Expensive Multiobjective Optimization with a Robotics Application

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Multiobjective Optimization

- Multiple independent, often competing, objectives
- If there is no clear preference, goal is a *Pareto set* of unrankable solutions -- not a single optimal solution

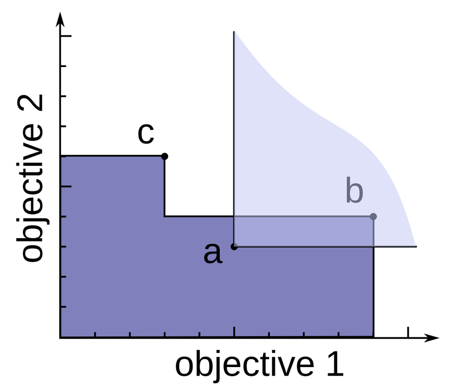
Pareto Optimality

Pareto Dominance:

$a, b \in \mathbb{R}^k$

$a \succ b$ iff $(\forall i \in 1 \ldots k, a_i \geq b_i) \wedge (\exists j \in 1 \ldots k \,|\, a_j > b_j)$

$a \sim b$ iff $\neg(a \prec b) \wedge \neg(a \succ b)$

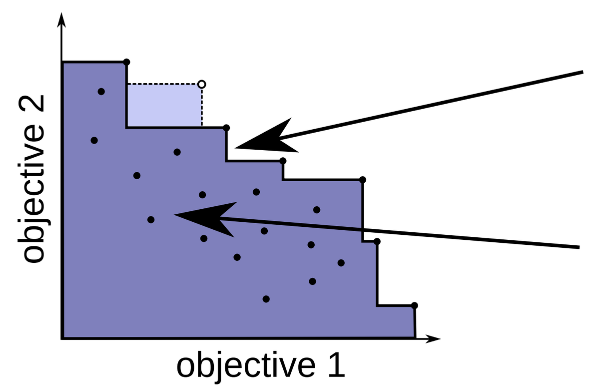


Pareto optimal subset of a set of points P:

$$\mathrm{PAR}(P) = \{p \in P \,|\, \forall q \in P,\ (p \sim q) \vee (p \succ q)\}$$

$$a \prec b, a \sim c, b \sim c$$

Pareto Front Hypervolume



Pareto optimal subset forms **Pareto front** in objective space

Hypervolume of Pareto front: region in objective space dominated by points in the Pareto front

Existing Methods

NISE [1]: fast approximate solutions; only for linear functions;

Simplex methods [2]: exact solutions, only for linear functions

NSGA-II [3]: Gold standard for nonlinear functions; GA based, takes thousands of samples

Applications

- Fast snake robot locomotion with a stable head camera view

- Animal Behavior modeling
- Engineering design
- Spacecraft trajectories

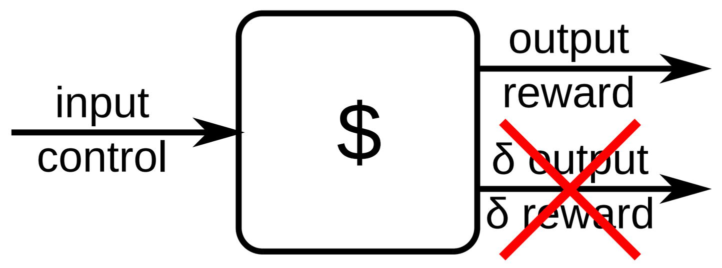
Expensive Multiobjective Optimization: Problem Definition

Goal: Given a budget of experiments (samplings of the objective functions), sequentially sample the objectives to obtain a set which has the highest hypervolume.

