Robust detection using sparse measurements

Balakrishnan N
Carnegie Mellon University

Joint work with Rohit Negi and Pradeep Khosla

Allerton, October 2009

There's Nothing So Practical As a Good Theory
-Kurt Lewin

Motivating Application : Distributed Sensor Networks

Monitor a large area

Motivating Application : Distributed Sensor Networks

Monitor a large area

Many cheap, low power nodes

Motivating Application : Distributed Sensor Networks

Monitor a large area

Many cheap, low power nodes

Correlated, imprecise measurements

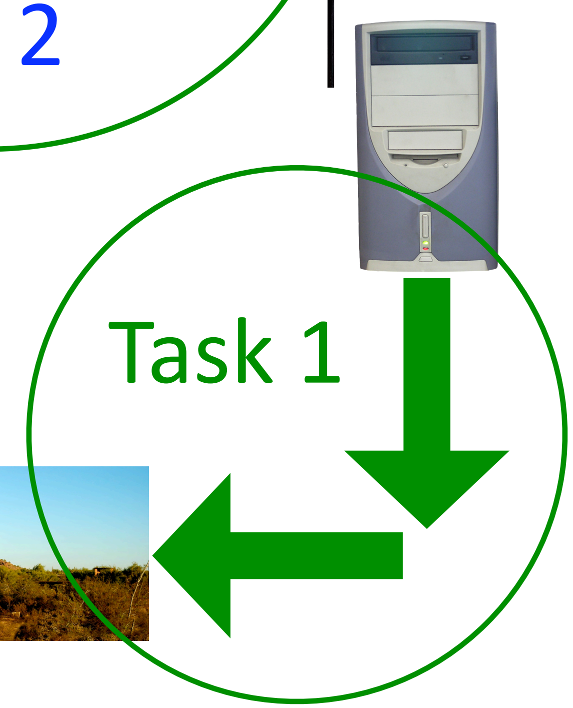
Distributed Sensor Networks : Task 1 : Detection

1. Detect state of environment

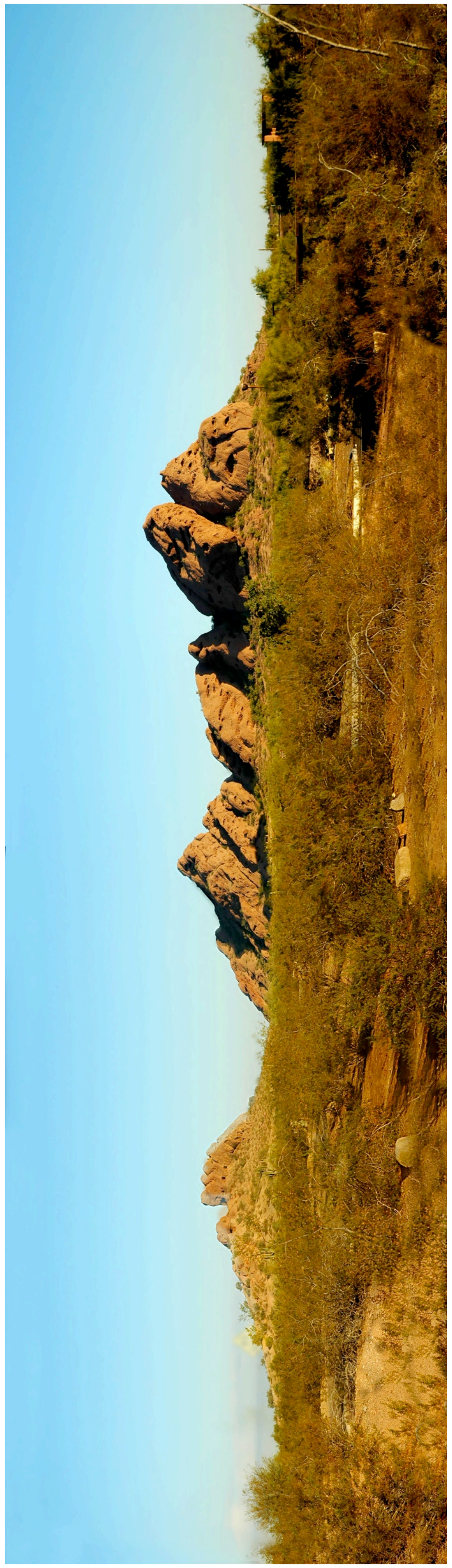
Distributed Sensor Networks : Task 2 : Communication

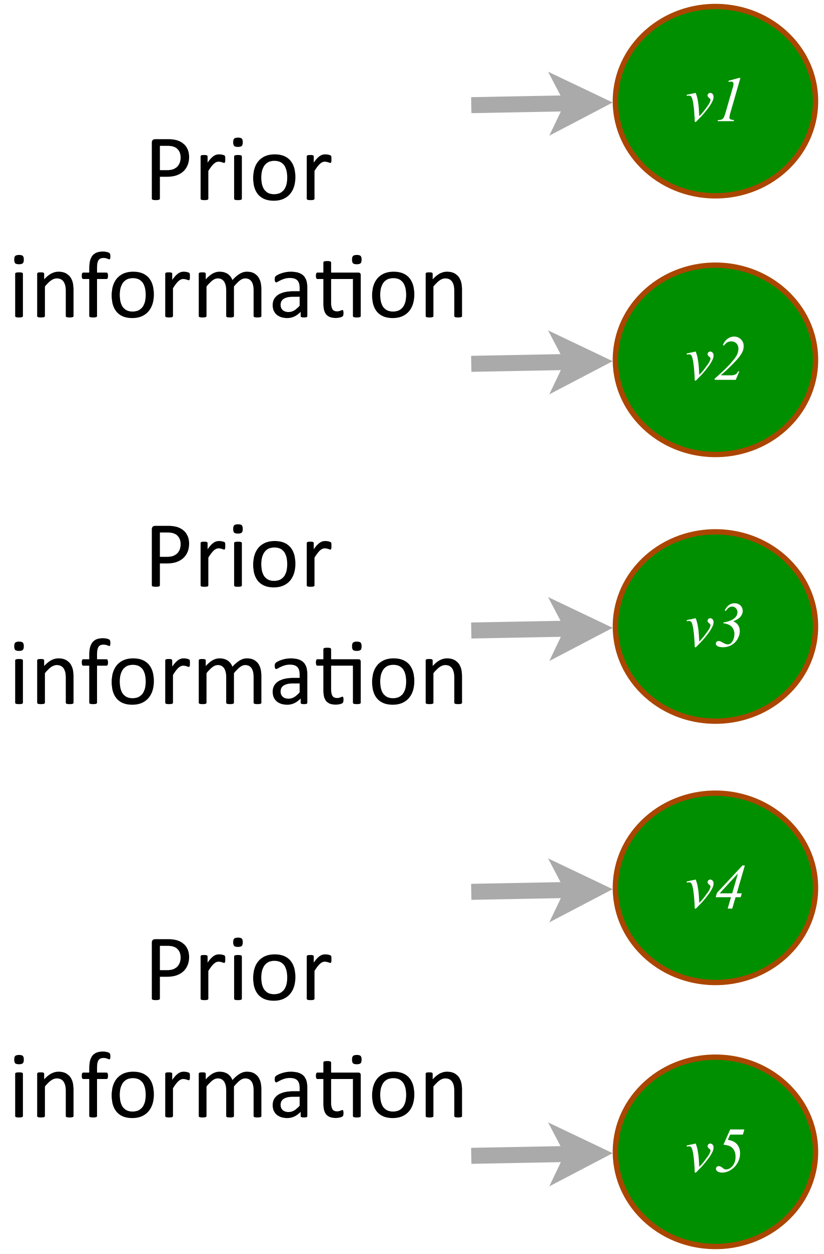
1. Detect state of environment

2. Communicate measurements

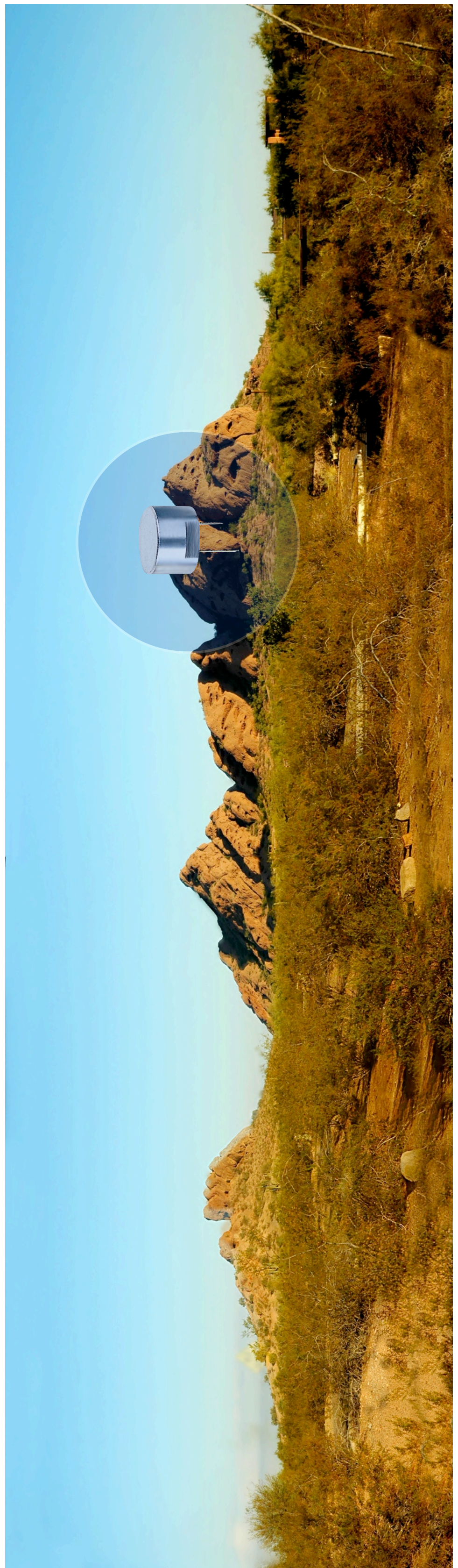


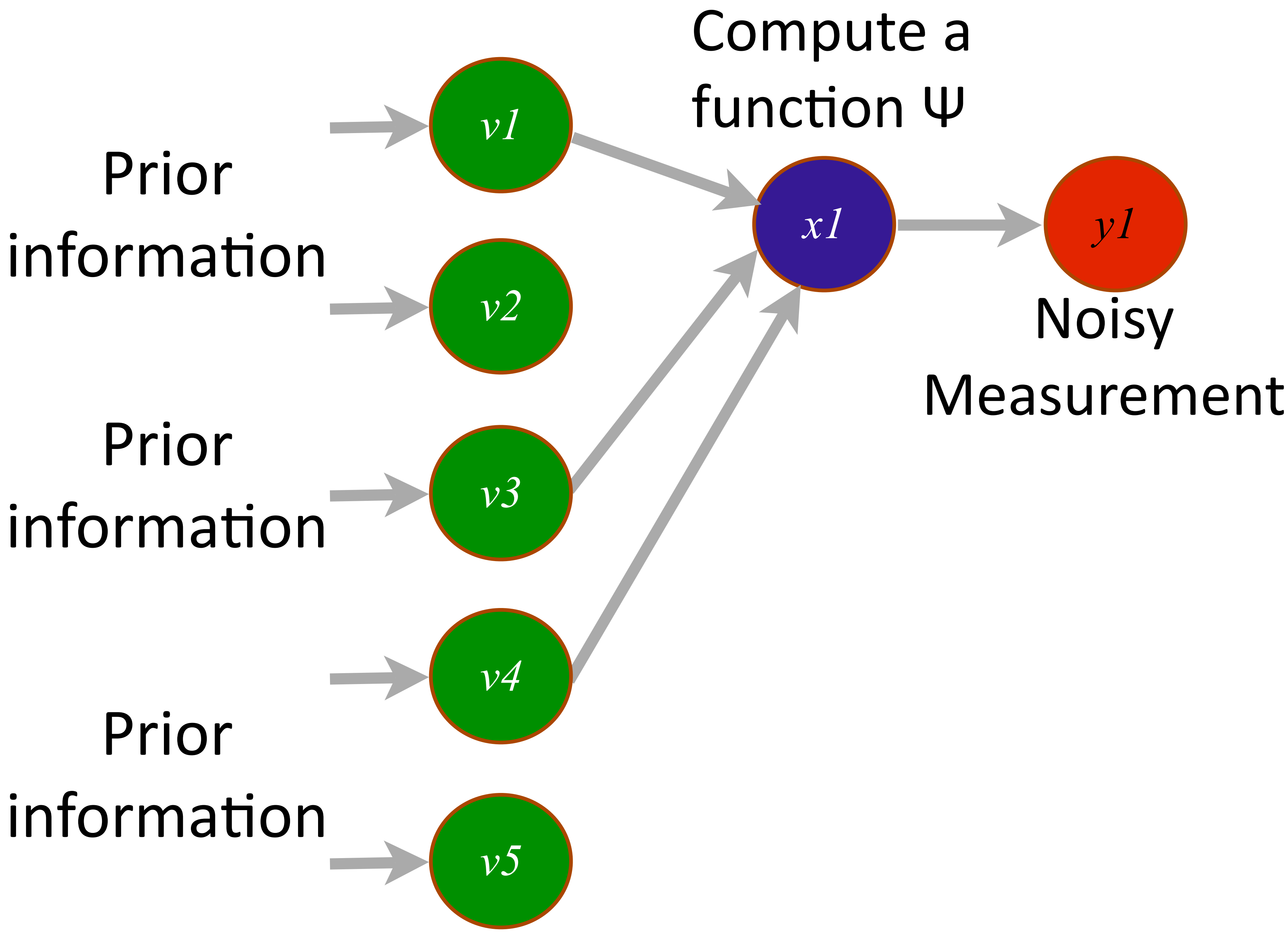
Task 1 : Detection : Modeling the problem



