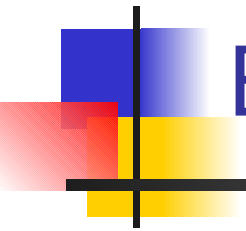
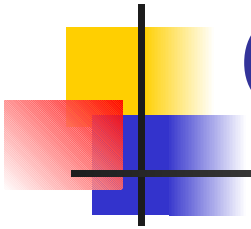


Chaff:
Engineering an Efficient SAT Solver

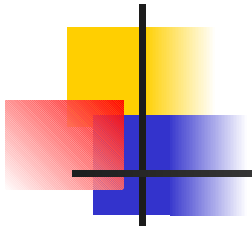
Matthew W.Moskewicz,
Concor F. Madigan, Ying Zhao, Lintao Zhang, Sharad Malik
Princeton University

Modified by T. Heyman and E. Clarke



Chaff's Main Procedures

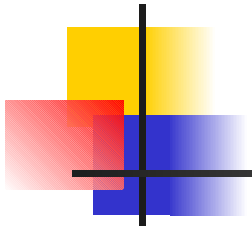
- Efficient BCP
  - Two watched literals
  - Fast backtracking
- Efficient decision heuristic
  - Localizes search space
- Random Restarts
  - Increases robustness



Implication

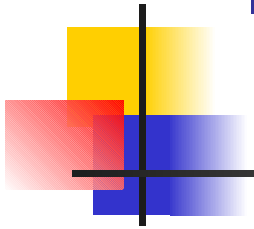
- What “causes” an implication?
- When can it occur?
- All literals in a clause but one are assigned False

Implication example



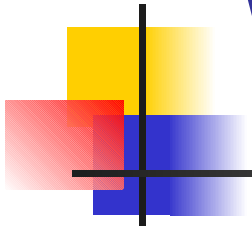
- The clause (v1 + v2 + v3) implies values only in the following cases
- In case (F + F + v3)
  - implies v3=T
- In case (F + v2 + F)
  - implies v2=T
- In case (v1 + F + F)
  - implies v1=T

Implication for N-literal clause



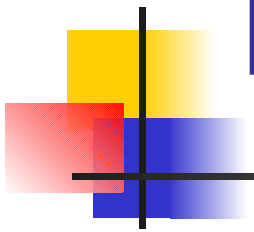
- Implication occurs after N-1 assignments of False to its literals
- Theoretically, we could ignore the first N-2 assignments to this clause
- The first N-2 assignments won't have any effect on the BCP

Watched Literals



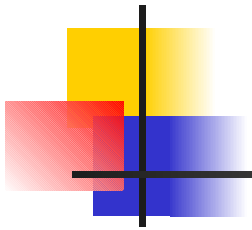
- Each clause has two watched literals
- Ignore any assignments to the other literals in the clause.
- BCP Maintains the following invariant
  - By the end of BCP, one of the watched literal is true or both are undefined.
- Guaranteed to find all implications

BCP with watched Literals



- Identify conflict clauses
- Identify unit clauses
- Identify associated implications
- Maintain “BCP Invariant”

Example (1/13)



v2 + v3 + v1 + v4
v1 + v2 + v3’
v1 + v2’
v1’+ v4

Example (2/13)

Watched literals

v2 + v3 + v1 + v4
v1 + v2 + v3’
v1 + v2’
v1’+ v4

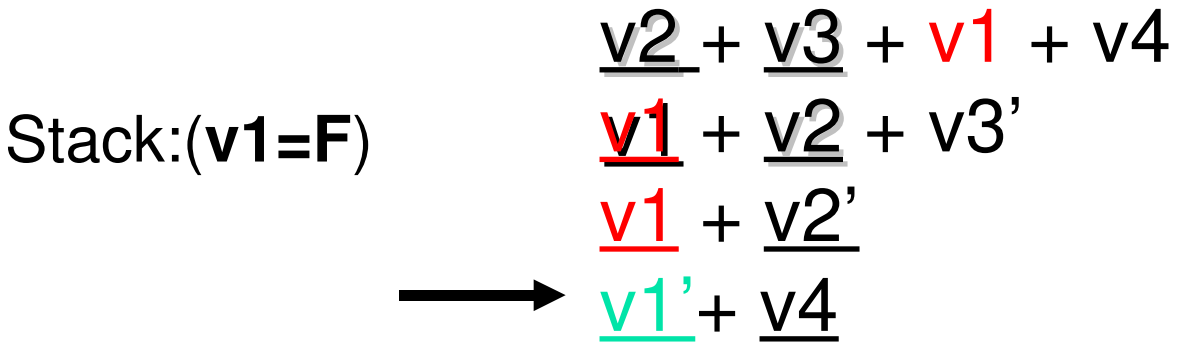
Example (3/13)