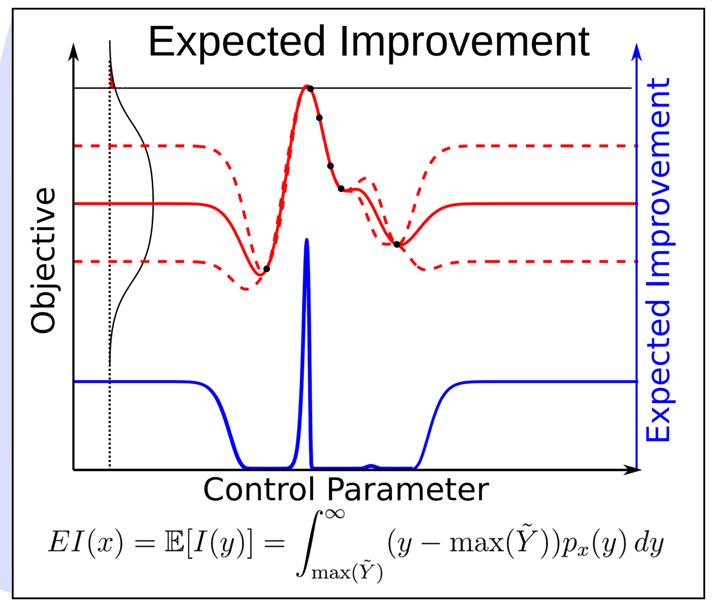
Parameter space: $X \subset \mathbb{R}^m$
Objectives: $\{f^1, f^2, \ldots f^k\}, k \geq 2, f^i : X \to \mathbb{R}$
Sample locations: $\{x_1, x_2, \ldots x_n \,|\, x_i \in X\}$
Objective evaluations at X: $\tilde{Y} = \{f^i(x_j)\}$

Goal: Choose x_i sequentially to maximize $\mathrm{HV}(\mathrm{PAR}(\tilde{Y}))$

Bayesian Approach to Expensive Black-Box Optimization



Surrogate Function Method:
1) Initial objective samples
2) Fit function (Gaussian process)
3) Use fit to select next sample
4) Repeat steps 2 and 3



Prior Extensions to Multiple Objectives

Common: Aggregate objectives, and treat problem as a single objective

ParEGO [4]: Current state of the art; optimizes random aggregate each step

Keane [5]: Approximation to method described below

Multiobjective Optimization using Expected Improvement in Hypervolume

Work in objective space to focus on quantity we are maximizing -- hypervolume

Updated Algorithm Outline:
1) Initial objective samples
2) Fit functions (GPs)
3) Next sample: `argmaxXEIHV(x)`
4) Repeat steps 2 and 3

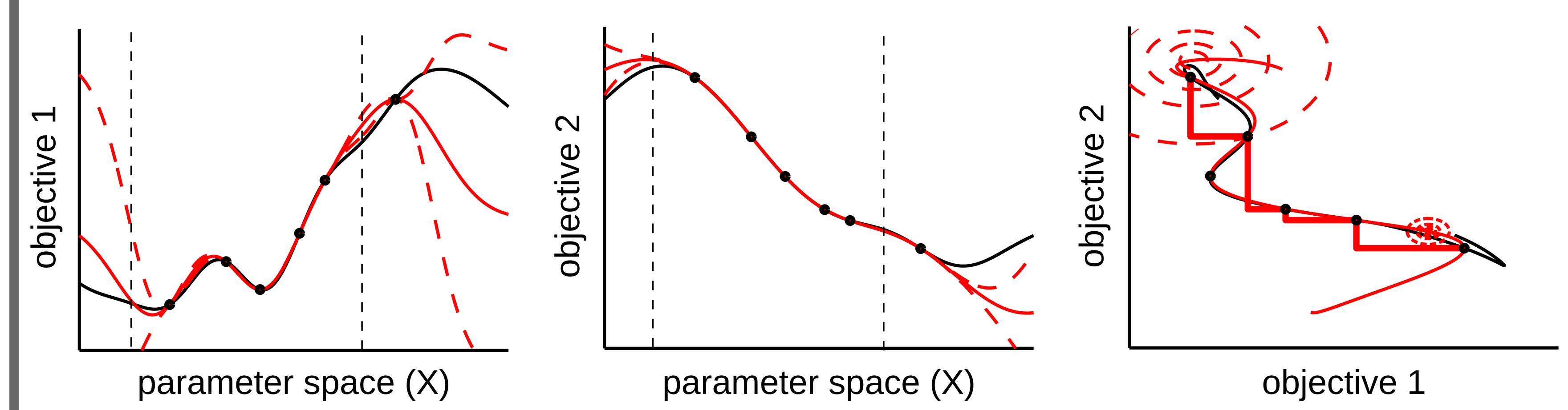
Define improvement as:

$$\mathrm{IHV}(y) = \mathrm{HV}(\tilde{Y} \cup y) - \mathrm{HV}(\tilde{Y})$$

Update selection metric (see [6]) to:

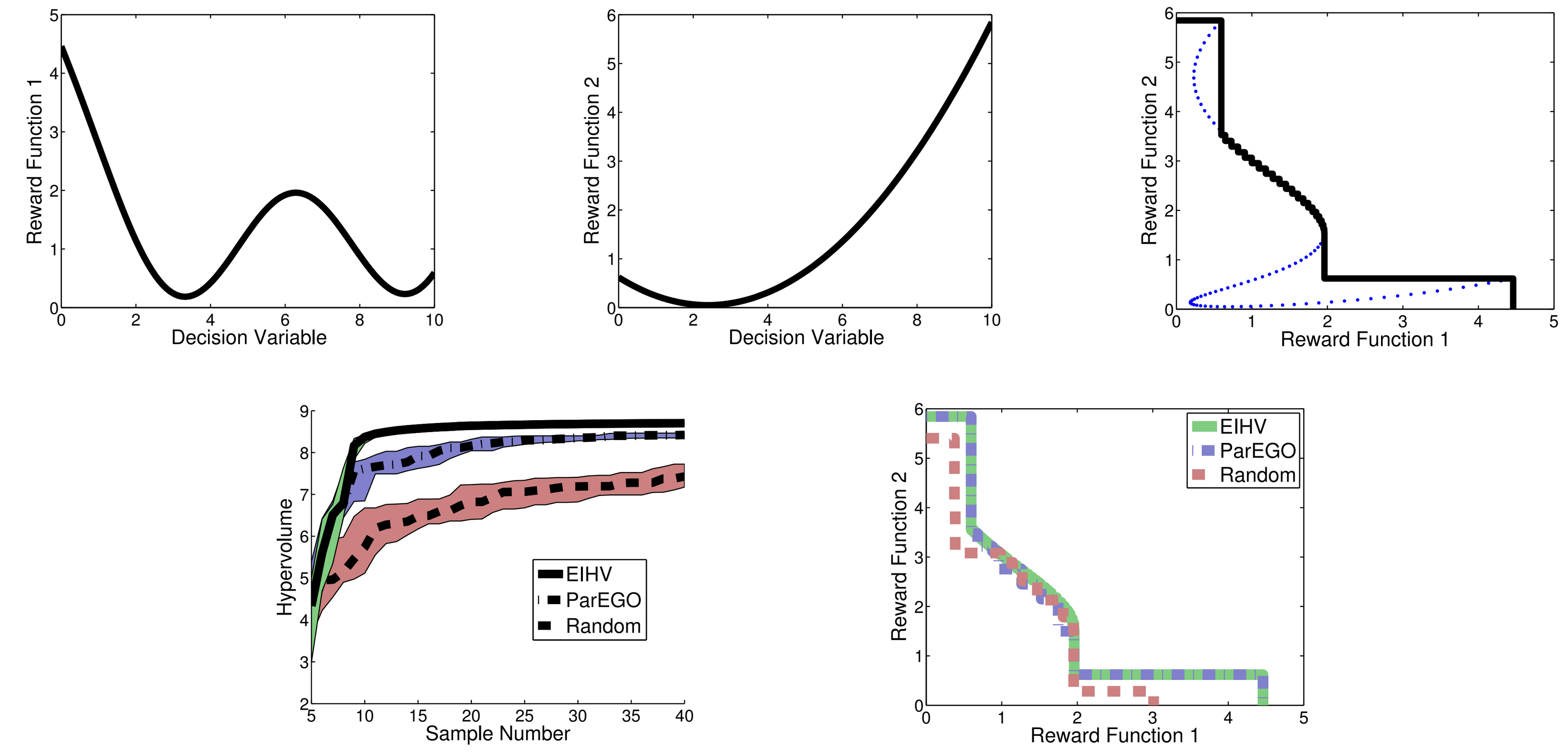
$$\mathrm{EIHV}(x) = \mathbb{E}[\mathrm{IHV}(y)] =$$
$$\int \cdots \int_{\mathbb{R}^k} \left(\mathrm{IHV}(y)\, p_x(y^1) \cdots p_x(y^k)\right) dy^1 \cdots dy^k$$

Projecting GP regression into objective space for EIHV calculation:

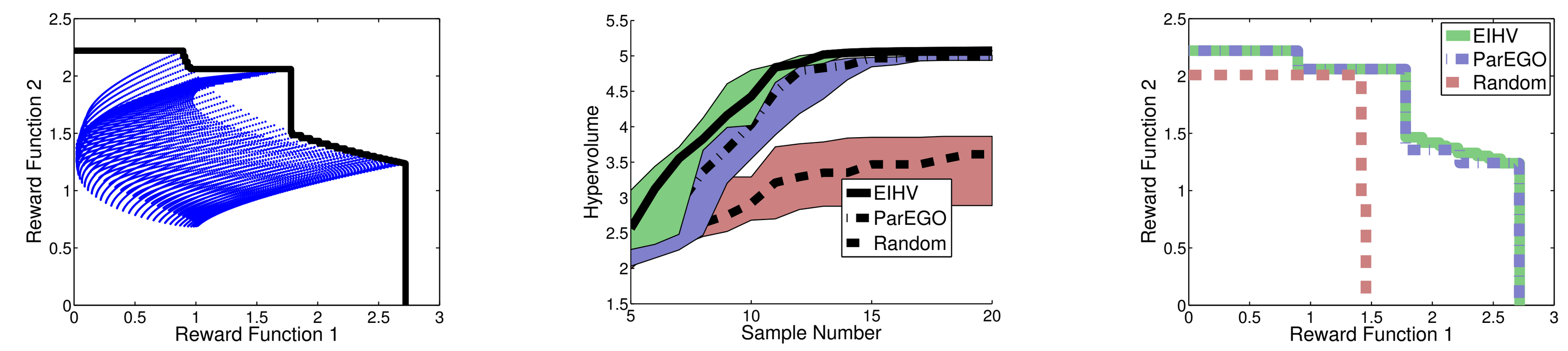


Results

1-D test problem

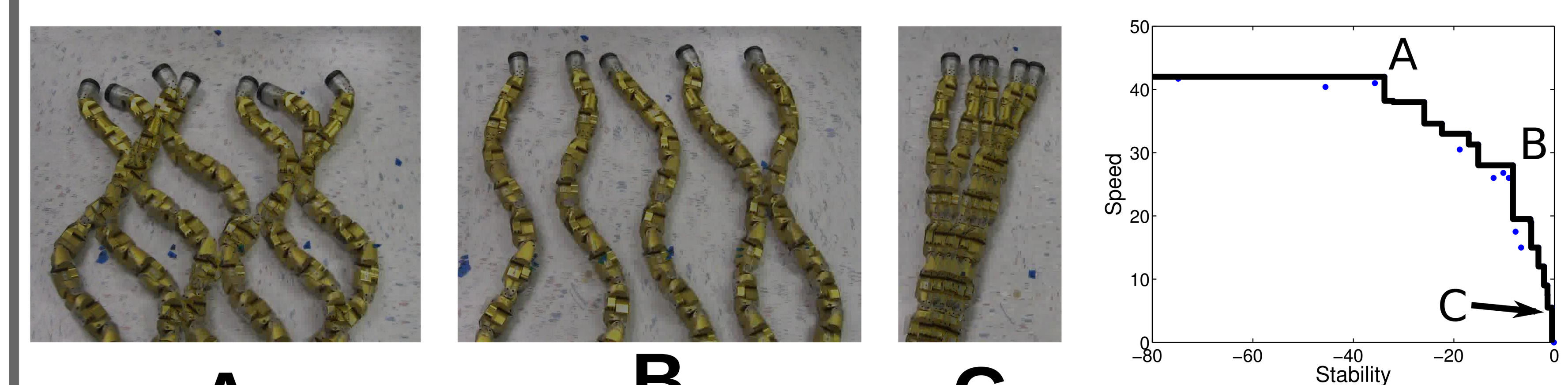


2-D test problem



Each algorithm ran 20 times for each problem.
Median and middle 50% of results shown.

Snake robot: speed vs. head stability



A B C

25 experiments; searched over head amplitude and offset parameters

Conclusions

- EIHV provides direct extension of EI to multiobjective problems
- Empirically produces better distribution over the Pareto front than ParEGO
- Demonstrated results on robotic snake

Future Work

- Better handling of noisy function evaluations
- Developing convergence guarantees and rates
- Discover upper dimensionality bound

[1] Solanki et. al., *Approximating the noninferior set in muliobjective linear programming problems*
[2] M. Zeleny, *Linear Multiobjective Programming*
[3] K. Deb et. al., *A Fast Elitist Multi-Objective Genertic Algorithm: NSGA-II*
[4] Knowles, J., *ParEGO: a hybrid algorithm with on-line landscape approximation for expensive multiobjective...*
[5] Keane. A.J., *Statistical improvement criteria for use in multiobjective design optimization*
[6] Emmerich et. al., *The computation of the expected improvement in dominated hypervolume of Pareto...*