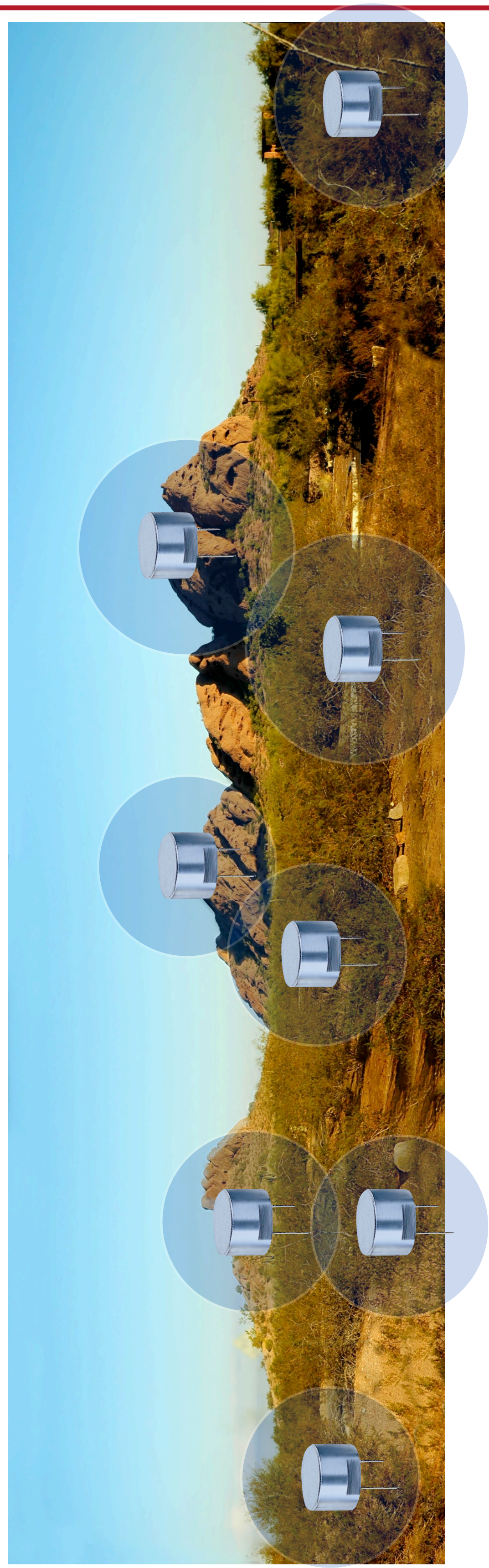


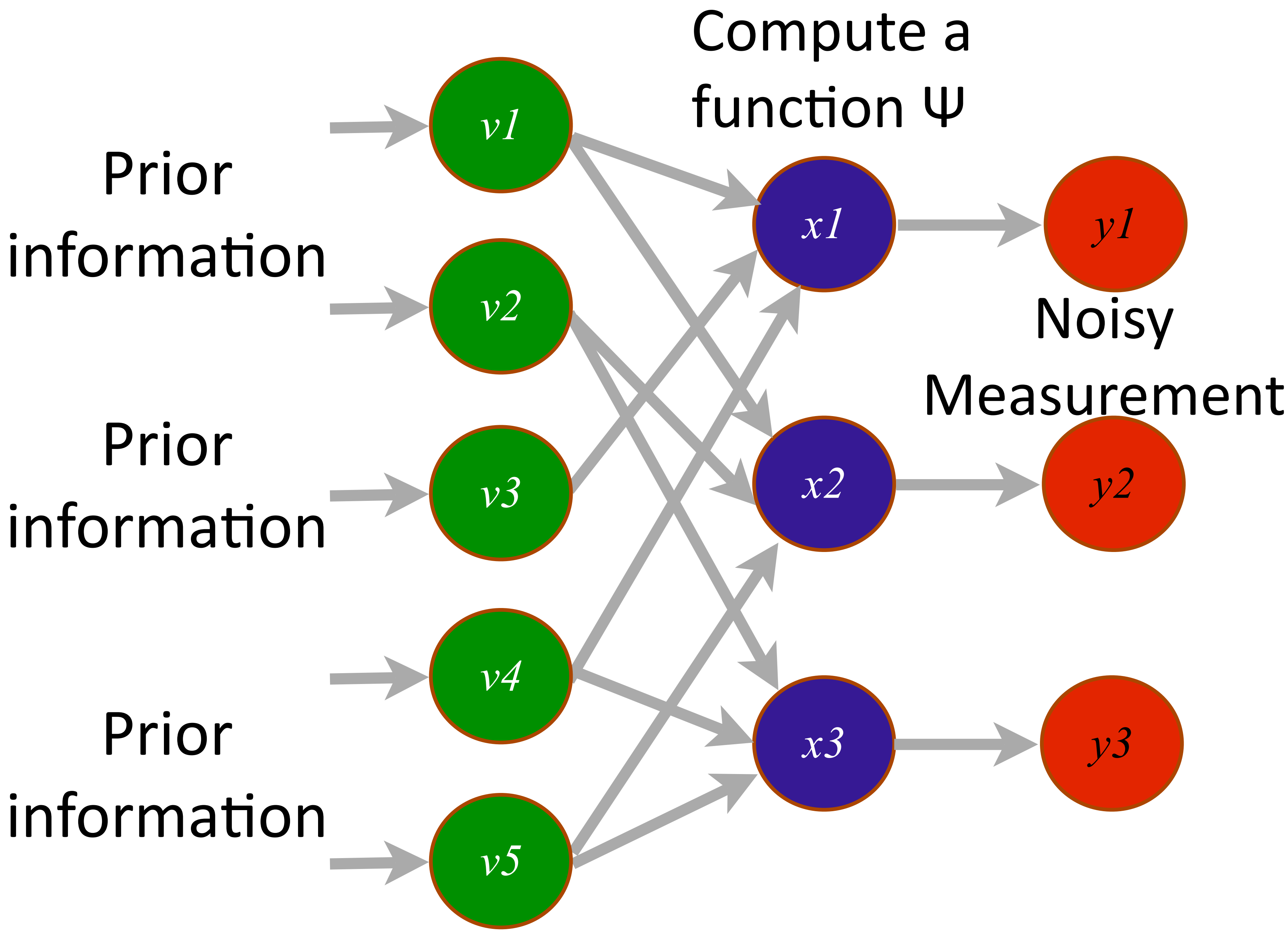
Task 1 : Detection : Modeling the problem



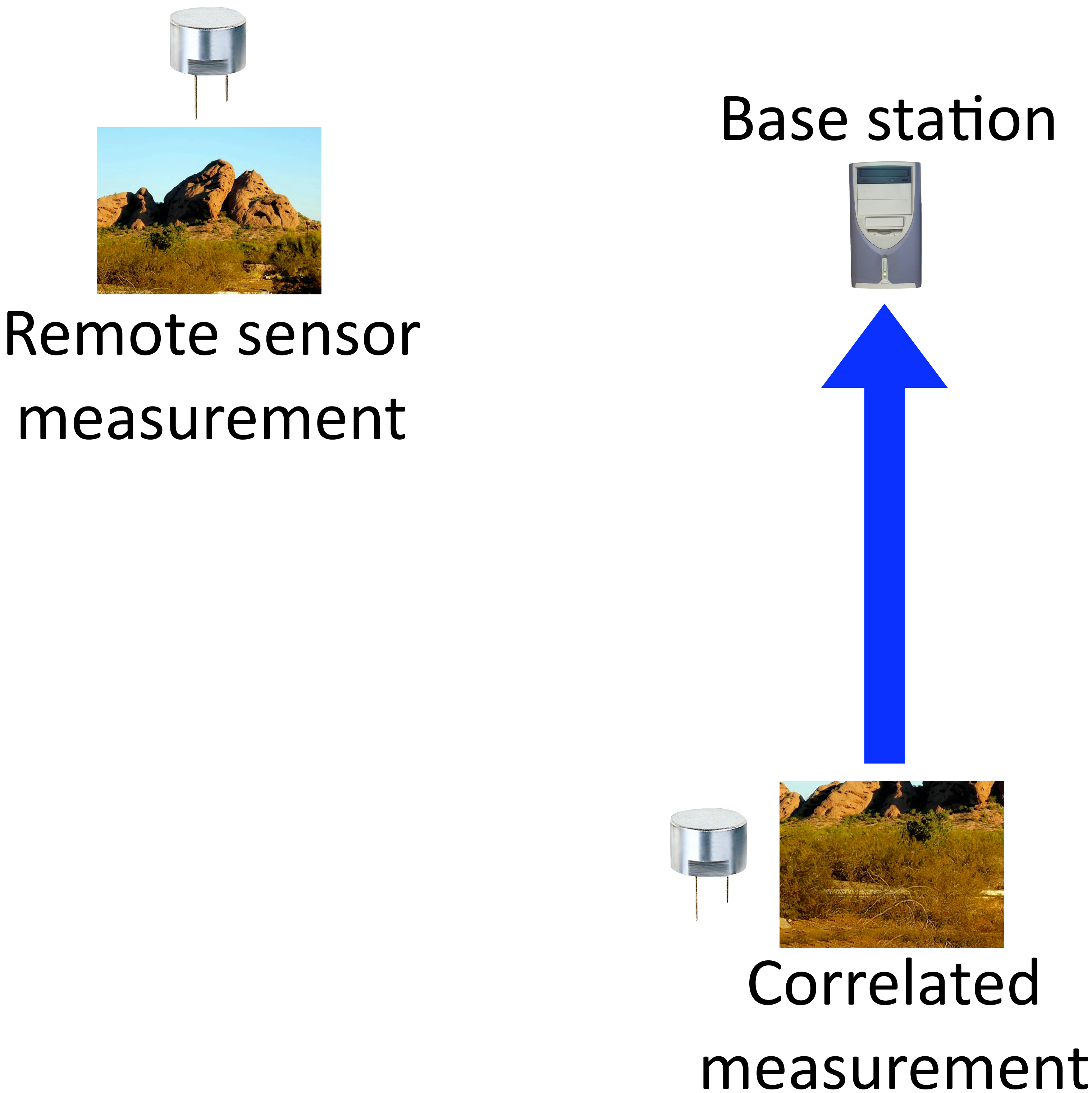


Task 1 : Detection : Graphical Model [1]

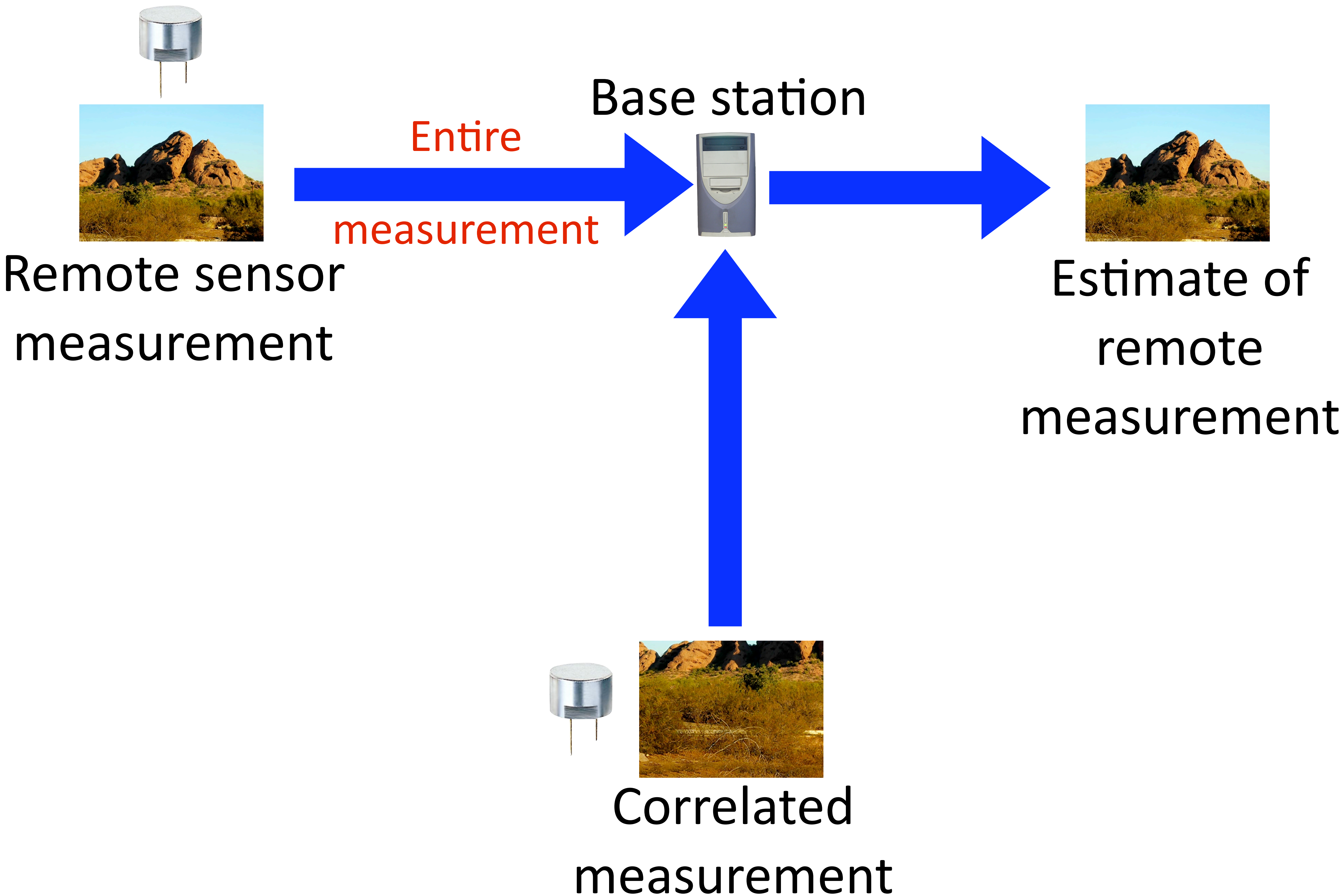




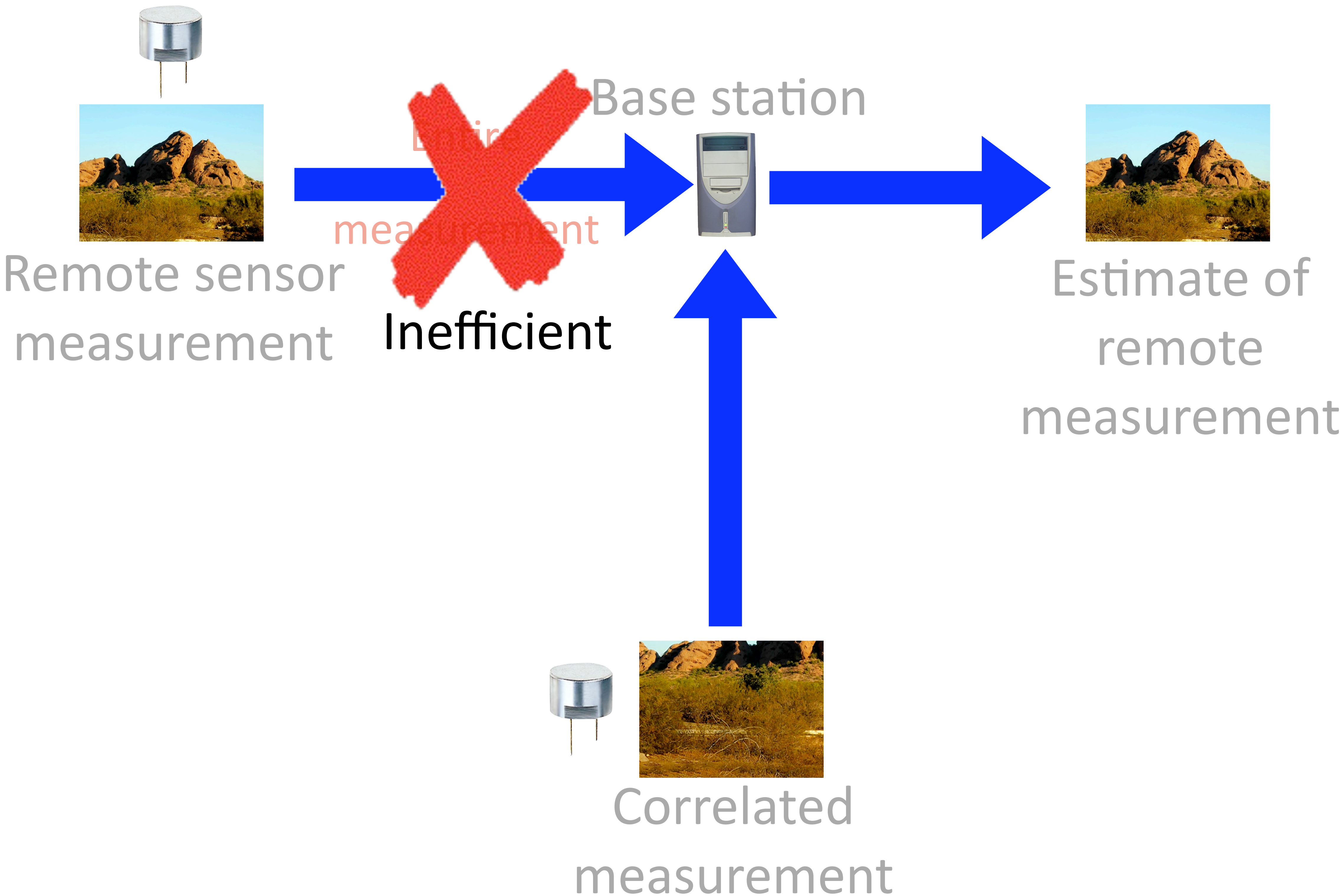
Task 2 : Distributed Source Coding : Problem Setup



