Expensive Multiobjective Optimization and Validation with a Robotics Application

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Abstract

Many practical optimization problems, especially in robotics, involve *multiple competing objectives*, e.g. performance metrics such as speed and energy efficiency. Proper treatment of these objective functions is often overlooked. Additionally, optimization of the performance of robotic systems can be restricted due to the *expensive* nature of testing control parameters on a physical system. This paper presents a multi-objective optimization (MOO) algorithm for expensive-to-evaluate functions which generates a *Pareto set* of solutions. This algorithm is compared against another leading MOO algorithm, and then used to optimize the speed and head stability of the sidewinding gait for a snake robot.

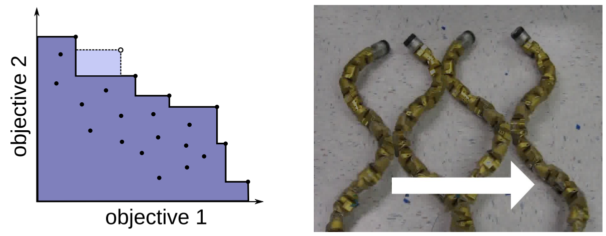
1 Introduction

Many problems in robotics require optimization of multiple conflicting criteria, such as the speed of a system and its energy efficiency. Often, it is tempting to simply consider a scalar combination of these criteria, e.g., to optimize a linear combination of speed and efficiency. Unfortunately, such combinations limit the solution to a single point based on preferences implied by the aggregate function. In the *multi-objective optimization* (MOO) community, these objectives are treated explicitly as independent unless the user has a clear prioritization. Instead of a single optimum, this gives rise to a set of *Pareto optimal* solutions, termed the *Pareto set* (Figure 1(a)).

This full set of solutions is useful in real-world situations. Our application considers teleoperation of a snake robot ([13]) equipped with an on-board camera. This camera is fixed to the undulating head of the robot, which causes difficulty in maintaining situational awareness during locomotion (Figure 1(b)). One can stabilize the camera by controlling the robot with small-amplitude low-frequency motions; however, such motions also reduce the speed of the system, creating conflicting notions of good performance. During field operation of the system, the relative importance of objectives can change – speed may be important when the robot is teleoperated through an easy to comprehend space, but as the robot enters a more complex passage camera stability becomes paramount. Other applications where finding a full optimal set is important include modeling animal behavior [12], rehabilitation of water distribution networks [1], airfoil design [9], and spacecraft trajectories [3].

Another important consideration when optimizing the performance of a robotic system is that the system can be *expensive* to run – where expense can be measured in terms of time, financial cost, or use of computational resources. This means that when attempting to optimize the performance of such a system, its use must be budgeted. In this work we consider an *experiment* to be a single objective function evaluation of a set of parameters on the robot itself. In expensive MOO, the goal is to reduce the number of experiments needed to find the *Pareto set* of solutions.

In this paper we describe an algorithm for obtaining a Pareto optimal set of solutions while budgeting the number of tests on the robot; this method seeks global optima on nonlinear and nonconvex



(a) (b)

Figure 1: **(a)** The set of Pareto optimal solutions are those not *Pareto dominated* by any other points in the full set of solutions. In this figure, they are on the stair-stepped line. The hypervolume of a Pareto set is the volume which is dominated by the Pareto set, shown as the dark shaded region. The light shaded region is the potential improvement to the hypervolume if the point indicated by the circle were added to the set. **(b)** When teleoperating our snake robot, the large movement of the head (which houses the camera) can cause operators to become disorientated. In this paper, we simultaneously optimize for the conflicting objectives of head stability and speed for these robots.

objective functions. We compare its performance to a state of the art algorithm, and provide a concrete real world MOO example on a snake robot.

2 Expensive Multiobjective Optimization

We consider the global, constrained, expensive multiobjective optimization problem as follows: given a parameter space $X \subset \mathbb{R}^m$, objective functions $\{f^1, f^2, \ldots f^k\}$ where $k \geq 2$ and $f^i \colon X \to \mathbb{R}$, and the opportunity to sequentially choose $\{x_1, x_2, \ldots x_n \,|x_i \in X\}$ to evaluate on all objectives simultaneously (generating $\tilde{Y} = \{y^i_j | 1 < i < k, 1 < j < n\}$), maximize the hypervolume of the Pareto set of $\tilde{Y}$, or $\mathrm{HV}(\mathrm{PAR}(\tilde{Y}))$ with respect to a reference point p. We use the convention of maximizing f^i rather than minimizing.

