Interreflections and Radiosity:
The Forward Problem

Lecture #8

Thanks to Kavita Bala, Pat Hanrahan, Doug James, Ledah Casburn
Light Transport Pathways

Light Source

Scene

Direct Component

Viewer
Light Transport Pathways

Light Source → Scene → Inter-reflections → Direct Component → Viewer

Slide adapted from Nayar et al
Light Transport Pathways

- Light Source
- Scene
- Viewer

Direct Component
Inter-reflections
Sub-surface scattering

Slide adapted from Nayar et al
Light Transport Pathways

Light Source → Volumetric Medium → Scene → Viewer

Direct Component
Inter-reflections
Sub-surface scattering
Volumetric Scattering

Slide adapted from Nayar et al
Light Transport Pathways

Light Source → Volumetric Medium → Scene → Viewer

Direct Component

Inter-reflections
Sub-surface scattering
Volumetric Scattering

Global Component

Slide adapted from Nayar et al
Global illumination around us
Cost of Ignoring Global Illumination

Photometric Stereo  
Shape from Focus  
Structured Light

Measured
Groundtruth
3D scanning of metals for Industrial Inspection

Strong inter-reflections

Phase Shifting (162 images)
Specular Interreflections

Mirror

Reflection causes errors

Regular Depth Map
Mies Courtyard House with Curved Elements

Modeling: Stephen Duck; Rendering: Henrik Wann Jensen
Lighting Effects

Hard Shadows

Soft Shadows

Caustics

Indirect Illumination
The ambient lighting in the upper-right image is approximated by a constant value. This is typical of most scanline algorithms. The middle and lower-left images were rendered with a ray tracing global illumination algorithm.

The middle image was rendered with no ambient light calculations. The lower-left image was rendered with several levels of diffuse re-reflection to give a better approximation of the ambient light in this scene.
Phong Shading

- Plastic looking scene
- No object interactions
- No shadows
Ray Tracing

Scene doesn’t look realistic enough.

• where is the corner of room?
• is window flush with wall?
• is the carpet and wood supposed to be this dark?
Radiosity – today’s topic

Indirect lighting affects realism.

- room has a corner
- window has depth
- carpet and wood on table is lighter
- walls look more pink
The Rendering Equation – Graph Style

\[ i(p, p') = v(p, p')(\epsilon(p, p') + \int \rho(p, p', p'')i(p', p'') dp'') \]

- Light passing from \( p' \) to \( p \)
- Visibility (shadows)
- Emission (light source)
- Reflectance from Surfaces
Conservation of Energy

\[ \text{Emitted power} = \text{self-emitted power} + \text{received & reflected power} \]
Diffuse Interreflections - Radiosity

- Consider lambertian surfaces and sources.
- Radiance independent of viewing direction.
- Consider total power leaving per unit area of a surface.
- Can simulate soft shadows and color bleeding from diffuse surfaces.
- Used abundantly in heat transfer literature
Irradiance, Radiosity

- Irradiance $E$ is the power **received** per unit surface area
  - Units: $W/m^2$

- Radiosity
  - Power per unit area **leaving** the surface (like irradiance)

**Figure 2.8:** Projection of differential area.
Planar piecewise constancy assumption

• Subdivide scene into small “uniform” polygons

Table in room sequence from Cohen and Wallace
Power Equation

- Power from each polygon:

\[ \forall i : \Phi_i = \Phi_{ei} + \rho_i \sum_{j=1}^{N} \Phi_j F(j \rightarrow i) \]

- Linear System of Equations:

- \( \Phi_i \): power of patch i (unknown)
- \( \Phi_{ei} \): emission of patch i (known)
- \( \rho_i \): reflectivity of patch i (known)
- \( F(j \rightarrow i) \): form-factor (coefficients of matrix)
Form Factor

- $\text{F}_{j \rightarrow i} = \text{the fraction of power emitted by } j, \text{ which is received by } i$
- **Area**
  - if $i$ is smaller, it receives less power
- **Orientation**
  - if $i$ faces $j$, it receives more power
- **Distance**
  - if $i$ is further away, it receives less power
Form Factor

\[ F(j \rightarrow i) = \frac{1}{A_j} \int_{A_i} \int_{A_j} \frac{\cos \theta_x \cos \theta_y}{\pi r_{xy}^2} V(x, y) \, dA_y \, dA_x \]

- Equations for special cases (polygons)
- In general hard problem
- Visibility makes it harder
Form Factors Invariant

\[ F(j \rightarrow i) = \frac{1}{A_j} \int_A \int_{A_i} \frac{\cos \theta_x \cos \theta_y}{\pi r_{xy}^2} V(x, y) \, dA_y \, dA_x \]

\[ F(i \rightarrow j) = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos \theta_x \cos \theta_y}{\pi r_{xy}^2} V(x, y) \, dA_x \, dA_y \]

\[ F(i \rightarrow j) A_i = F(j \rightarrow i) A_j \]
Form Factor Computation

\[ F(j \rightarrow i) = \frac{1}{A_j} \int \int A_i A_j \frac{\cos \theta_x \cos \theta_y}{\pi r_{xy}^2} V(x, y) \, dA_y \, dA_y \]

• Schroeder and Hanrahan derived an analytic expression for polygonal surfaces.

• In general, computing double integral is hard.