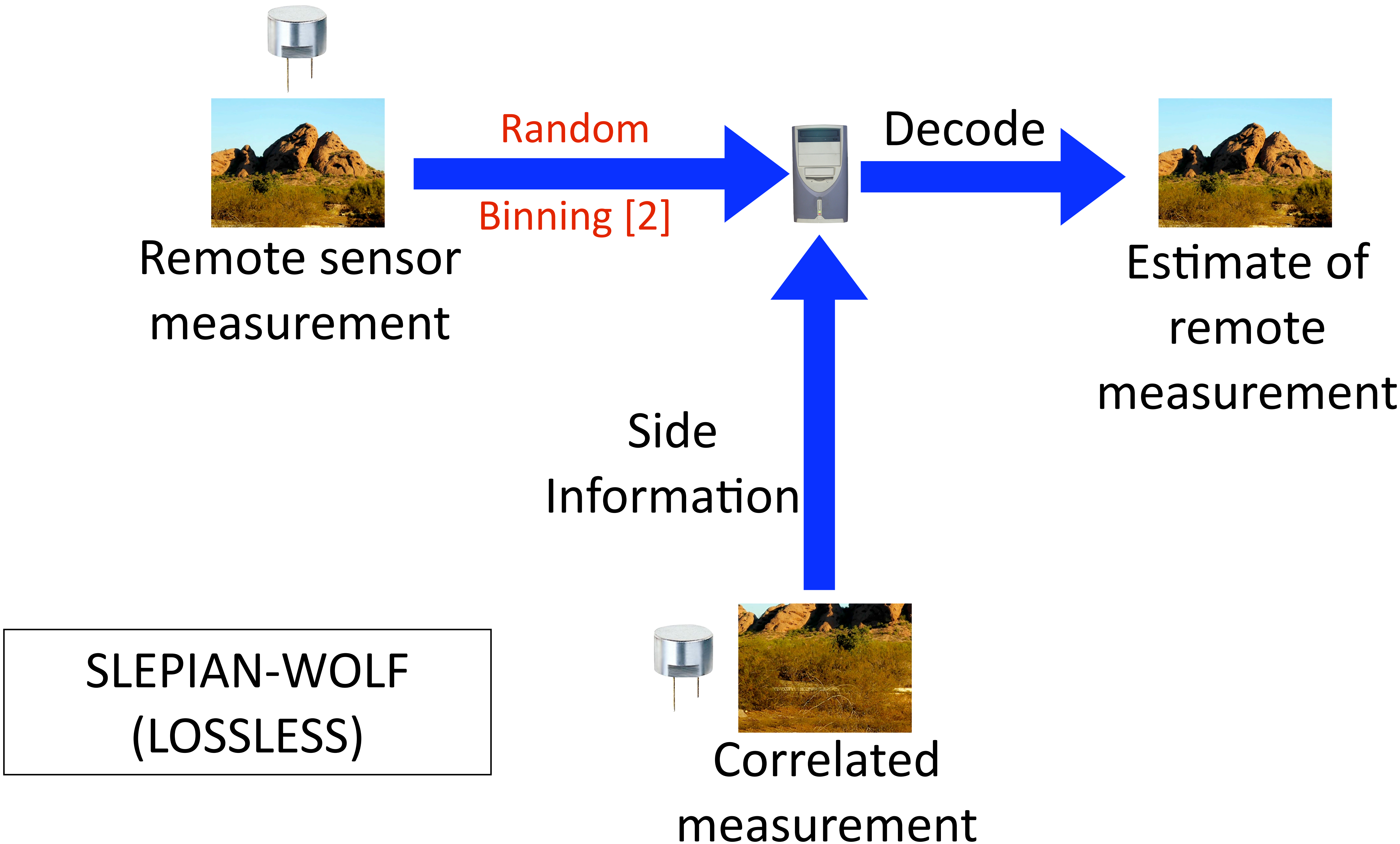
Task 2 : Distributed Source Coding : Naive Strategy



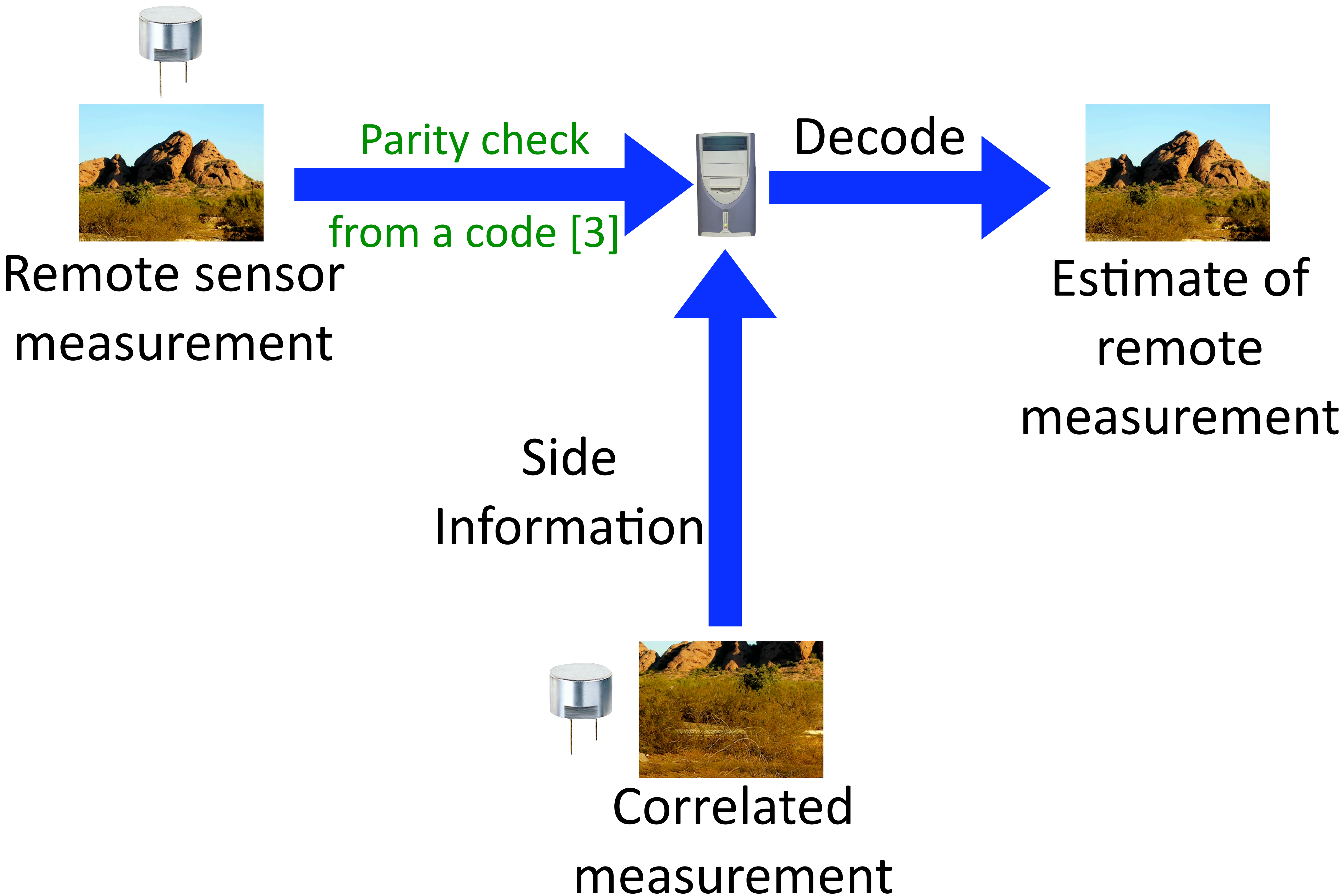
Task 2 : Distributed Source Coding : Naive Strategy