Stack:(**v1=F**)

v2 + v3 + v1 + v4
v1 + v2 + v3’
v1 + v2’
v1’+ v4

Assume we decide to set v1 the value F

Example (4/13)



- Ignore clauses with a watched literal whose value is T

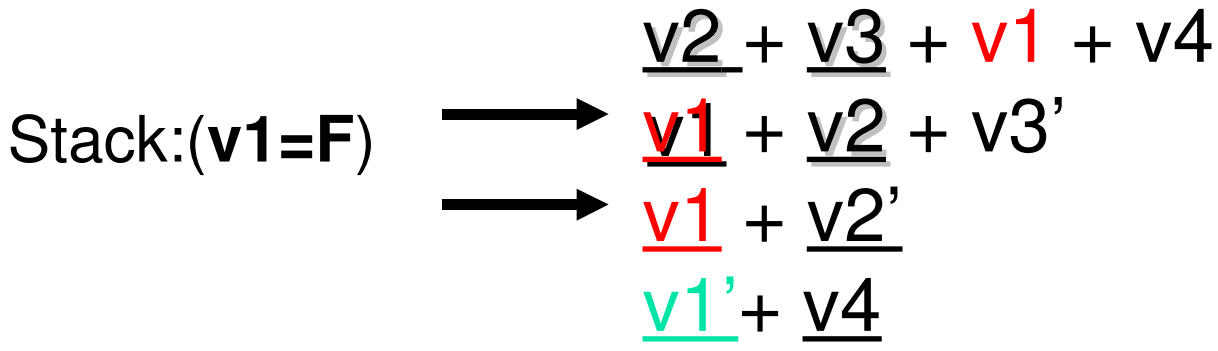
Example (5/13)

v2 + v3 + v1 + v4
v1 + v2 + v3’
v1 + v2’
v1’+ v4

Stack:(**v1=F**)

- Ignore clauses where neither watched literal value changes

Example (6/13)



- Examine clauses with a watched literal whose value is F

Example (7/13)

v2 + v3 + v1 + v4
v1 + v2 + v3'
v1 + v2'
v1'+ v4

Stack:(**v1=F**)

v2 + v3 + v1 + v4
v1 + v2 + v3'
v1 + v2'
v1'+ v4

Stack:(**v1=F**)

- In the second clause, replace the watched literal v1 with v3'

Example (8/13)

v2 + v3 + v1 + v4
v1 + v2 + v3'
v1 + v2'
v1'+ v4

Stack:(**v1=F**)

v2 + v3 + v1 + v4
v1 + v2 + v3'
v1 + v2'
v1'+ v4

Stack:(v1=F)
Pending: (v2=F)

- The third clause is a unit and implies v2=F
- We record the new implication, and add it to a queue of assignments to process.

Example (9/13)

v2 + v3 + v1 + v4
v1 + v2 + v3'
v1 + v2'
v1'+ v4

Stack:(v1=F, **v2=F**)

→ v2 + v3 + v1 + v4
→ v1 + v2 + v3'
v1 + v2'
v1'+ v4

Stack:(v1=F, v2=F)
Pending: (v3=F)

- Next, we process v2.
- We only examine the first 2 clauses

Example (10/13)

v2 + v3 + v1 + v4
v1 + v2 + v3'
v1 + v2'
v1'+ v4

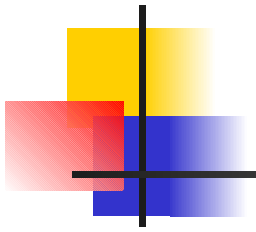
Stack:(v1=F, **v2=F**)

→ v2 + v3 + v1 + v4
→ v1 + v2 + v3'
v1 + v2'
v1'+ v4

Stack:(v1=F, v2=F)
Pending: (v3=F)

- In the first clause, we replace v2 with v4
- The second clause is a unit and implies v3=F
- We record the new implication, and add it to the queue

Example (11/13)



v2 + v3 + v1 + v4
v1 + v2 + v3'
v1 + v2'
v1'+ v4

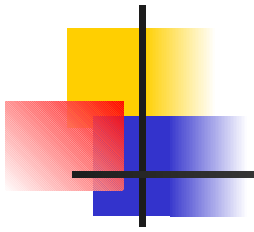
Stack:(v1=F, v2=F, **v3=F**)

v2 + v3 + v1 + v4
v1 + v2 + v3'
v1 + v2'
v1'+ v4

Stack:(v1=F, v2=F, v3=F)
Pending: ()

- Next, we process v3'. We only examine the first clause.

Example (12/13)



v2 + v3 + v1 + v4
v1 + v2 + v3'
v1 + v2'
v1' + v4

Stack:(v1=F, v2=F, **v3=F**)

v2 + v3 + v1 + v4
v1 + v2 + v3'
v1 + v2'
v1' + v4

Stack:(v1=F, v2=F, v3=F)
Pending: (v4=T)

- The first clause is a unit and implies v4=T.
- We record the new implication, and add it to the queue.

