GFSS substantially improves accuracy and timeliness of detection for elongated or irregular clusters, with similar performance for compact clusters.

While a naïve computation over the exponentially many subsets of the data is computationally infeasible, GFSS can efficiently and exactly compute the posterior probability map.

We can learn the distribution of the sparsity parameter p for multiple event types using a small amount of labeled training data.
Conclusions

Learning the distribution of the sparsity parameter not only improves detection power, but enables us to accurately differentiate between multiple, similar types of outbreak.

We are currently extending GFSS to simultaneously learn the distributions over the center location, neighborhood size, and the sparsity parameter $p$, using an EM-based approach.

In future work, we will also extend GFSS to the case of partially labeled training data, when only a small subset of affected locations are identified for each labeled event.

Thanks for listening! Any questions?