Task 2 : Distributed Source Coding : In Theory



Task 2 : Distributed Source Coding : In Practice



Task 2 : Distributed Source Coding : In Practice

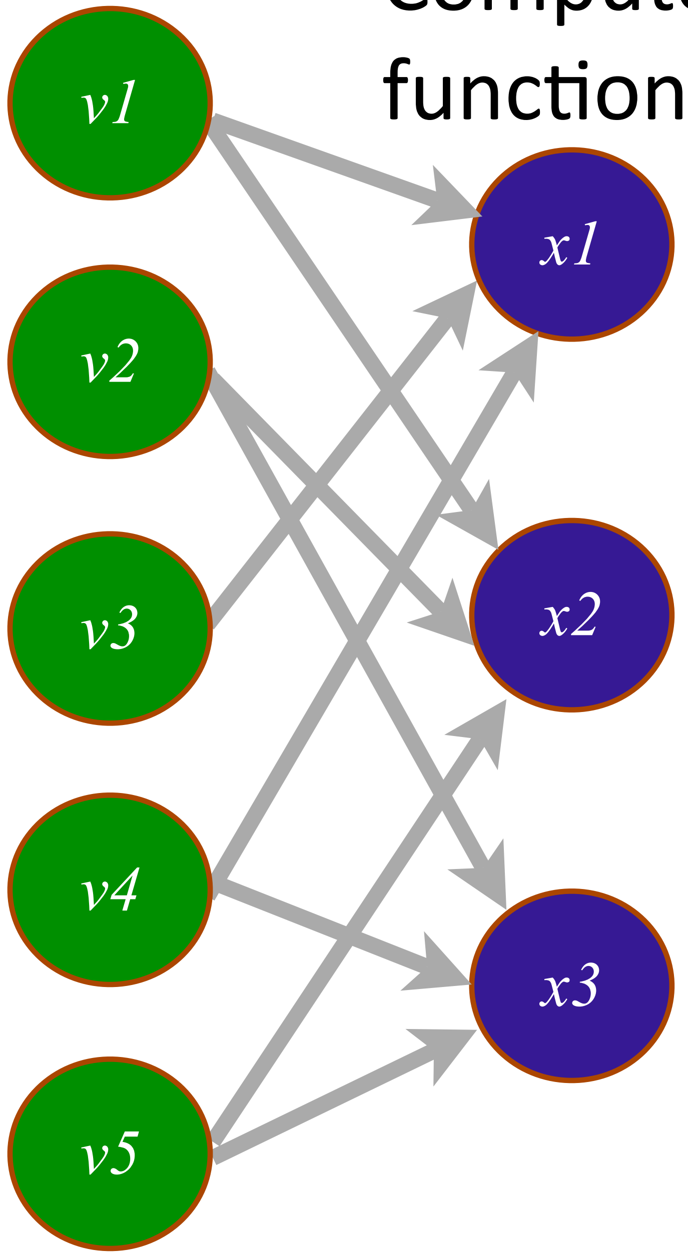
Remote sensor measurement

Task 2 : Distributed Source Coding : In Practice

Remote sensor measurement

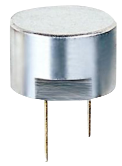
Parity check from a code

Compute A function Ψ



Task 2 : Distributed Source Coding : In Practice

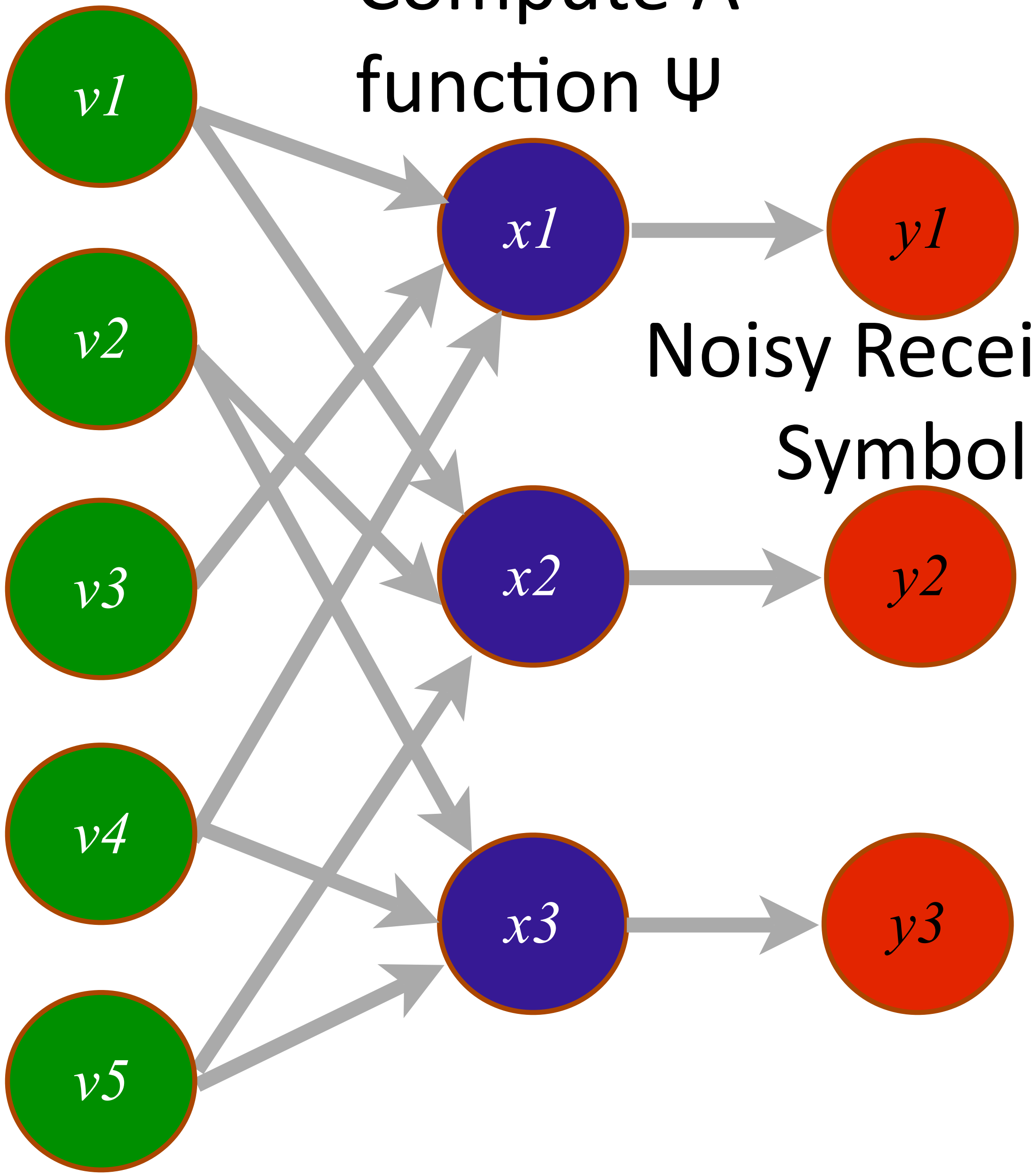
Remote sensor measurement



Parity check from a code

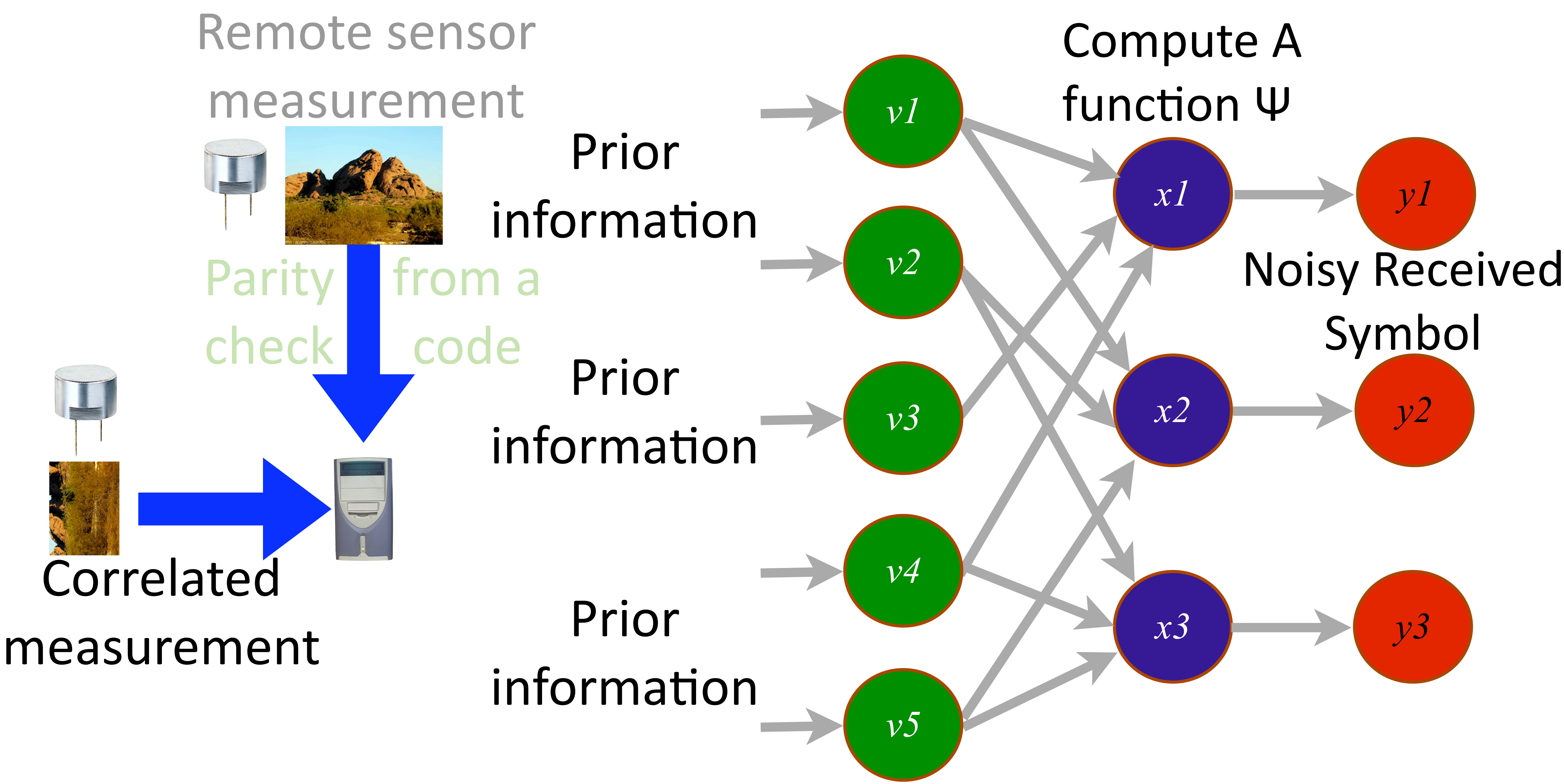
Base station

Compute A function Ψ



Noisy Received Symbol

Task 2 : Distributed Source Coding : Graphical Model

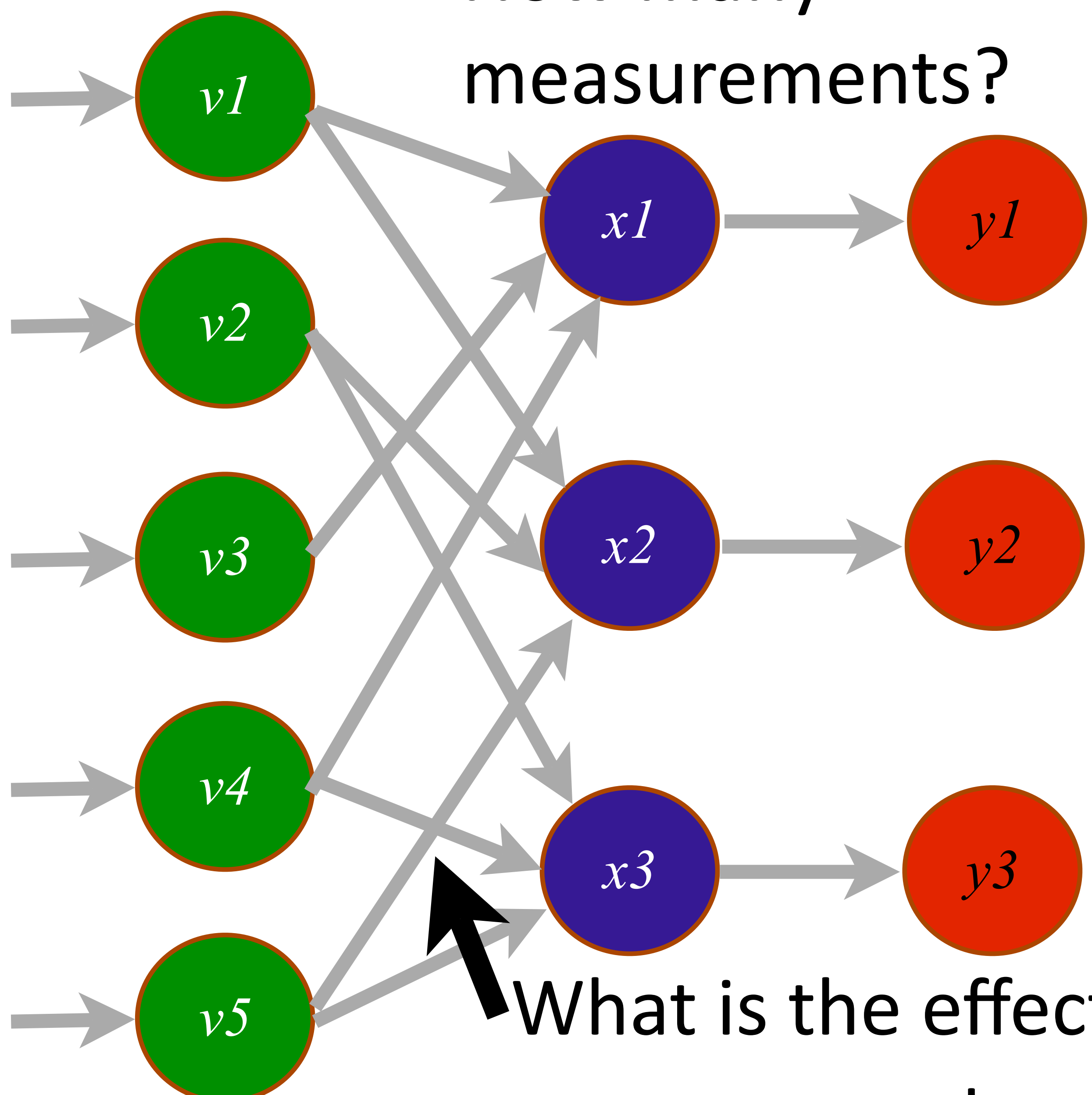


Important Questions in

Detection and Distributed Source Coding

What is the effect of prior information?

How many measurements?



What is the effect of noise?

What is the effect of choice of measurement configuration ?

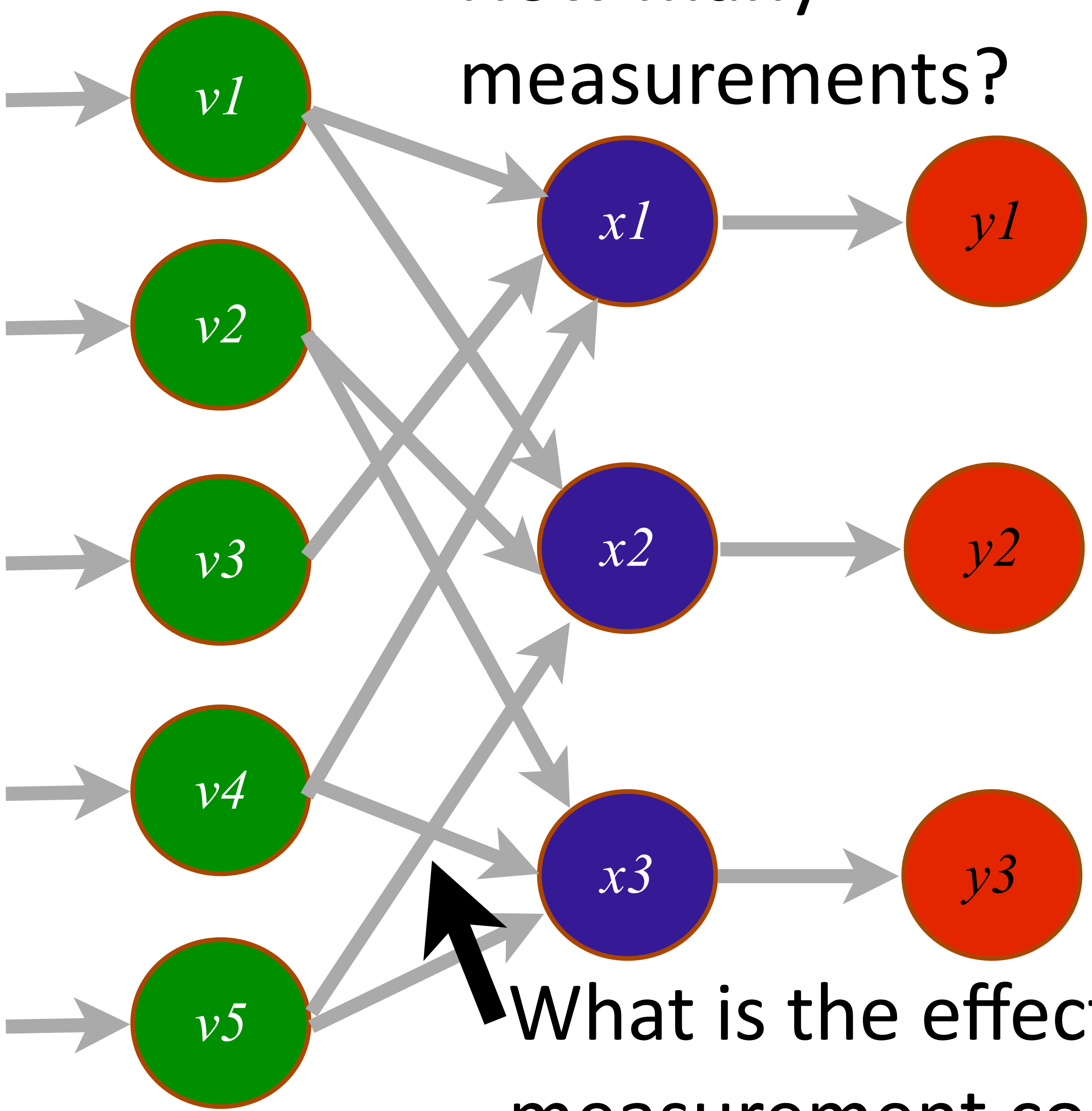
Important Questions in

Detection and Distributed Source Coding

Robustness to model mismatch and model uncertainty

How many measurements?

What is the effect of prior information?



What is the effect of noise?

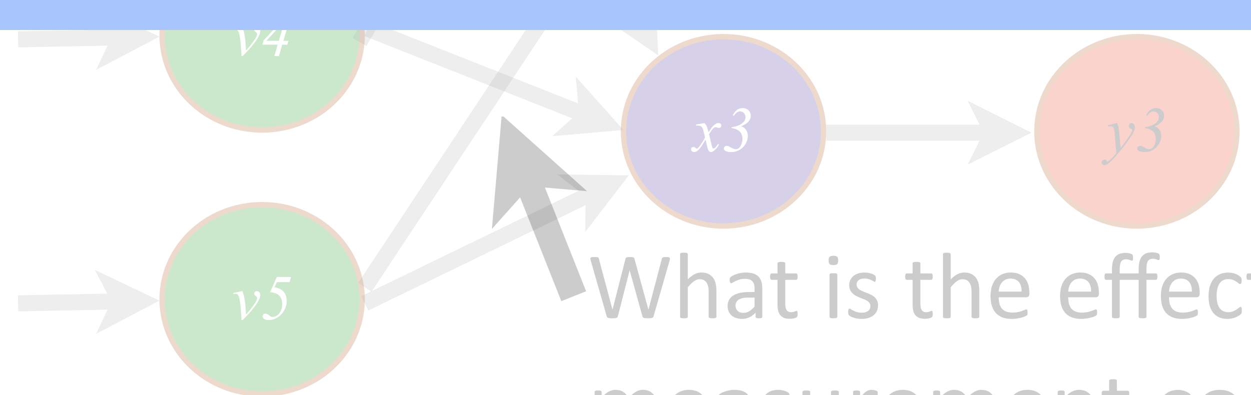
What is the effect of choice of measurement configuration ?