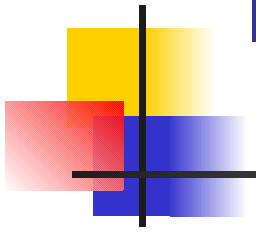
Example (13/13)

v2 + v3 + v1 + v4
v1 + v2 + v3'
v1 + v2'
v1' + v4

Stack:(v1=F, v2=F, v3=F, v4=T)

- There are no pending assignments, and no conflict
- Therefore, BCP terminates and so does the SAT solver

Identify conflicts

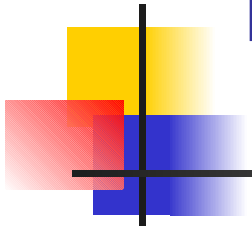


v2 + v3 + v1
v1 + v2 + v3’
v1 + v2’
v1’+ v4

Stack:(v1=F, v2=F, **v3=F**)

- What if the first clause does not have v4?
- When processing v3’, we examine the first clause.
- This time, there is no alternative literal to watch.
- BCP returns a conflict

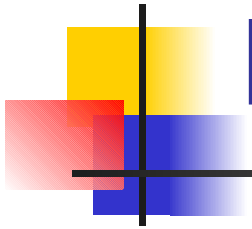
Backtrack



v2 + v3 + v1
v1 + v2 + v3’
v1 + v2’
v1’+ v4

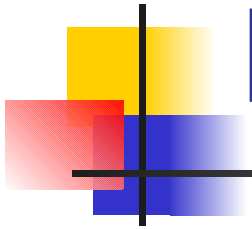
Stack:()

- We do not need to move any watched literal



BCP Summary

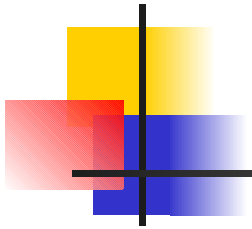
- During forward progress (decisions, implications)
  - Examine clauses where watched literal is set to F
  - Ignore clauses with assignments of literals to T
  - Ignore clauses with assignments to non-watched literals



Backtrack Summary

- Unwind Assignment Stack
- No action is applied to the watched literals
- Overall
  - Minimize clause access

Chaff Decision Heuristic VSIDS



- Variable State Independent Decaying Sum
  - Rank variables based on literal count in the initial clause database.
  - Only increment counts as new clauses are added.
  - Periodically, divide all counts by a constant.

VSIDS Example (1/2)

Initial data base

x1 + x4
x1 + x3' + x8'
x1 + x8 + x12
x2 + x11
x7' + x3' + x9
x7' + x8 + x9'
x7 + x8 + x10'

Scores:
4: x8
3: x1,x7
2: x3
1: x2,x4,x9,x10,x11,x12

watch what happens to x8, x7 and x1

New clause added

x1 + x4
x1 + x3' + x8'
x1 + x8 + x12
x2 + x11
x7' + x3' + x9
x7' + x8 + x9'
x7 + x8 + x10'
x7 + x10 + x12'

Scores:
4: x8,x7
3: x1
2: x3,x10,x12
1: x2,x4,x9,x11

VSIDS Example (2/2)

Counters divided by 2

x1 + x4
x1 + x3' + x8'
x1 + x8 + x12
x2 + x11
x7' + x3' + x9
x7' + x8 + x9'
x7 + x8 + x10'
x7 + x10 + x12'

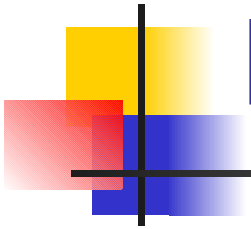
Scores:
2: x8,x7
1: x3,x10,x12,x1
0: x2,x4,x9,x11

watch what happens to x8, x10

New clause added

x1 + x4
x1 + x3' + x8'
x1 + x8 + x12
x2 + x11
x7' + x3' + x9
x7' + x8 + x9'
x7 + x8 + x10'
x7 + x10 + x12'
x12' + x10

Scores:
2: x8,x7,x12,x10
1: x3,x1
0: x2,x4,x9,x11



Restart

- Abandon the current search tree and reconstruct a new one
- Helps reduce runtime variance between instances- adds to robustness of the solver
- The clauses learned prior to the restart are *still there* after the restart and can help pruning the search space

TimeLine

1960
DP
≈10 var

1962
DLL
≈ 10 var

1986
BDD
≈ 100 var

1988
SOCRATES
≈ 3k var

1992
GSAT
≈ 300 var

1994
Hannibal
≈ 3k var

1996
Stålmarck
≈ 1000 var

1996
GRASP
≈1k var

1996
SATO
≈1k var

2001
Chaff
≈10k var