This Pareto set, named after economist Vilfredo Pareto, includes all solutions which cannot be improved in one objective without a decrease in another. In particular, a point in objective space a is said to *dominate* b, written $a \succ b$ or $b \prec a$, if a is at least as good in every objective, and better in at least one; $a \sim b$ denotes two points for which neither dominates the other. The *Pareto optimal* subset of a collection of points P is $\{p \in P \mid \forall\, q \in P,\, (p \sim q) \vee (p \succ q)\}$. For detailed coverage of these ideas, see [2].

In the problem setup described above, the *hypervolume indicator* [14] is chosen, a common quality measure of the Pareto set. This is the volume in objective space which is Pareto-dominated by at least one point in the Pareto set (see Figure 1(a)). This measure has many desirable properties; for example, it is not affected when a dominated point is added to a set of solutions, and the addition of a non-dominated point always increases a set's hypervolume.

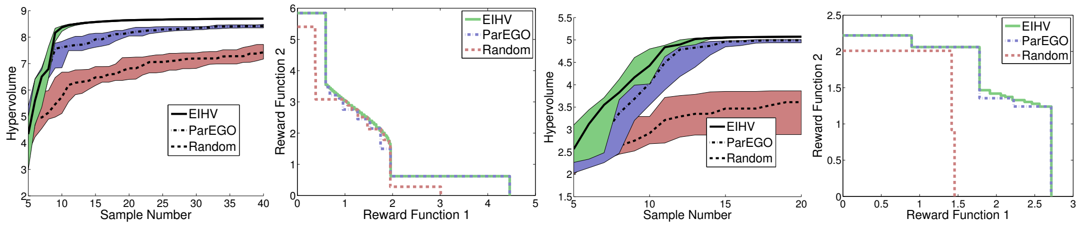
3 Method: Maximize Expected Improvement in Hypervolume

In expensive single-objective optimization, Jones et al.'s *efficient global optimization* (EGO) algorithm [6] has popularized the use of *expected improvement* (EI) [8] as an experiment selection metric with no tuning parameters. Given a sampled value set $\tilde{Y}$, the improvement $I(y)$ of a new sample at x with the value y is the increase in the maximum of this set with the addition of y. The expectation of this quantity over p_x, the surrogate's predictive distribution at x, is given in closed form as

$$EI(x) = \mathbb{E}[I(y)] = \int_{\max(\tilde{Y})}^{\infty} (y - \max(\tilde{Y})) p_x(y)\, dy, \tag{1}$$

and captures the intuition of selecting the next sample as the point which you expect to most improve your current solution.

As the EI metric has been a success in expensive single-objective optimization, the natural question is whether it can be applied to the MOO case. First, we define the notion of improvement in the multi-objective case as the improvement in hypervolume,



(a) (b) (c) (d)

Figure 2: Results of two expensive MOO methods – optimization of the expected improvement in hypervolume (EIHV) and ParEGO – and a random experiment selection baseline on the 1-D **(a)** and 2-D **(c)** test functions. We ran each algorithm 20 times; the repeated trials are needed because initial sample selection is random. The line indicates the median hypervolume of the 20 trials, while the shaded regions represent the middle 50%. Also shown are the resulting Pareto frontiers from each method on each test function (**(b)** and **(d)**). On both functions EIHV outperforms ParEGO for either tested setting of ParEGO's s-value (10 and 1000), and both methods are significantly better than random, demonstrating the importance of careful experiment selection.

$$\mathrm{IHV}(y) = \mathrm{HV}(\tilde{Y} \cup y) - \mathrm{HV}(\tilde{Y}), \tag{2}$$

and the expected improvement in hypervolume at x is:

$$\mathrm{EIHV}(x) = \mathbb{E}[\mathrm{IHV}(y)] = \int \cdots \int_{\mathbb{R}^k} \left(\mathrm{IHV}(y)\, p_x(y^1) \cdots p_x(y^k)\right) dy^1 \cdots dy^k. \tag{3}$$

Although a simple closed-form solution has been derived for EI in the single objective case, evaluation of $\mathbb{E}[\mathrm{IHV}(y)]$ is more complex. Fortunately, Emmerich et al. provide the outline of a method to compute this quantity analytically [5]. Our MOO algorithm using the *expected improvement in hypervolume* (EIHV) metric can be summarized in a form parallel to that of surrogate-based single objective algorithms:

1. Sample the objectives at a set of initial points.
2. Fit surrogates to the sampled points to obtain mappings $X \to P(Y)$; each $x \in X$ maps to a $p_x(y^i)$ for each objective y^i. We use Gaussian proccesses (GPs) for these surrogates.
3. Select the next sample location as one point in the set $\mathrm{argmax}_X(EIHV(x))$.
4. Repeat steps 2-3 until convergence.