Important Questions in

Detection and Distributed Source Coding

Robustness to model mismatch and model uncertainty

We develop
a theoretical analysis
of the robustness of
practical encoders

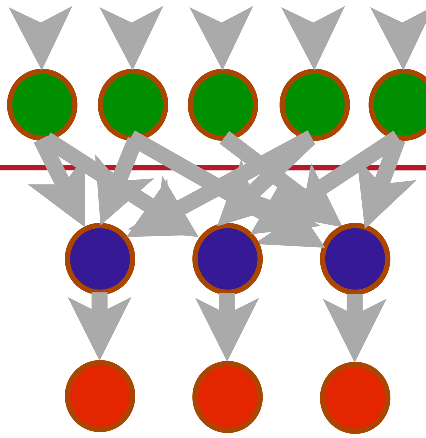


What is the effect of choice of measurement configuration ?

Related Problems

- Group Testing
- Sketching / Streaming in networks
- LT codes / LDGM codes
- Multi-user detection
- Compressed sensing with sparse measurements

Message



Detection and source coding can be cast into a common framework

We analyze the robustness to noise, mismatch and uncertainty using techniques from information theory

We use the theory to make design decisions

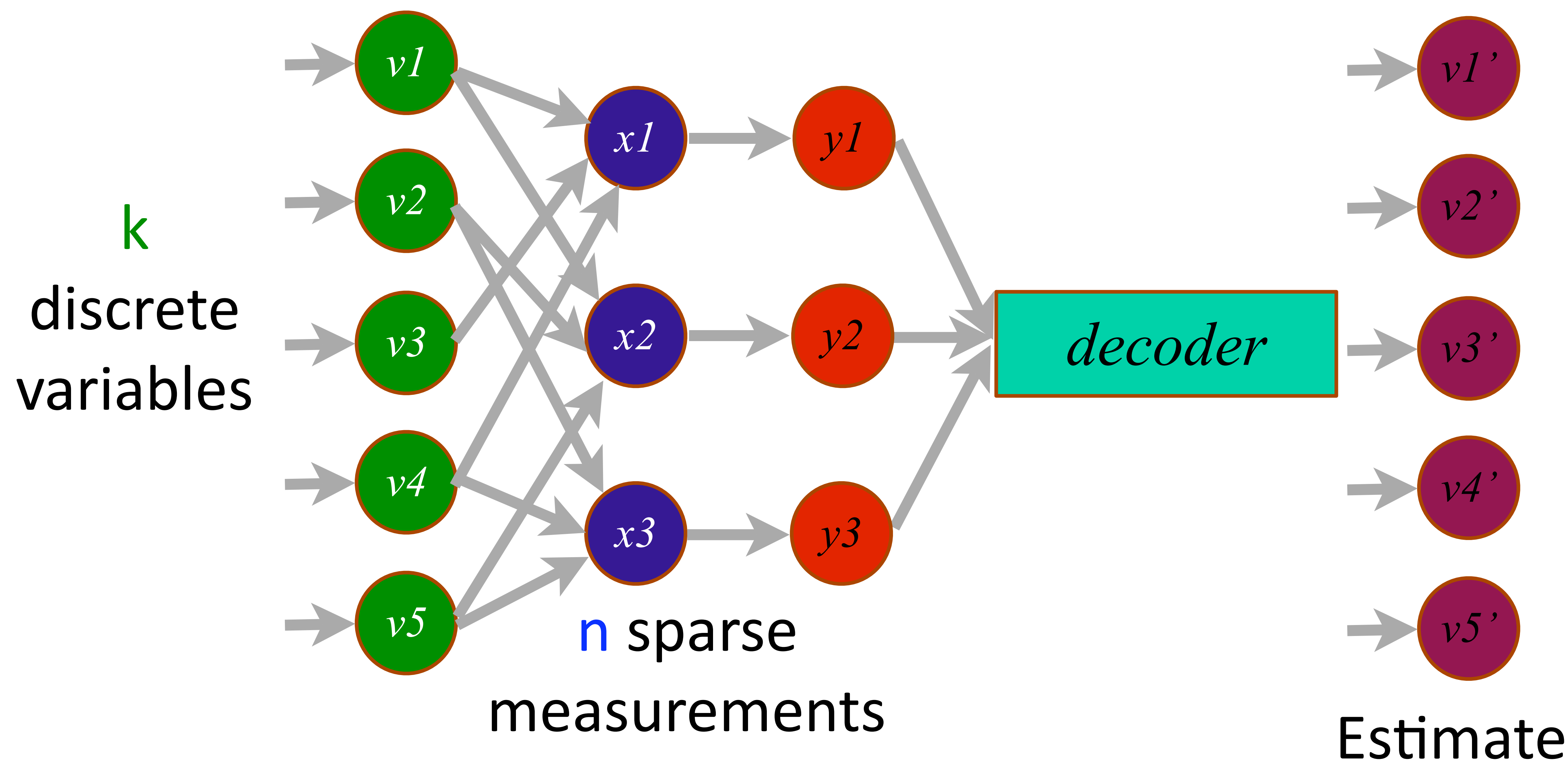
Outline

- Motivating applications
- Problem statement
- Intuition behind the analysis
- An application

Problem Statement

Rate R=k/n= $\frac{\text{\# of inputs}}{\text{\# of measurements}}$

Distortion = (1/k) Hamming Distance(v,v')



Problem Statement

$$\text{Rate } R = k/n = \frac{\text{\# of inputs}}{\text{\# of measurements}}$$

Distortion = (1/k) Hamming Distance(v,v')

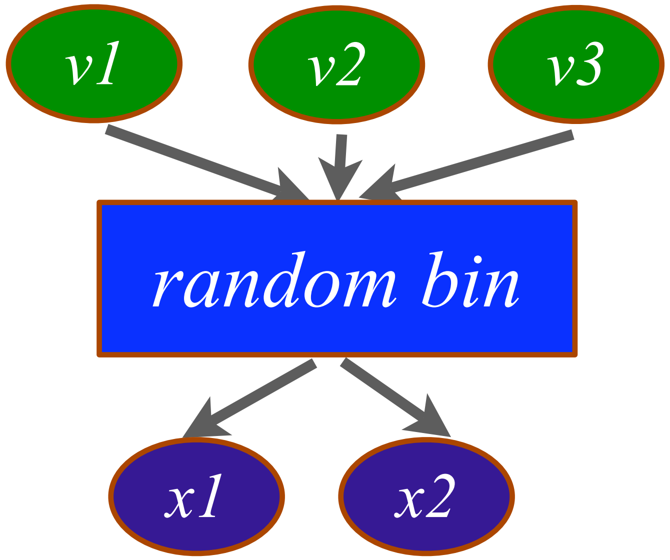
Error if Distortion > D

Sensing Capacity [4] : C(D) : Maximum R such that Pr(Error) → 0 as n →∞

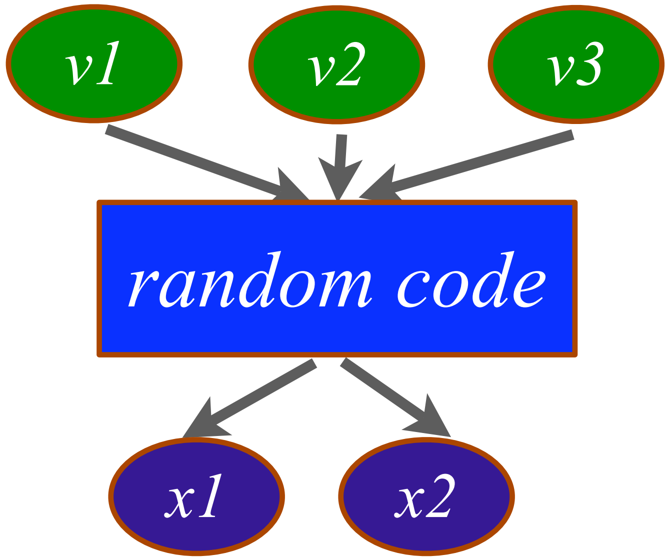
We lower bound the Sensing Capacity : $C_{LB}(D)$

Insight : Parallels to Information Theory

- Random Binning methods in source coding



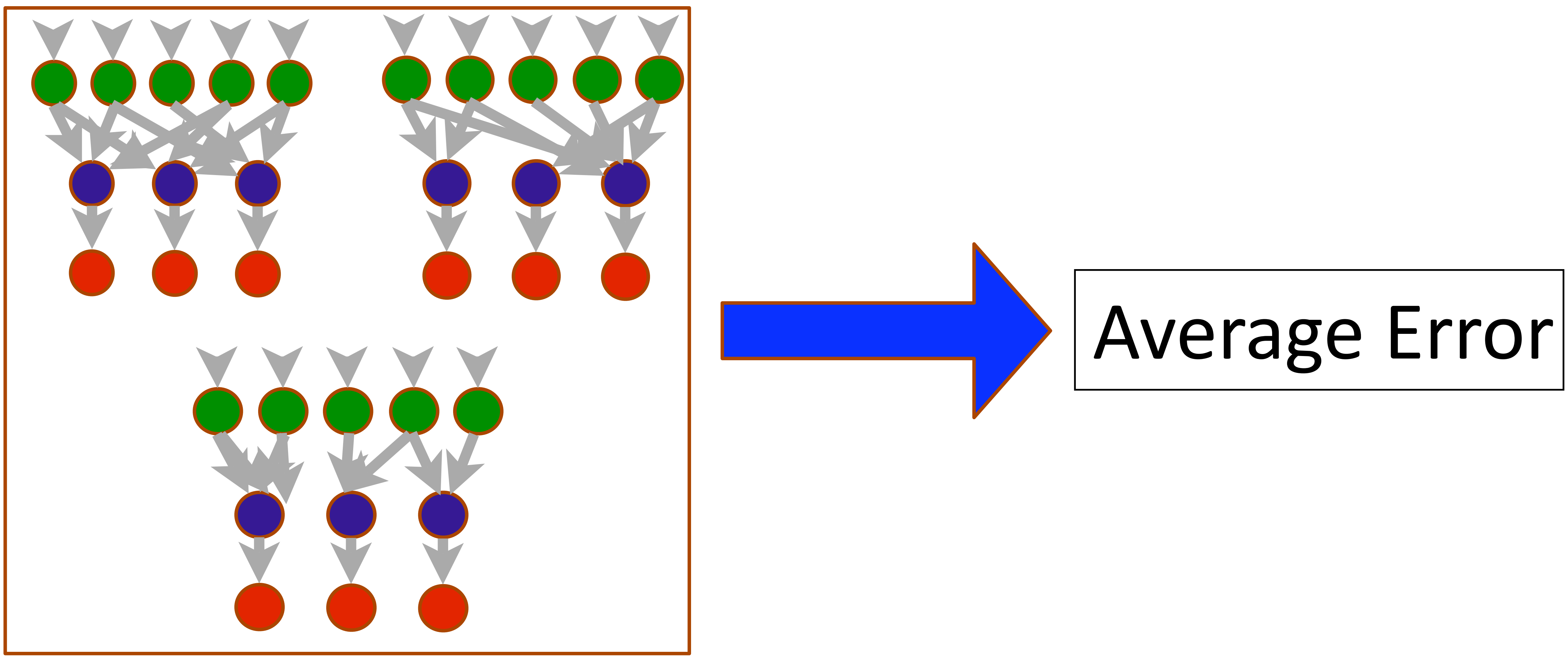
- Random Coding methods in communication



- Random Measurements ?

Insight : Parallels to Information Theory

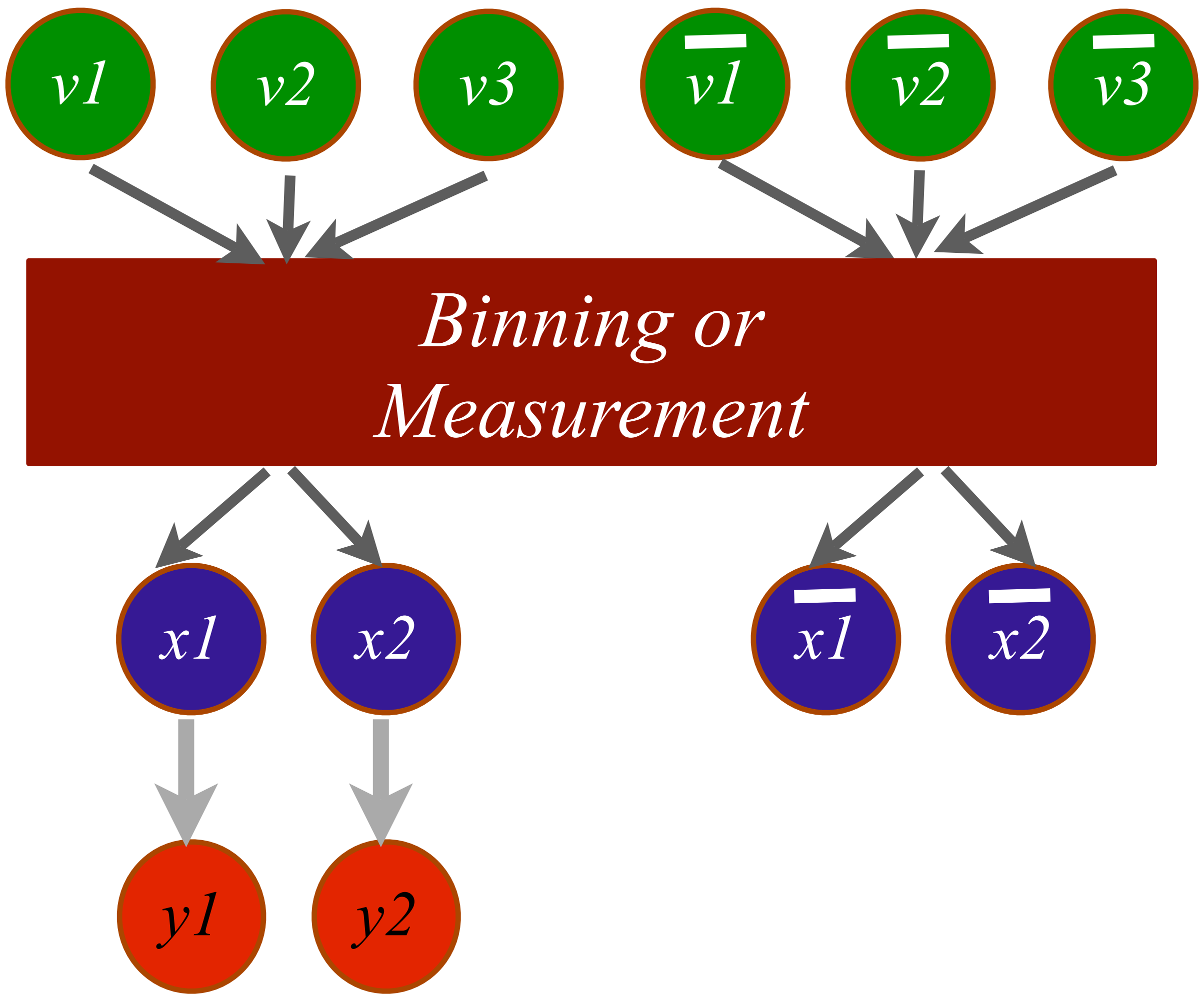
- **Proof using Random Measurements**
  - Random measurement configuration → generates codebook
  - Calculate average error across random ensemble of measurements
  - If average error → 0, then for some configuration error → 0



Outline

- Motivating applications
- Problem statement
- Intuition behind the analysis
- An application

Description of the decoder



A similarity metric
S(x, y)

find measurement vector x and corresponding environment v that maximizes S(x, y)

Union bounding - Gallager-Fano bounding technique [5]

$$\Pr[\text{Error} \mid v \text{ is true}] = \Pr[\text{Decode to } v' \text{ s.t distortion}(v,v') > D \mid v \text{ is true}]$$

$$= \sum_{\text{dist}(v') > D} \Pr[\text{Decode to } v' \mid v \text{ is true}]$$

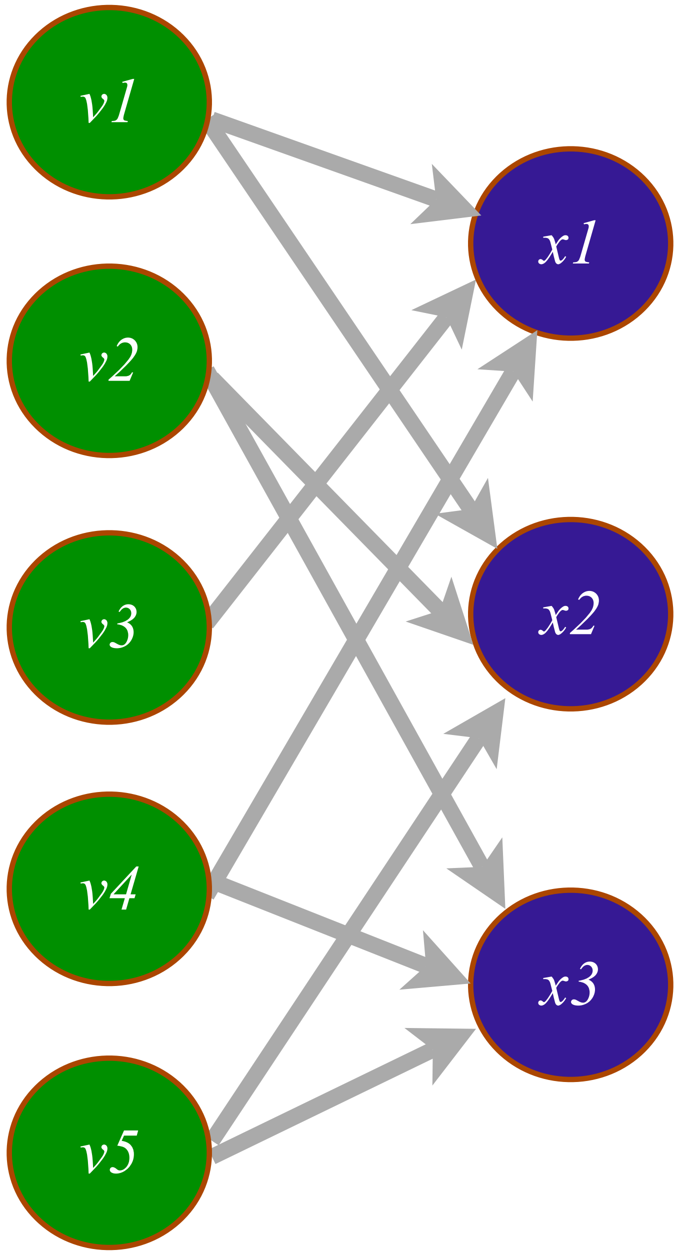
Exponential number of terms !!!

Group terms into polynomial number of groups g using symmetry

$$\Pr[\text{Error} \mid v \text{ is true}] = \sum_{g} (\text{number of } v' \text{ in } g) \Pr[\text{Decode to } v' \text{ in } g \mid v \text{ true}]$$

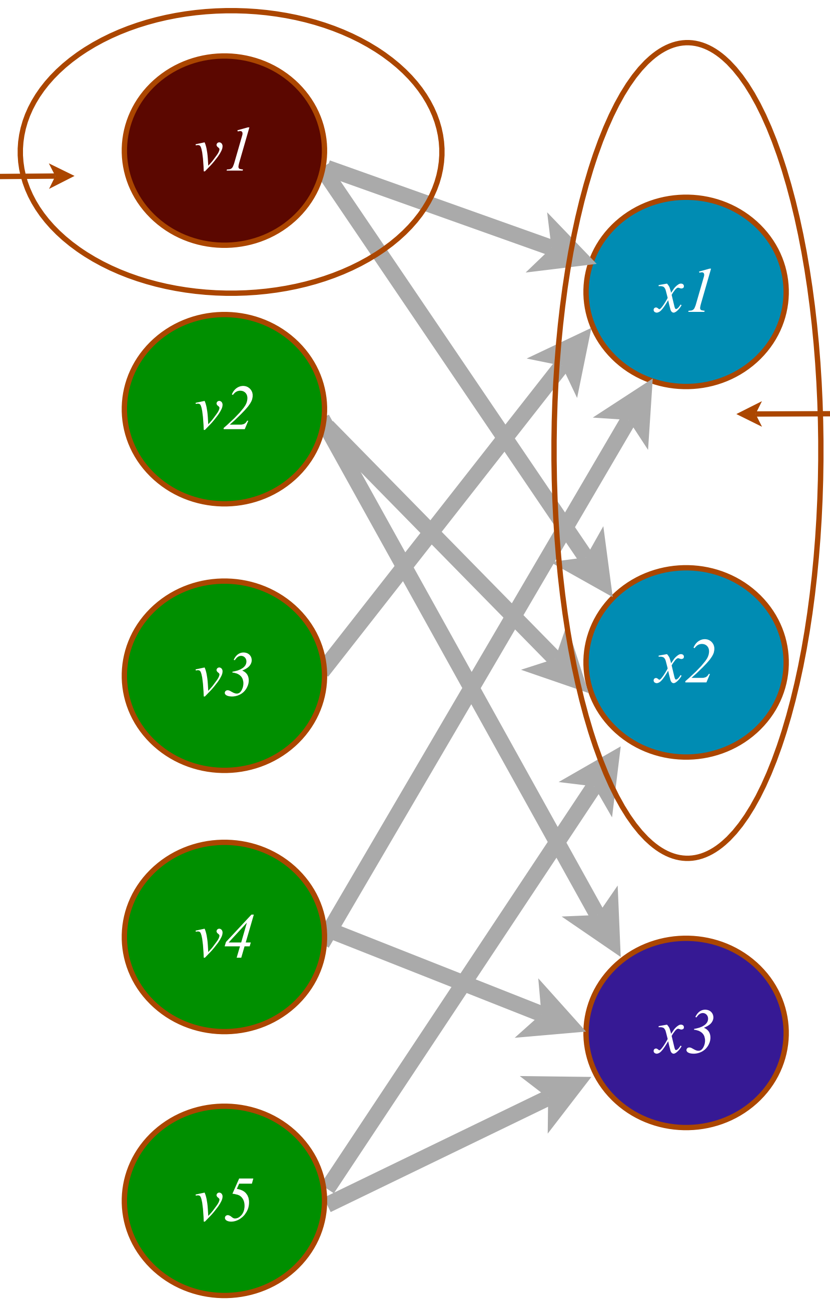
$$\leq |g| \max_{g} (\text{number of } v' \text{ in } g) \Pr[\text{Decode to } v' \text{ in } g \mid v \text{ true}]$$

Non-i.i.d codewords



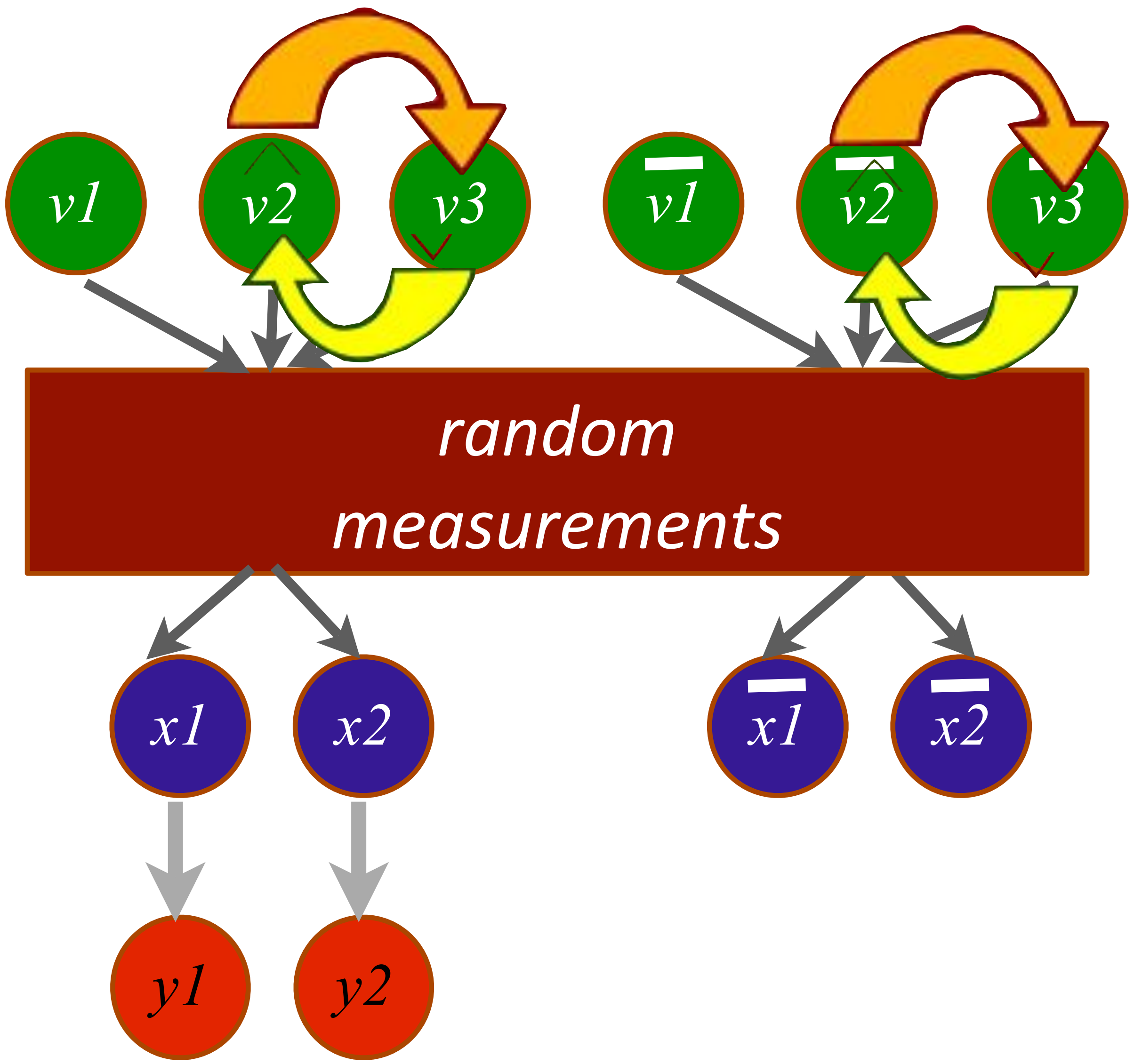
Non-i.i.d codewords

Change one input



Changes only a few outputs

Permutation invariant measurement ensembles



P(X)

depends only on type Υ of v

$Q(\bar{X} \mid X)$

depends only on joint type λ of v and $\bar{v}$

[1/2 1/2]

Υ

[1/4 1/4 1/4 1/4]

λ

Large deviations

$$\Pr[\text{Error given } v] \approx \max_{\lambda\,:\,Distortion(\lambda)>D} (\text{number of } v' \text{ at } \lambda)\ \Pr[\text{Decode to } v' \text{ at } \lambda]$$

$$\text{Number of } v' \text{ at } \lambda \leq 2^{k[H(\lambda)-H(\Upsilon)]}$$

$$\Pr[\text{Decode to } v' \text{ at } \lambda] = \Pr[S(x', y) > S(x, y)]$$

$$\leq 2^{-nT(\lambda)}$$

Heart of the main theorem

$$\frac{1}{N}\log\left(\frac{S(x, y)}{E[S(x', y)]}\right) \longrightarrow T(\lambda)$$

Lower Bound on Sensing Capacity

A rate R is achievable (for a joint type λ) if,

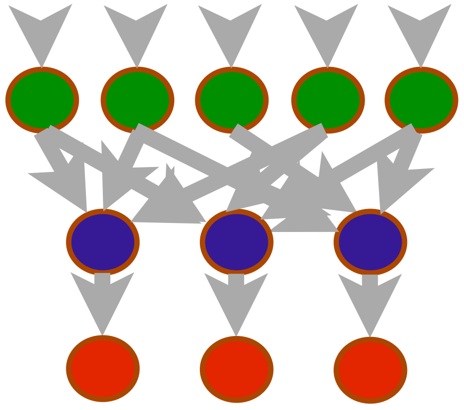
$$R < \frac{T(\lambda)}{[H(\lambda)-H(\Upsilon)]}$$

A rate R is achievable (for a distortion *D*) if,

$$R < C_{LB}(D) = \min_{\lambda : Dis(\lambda) > D} \frac{T(\lambda)}{[H(\lambda)-H(\Upsilon)]}$$