The performance of surrogate-function based algorithms rely on the function regression method to provide a reasonable estimate of the objective and the uncertainty of that estimate; we recommend the careful tuning of the *hyperparameters* that describe the GP to obtain a realistic and non-trivial fit. As recommended by [10], we find hyperparameters that maximize the log likelihood of the data; we use hundreds of random hyperparameter seed points to robustly solve this optimization. Also, to further improve the fit, we choose to run this hyperparameter selection process over a number of different sets of covariance functions for the GP (effectively, model selection over a number of different function forms for regression); using the log likelihood of the data as an optimization criterion allows the complexity of the model to match the complexity of the data.

4 Results

To provide validation of the EIHV selection metric we compare against a start-of-the-art MOO algorithm, ParEGO [7], which aggregates the objectives into a single function at each step by using an augmented Tchebycheff function; it requires the user to tune a parameter controlling the number of potential aggregate functions. It then reverts to single-objective experiment selection method (the previously mentioned EGO algorithm) to choose the next sample. Finally, we also use random experiment selection to provide a baseline for the importance of any careful experiment selection.

As extensive testing on physical robotic systems is expensive, we compare these methods on two simple analytic functions (see Figure 2); one test function has a 1-D parameter space and the other a 2-D parameter space. Since many of the multi-objective test functions in the literature are particularly designed to confound existing multi-objective evolutionary algorithms (MOEAs) and are less appropriate for low-dimensional expensive optimization situations, we chose to use regions of the Branin test function, a common benchmark for global single objective optimization from the Dixon-Szegö test problem set [4]. Given B as the original Branin function and all inputs between 0 and 10 inclusive, the one dimensional test problem objectives are defined as $B(x, 1)/10$ and $B(3, x)/10$; the two dimensional objectives are $B(x_1, 2 + 0.5x_2)/20$ and $B(0.4x_1, 5 + 0.1x_2)/10$.

(a) Point A, 1.25Hz frame rate

(b) Point B, 0.4Hz frame rate

(c) Point C, 0.4Hz frame rate

(d)

Figure 3: **(a)-(c):** Time lapse images showing the head and snake motion of three different points in the head stability/speed Pareto set. The snake in **(a)** moves quickly, but the head rotates back and forth. The snake in **(b)** moves at a slower rate, but with noticeably less head sway. Finally, **(c)** is a very slow but very head-stable motion. As can be seen in **(d)**, these points are distributed on the Pareto front, and none dominate the others. The dots are all experimental evaluations during the optimization.

After the empirical validation of EIHV, we used this method to generate a set of Pareto optimal solutions for competing objectives of head stability and speed on the robot (Figure 3). In order to do so, we first needed to define our objective functions and parameter space. The speed objective definition is the center of mass displacement after running the snake for 10 seconds. The head stability objective was more complicated; we filtered onboard sensor data through a Kalman filter sensor-fusion state estimation technique [11] to estimate the motion of a point at a distance of 18 inches normal to the center of the camera lens plane as the desired focal point; the estimated area through which this point swept was given as the cost to minimize. Intuitively, this captures how much the camera image changes for an operator.

This optimization was restricted to running 25 experiments. We chose to optimize over two parameters of an augmented gait model that would add more position control of the head module while still keeping a low-dimensional parameter space. We take the basic form of the gait model from [13], restrict it to the sidewinding parameter space, and add an additional offset ϕ for the head module. We then optimize over the amplitude and ϕ, with a fixed ratio between A_{odd} and A_{even}.

5 Conclusions and Future Work

Multi-objective optimization is often a reasonable alternative to creating a single aggregate in the case of competing system performance objectives; this occurs frequently in fields such as design, decision theory, and economics. Instead of aggregation a Pareto optimal set should be found, containing all solutions which are not *dominated* by another solution. The generation of Pareto optimal solutions sets is difficult when sampling the performance of a system is expensive, but once accomplished these solutions can be selected from to provide real-time trade-offs between objectives.

In this paper, we have created and tested a MOO approach based on maximization of the expected improvement in hypervolume of the Pareto set, compared this to a leading MOO algorithm (ParEGO), and applied the former algorithm to a practical application, the task of finding snake robot gait parameters for fast and head-stable sidewinding. Future work involves testing these methods with higher dimensional parameter spaces and use of these methods on other robotic systems. In addition, the explicit handling of noisy objective evaluations and guarantees of convergence to a dense covering of the full Pareto optimal solution set are open problems. Both of these should be addressed in order to give potential adopters more confidence in the results of these methods.

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