Generality of the result

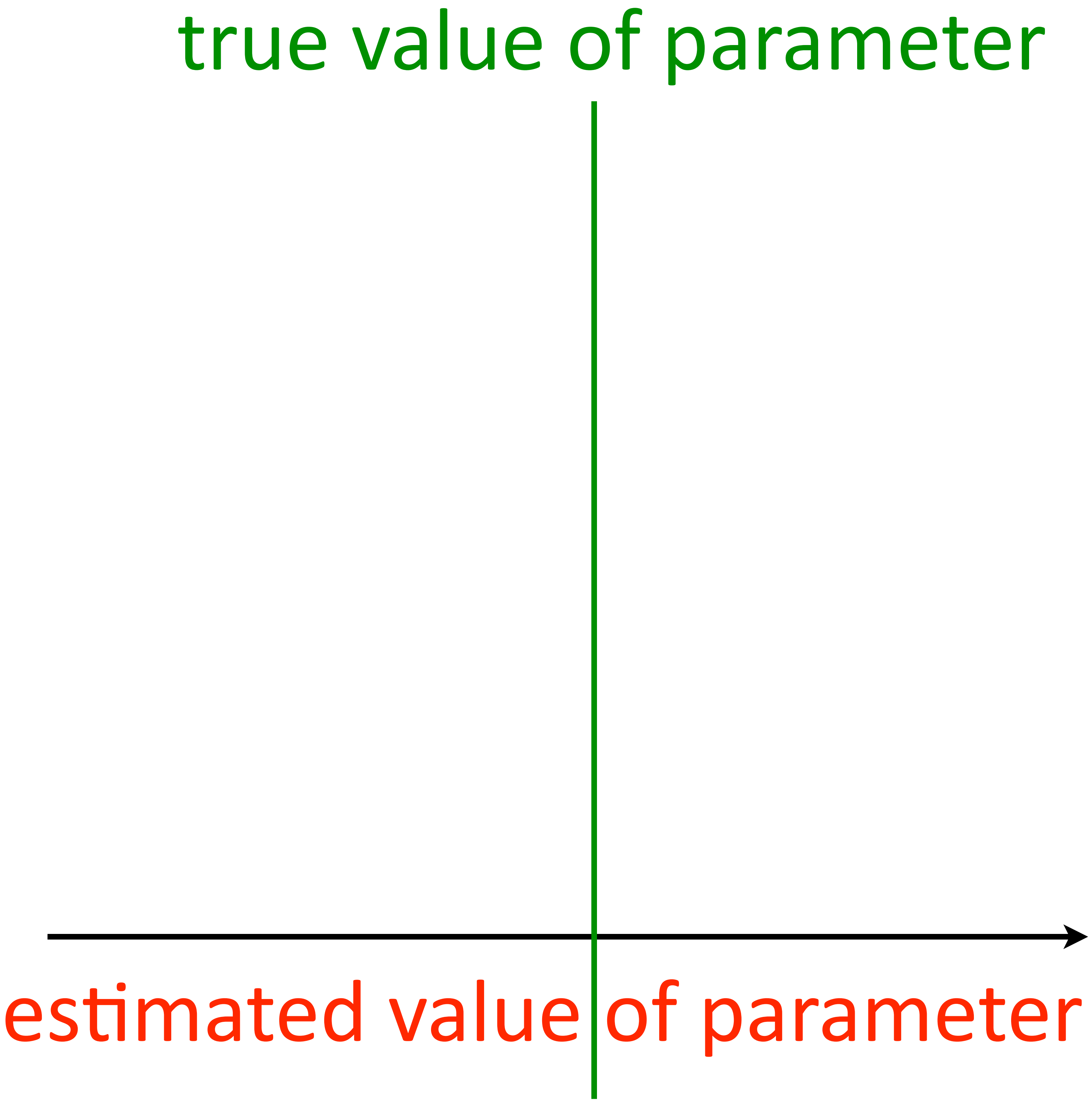
- **Different similarity metrics**
  - –ML decoder - $S(x, y) = \prod P(y_i | x_i)$
  - –Mismatch - $S(x, y) = \prod P_\theta(y_i | x_i)$
  - –Uncertain - $S(x, y) = \sum_\theta \prod P_\theta(y_i | x_i)$
- **Different random measurement ensembles**
  - –Check regular ensembles
  - –Check and bit regular ensembles



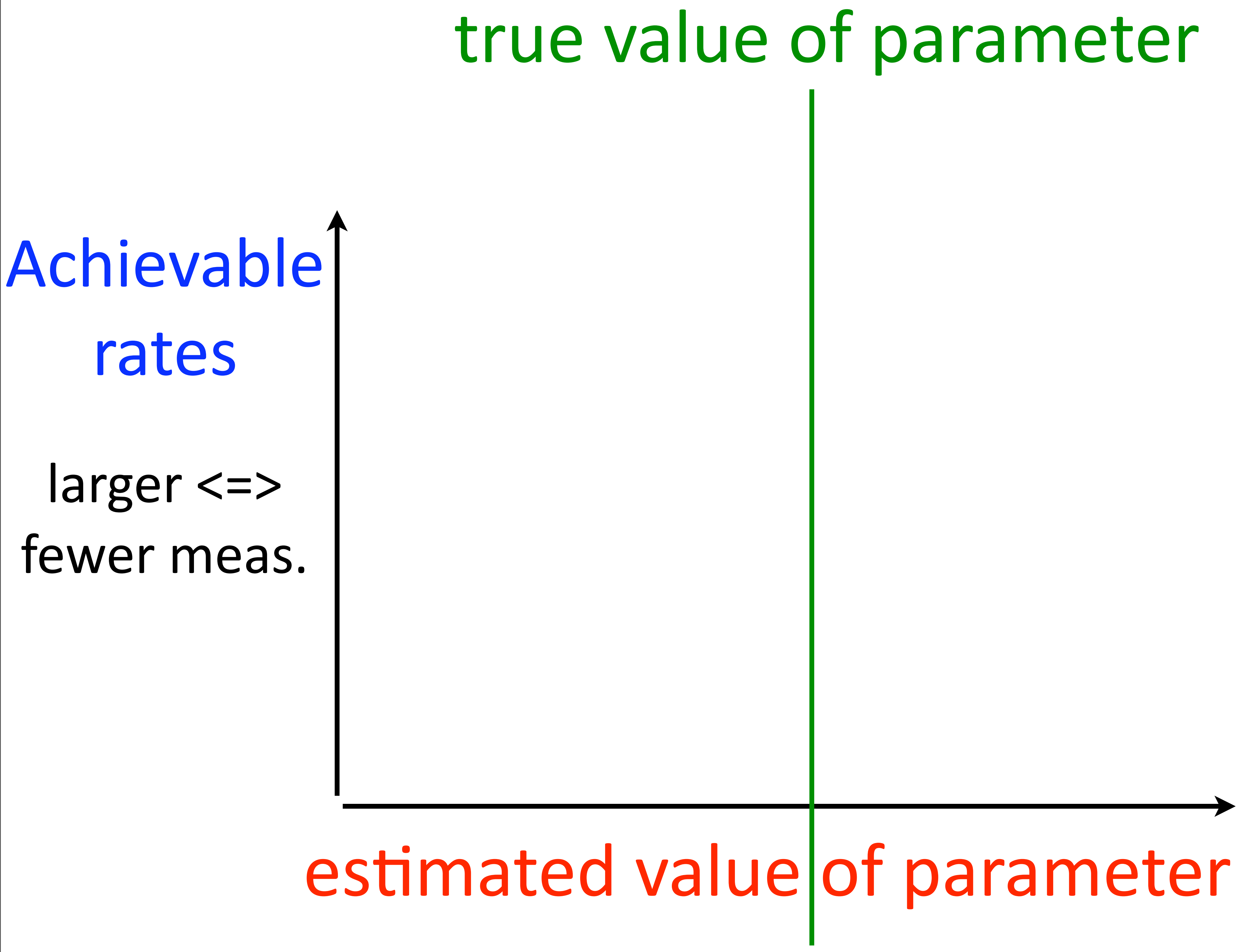
Outline

- Motivating applications
- Problem statement
- Intuition behind the analysis
- An application

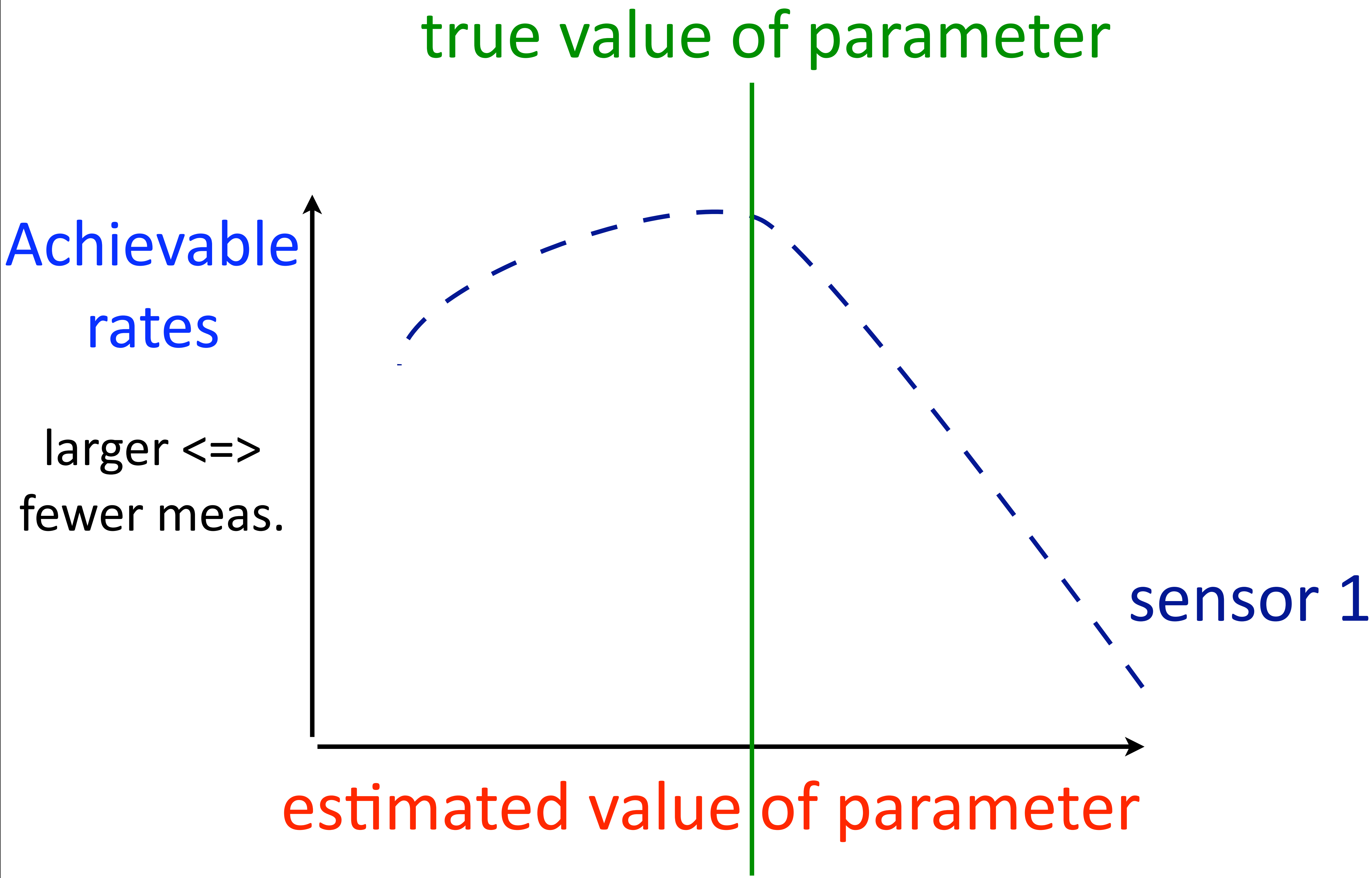
Performance of mismatched decoders



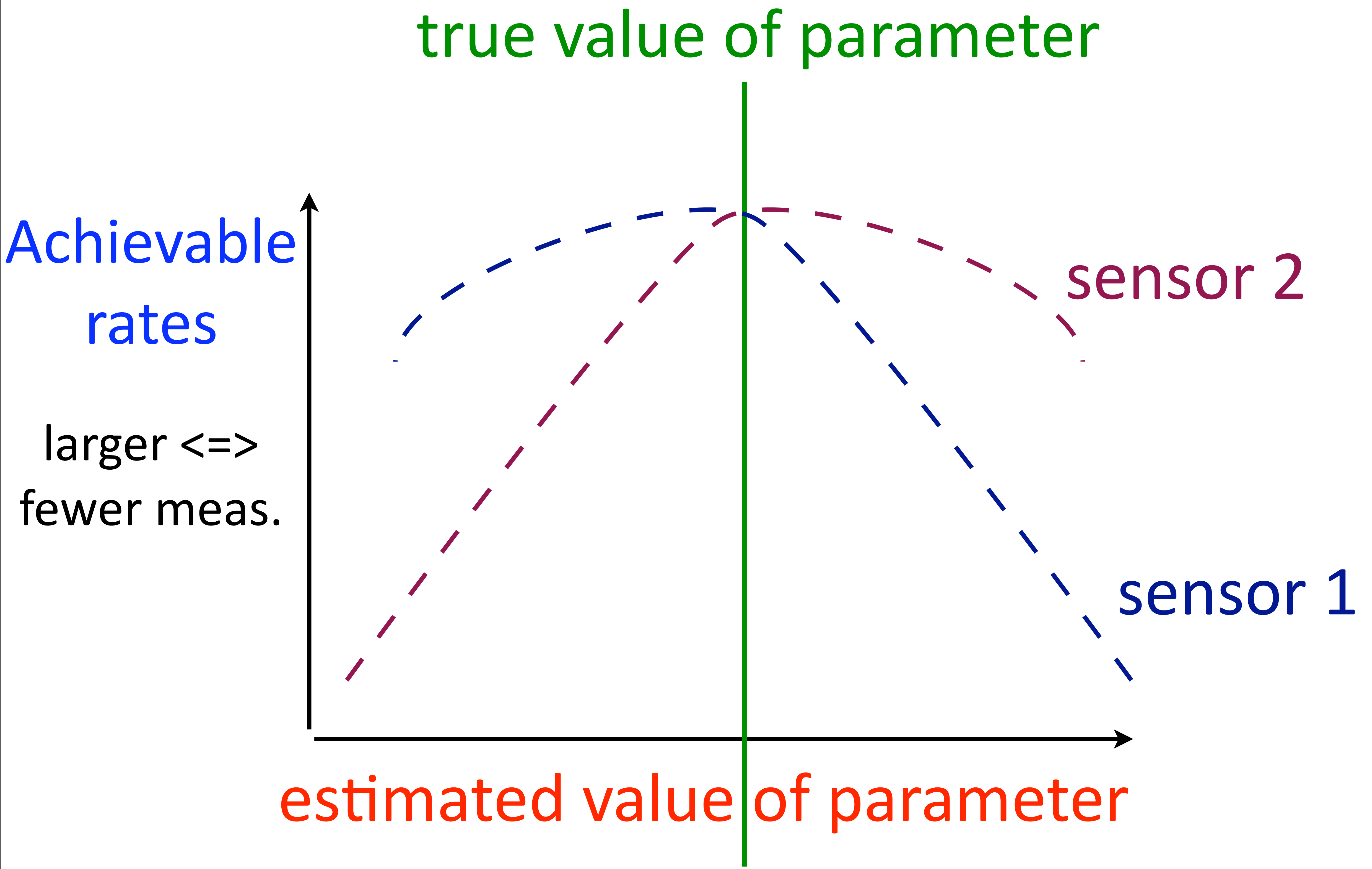
Performance of mismatched decoders



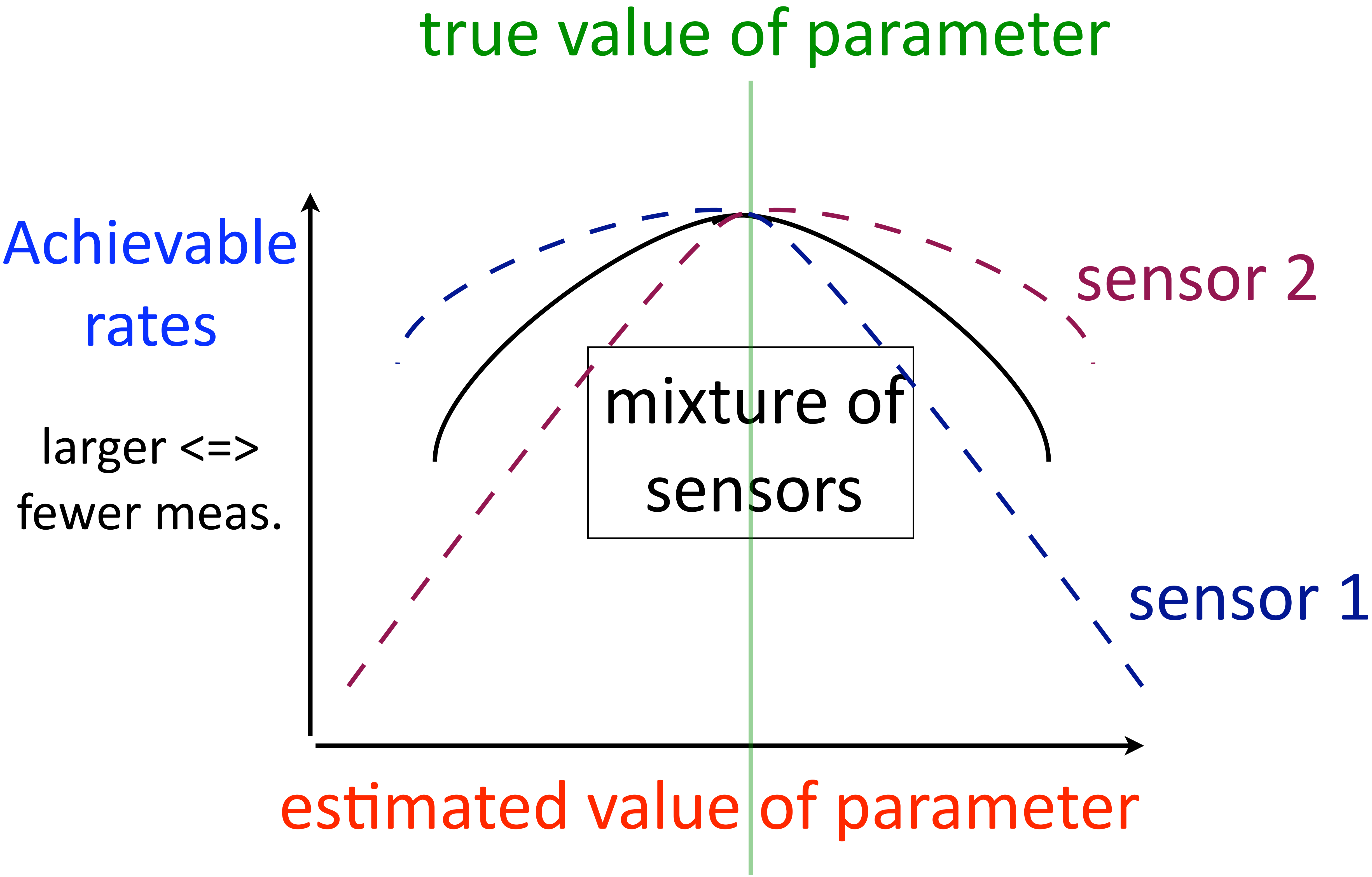
Performance of mismatched decoders



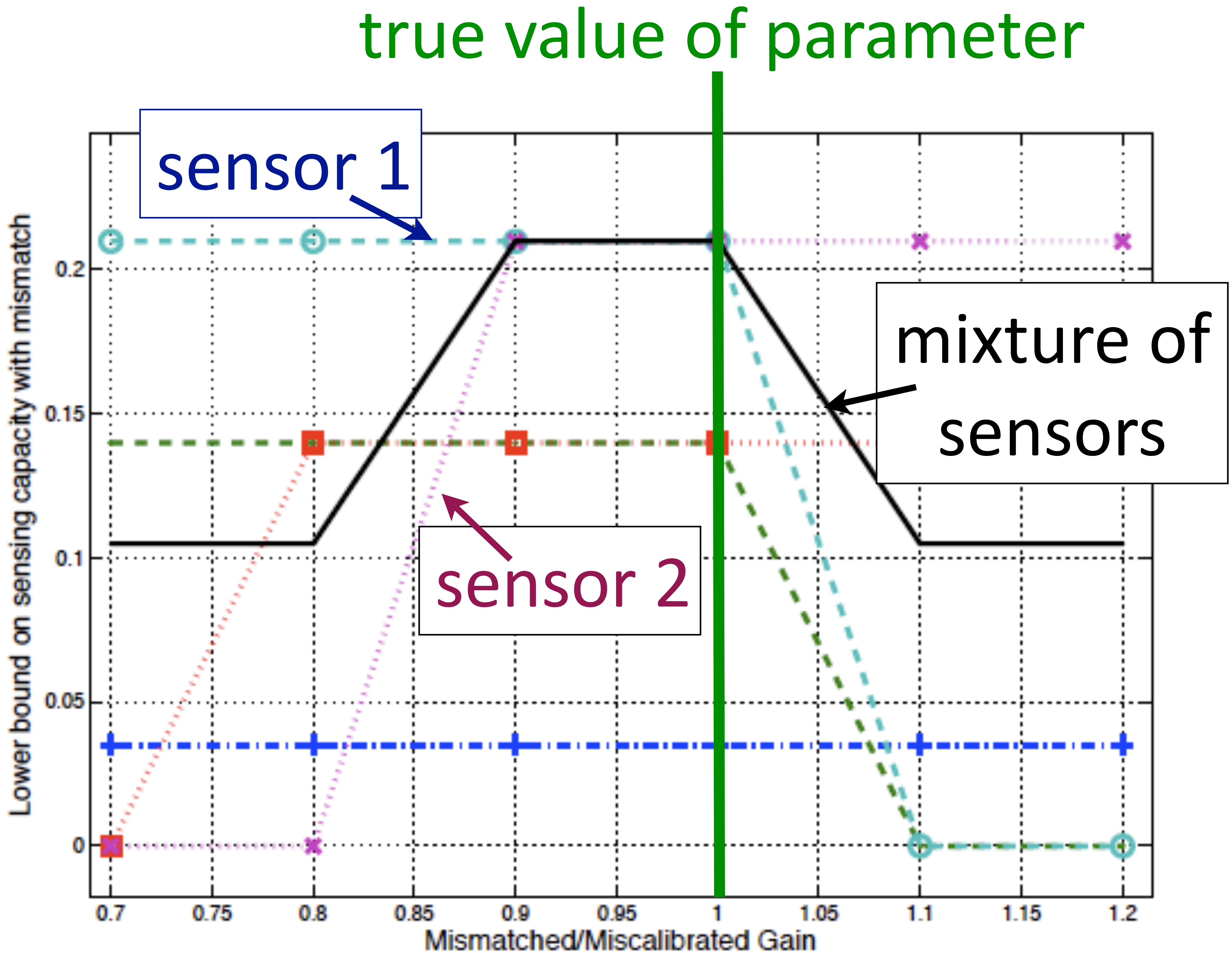
Performance of mismatched decoders



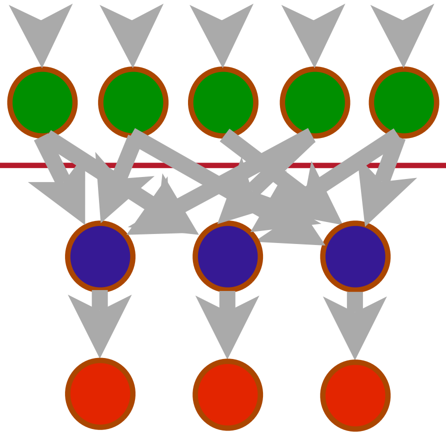
Design of robust measurements



Design of robust (threshold) measurements



Take-away / Conclusions



Detection and source coding can be cast into a common framework

We analyze the robustness to noise, mismatch and uncertainty using insights from information theory

We use the theory to make design decisions

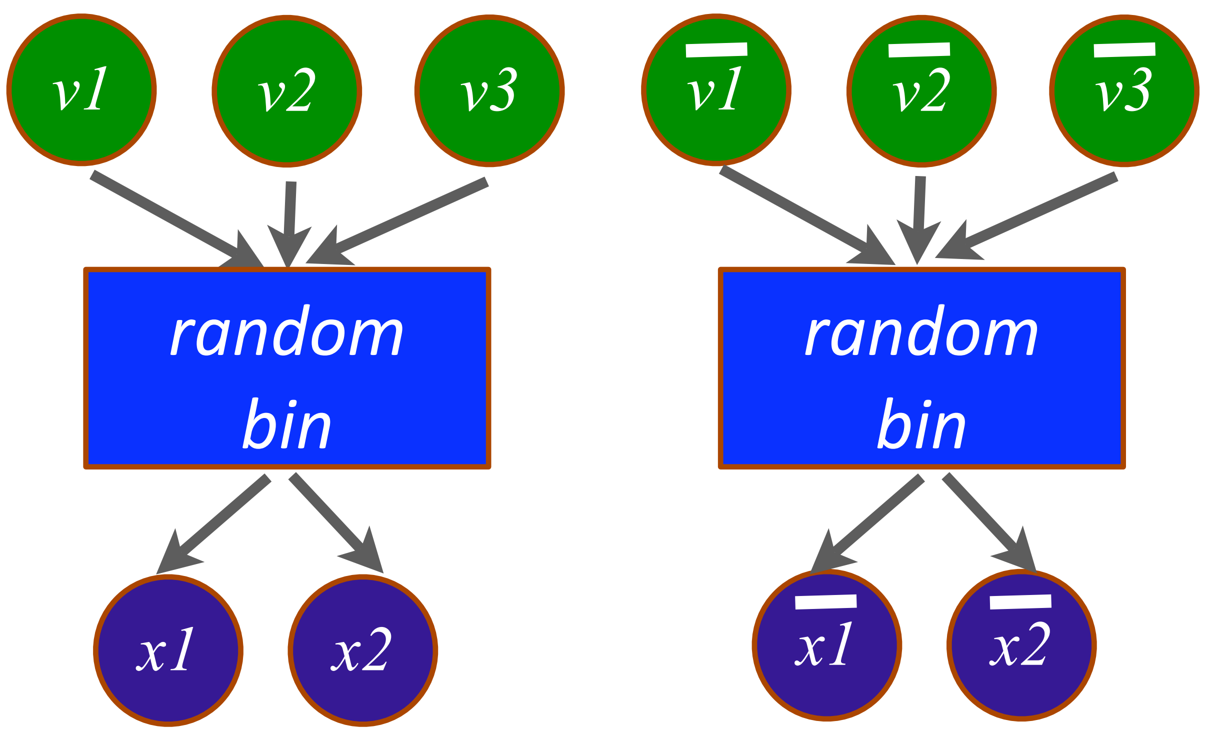
References

- [1]J. Moura, R. Negi, and M. Pueschel, “The network as the sensor, distributed sensing and processing: a graphical model approach,” DARPA ISP Review, St. Petersburg,FL., October 2003.
- [2]Slepian, D and Wolf, J K (1973). Noiseless coding of correlated information sources. IEEE Transactions on information Theory 19: 471-480.
- [3]Wyner, A D (1974). Recent results in the Shannon theory. IEEE Transactions on information Theory 20: 2-10.
- [4]Y. Rachlin, R. Negi, and P. Khosla, “Sensing capacity for discrete sensor network applications,” in Proc. Fourth Int. Symp. on Information Processing in Sensor Networks,April 25-27 2005.
- [5]Sason, I. and Shamai, S. 2006. Performance analysis of linear codes under maximum-likelihood decoding: a tutorial. Commun. Inf. Theory 3,

- **Backup Slides**

Why do we need a different analysis ?

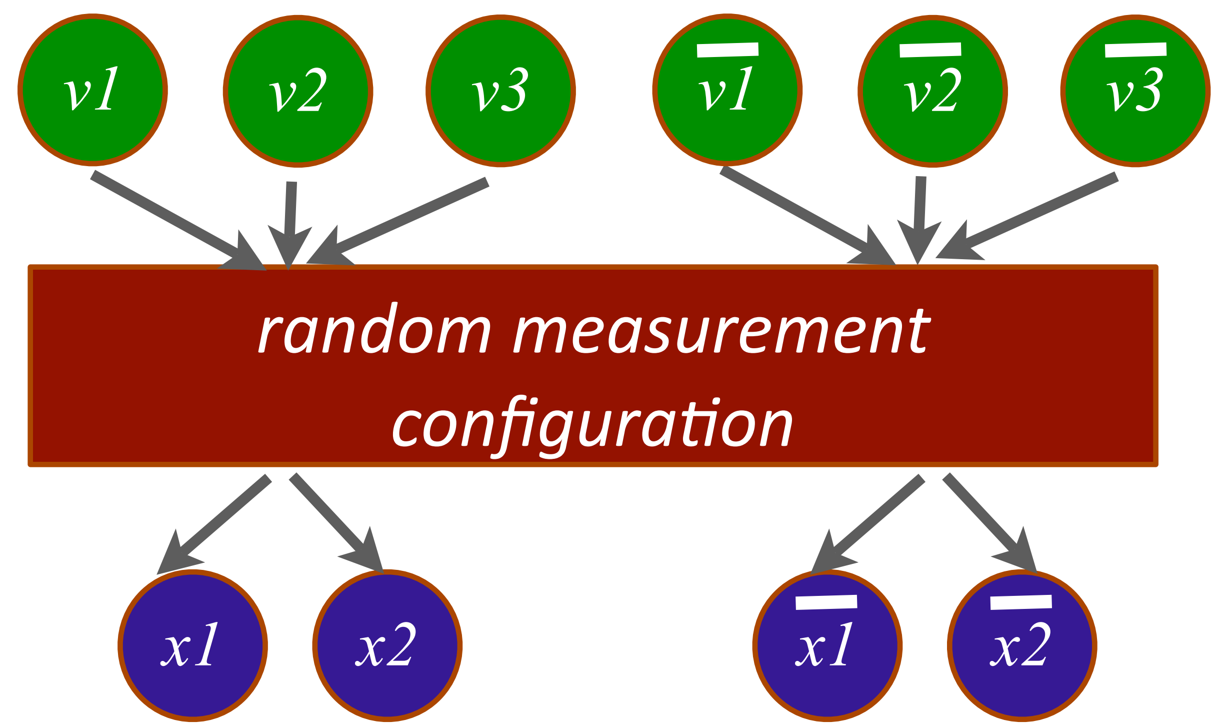
Random binning vs. Random measurements



any mapping

independent codewords

$$Q(\bar{X} \mid X)=P(\bar{X})$$



mappings constrained by kind of measurements and configurations

dependent codewords

$$Q(\bar{X} \mid X)$$