• Use Monte Carlo Integration.
Form Factor Computation

THE HEMICUBE APPROXIMATION

- The contribution of each cell on the surface of the hemicube to the form factor value is computed. This is the delta form factor for each cell.
- The polygon is projected onto the hemicube.
- The delta form factors for the covered cells are summed to get the approximation to the true form factor.
Form Factor Computation
Power $\rightarrow$ Radiosity

\[ \Phi_i = \Phi_{e,i} + \rho_i \sum_{j=1}^{N} \Phi_j F(j \rightarrow i) \]

Divide by $A_i$

\[ \frac{\Phi_i}{A_i} = \frac{\Phi_{e,i}}{A_i} + \rho_i \sum_{j=1}^{N} \frac{\Phi_j F(j \rightarrow i)}{A_i} \]

\[ B_i = B_{e,i} + \rho_i \sum_{j=1}^{N} \frac{\Phi_j F(i \rightarrow j) A_i}{A_j} \]

\[ B_i = B_{e,i} + \rho_i \sum_{j=1}^{N} \frac{\Phi_j F(i \rightarrow j)}{A_j} \]

\[ B_i = B_{e,i} + \rho_i \sum_{j=1}^{N} B_j F(i \rightarrow j) \]

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Linear System of Radiosity Equations

\[ \forall \text{patches } i: \quad B_i = B_{ei} + \rho_i \sum_{j} F_{i \to j} B_j \]

\[
\begin{bmatrix}
1 - \rho_1 F_1 \to 1 & -\rho_1 F_1 \to 2 & \cdots & -\rho_1 F_1 \to n \\
-\rho_2 F_2 \to 1 & 1 - \rho_2 F_2 \to 2 & \cdots & -\rho_2 F_2 \to n \\
\vdots & \vdots & \ddots & \vdots \\
-\rho_n F_n \to 1 & -\rho_n F_n \to 2 & \cdots & 1 - \rho_n F_n \to n
\end{bmatrix}
\begin{bmatrix}
B_1 \\
B_2 \\
\vdots \\
B_n
\end{bmatrix}
= 
\begin{bmatrix}
B_{e1} \\
B_{e2} \\
\vdots \\
B_{en}
\end{bmatrix}
\]

- Matrix Inversion to Solve for Radiosities.
Iterative approaches

• Jacobi iteration
• Start with initial guess for energy distribution (light sources)
• Update radiosity/power of all patches based on the previous guess

\[ B_i = B_{e,i} + \rho_i \sum_{j=1}^{N} B_j F(i \rightarrow j) \]

new value  old values

• Repeat until converged
Radiosity “Pipeline”

Scene Geometry → Form factor calculation → Solution of Radiosity Eq → Visualization → Viewing Conditions → Reflectance Properties → Radiosity Image
- Classical Approach
- No Interpolation
• Classical Approach

• Low Res
• Classical Approach
• High Res
• More accurate
• Classical Approach
• High Res
• Interpolated
PROGRESSIVE SOLUTION

The above images show increasing levels of global diffuse illumination. From left to right: 0 bounces, 1 bounce, 3 bounces.
Sample Scenes
Sample Scenes

From Cohen, Chen, Wallace and Greenberg 1988
Sample Scenes
Sample Scenes
Sample Scenes
Radiosity

Summary

Classic radiosity = finite element method

Assumptions

- Diffuse reflectance
- Usually polygonal surfaces

Advantages

- Soft shadows and indirect lighting
- View independent solution
- Precompute for a set of light sources
- Useful for walkthroughs
Interreflections :
The Inverse Problem

Thanks to Shree Nayar, Seitz et al, Levoy et al, David Kriegman
Vision: Estimating Shape of Concave Surfaces

Actual Shape

Shape from Photometric Stereo

Need to account for Interreflections!!
Shape and Interreflections: Chicken and Egg

• If we remove the effects of interreflections, we know how to compute shape.

• But, interreflections depend on the shape!!

So, which comes first?

Diagram: [A diagram showing a cycle between Shape and Interreflections with a question mark in the middle.]
Linear System of Radiosity Equations - RECAP

\[ B_i = B_{ei} + \rho_i \sum_j F_{i \rightarrow j} B_j \]

\[
\begin{bmatrix}
1 - \rho_1 F_1 \rightarrow 1 & -\rho_1 F_1 \rightarrow 2 & \cdots & -\rho_1 F_1 \rightarrow n \\
-\rho_2 F_2 \rightarrow 1 & 1 - \rho_2 F_2 \rightarrow 2 & \cdots & -\rho_2 F_2 \rightarrow n \\
& \cdots & \cdots & \cdots \\
-\rho_n F_n \rightarrow 1 & -\rho_n F_n \rightarrow 2 & \cdots & 1 - \rho_n F_n \rightarrow n
\end{bmatrix}
\begin{bmatrix}
B_1 \\
B_2 \\
\vdots \\
B_n
\end{bmatrix}
= 
\begin{bmatrix}
B_{e1} \\
B_{e2} \\
\vdots \\
B_{en}
\end{bmatrix}
\]

• Matrix Inversion to Solve for Radiosities.
Use Radiance instead of Radiosity

- Vision shape-from-intensity algorithms work on Radiance.
- Assume image pixel covers infinitesimal scene patch area.
- Form factor is called “Interreflection Kernel”, $K$ (better name).

$$L_i = L_s + \rho_i \sum_j L_j K_{ij}$$

Radiance of a facet is given by the linear combination of radiances from other facets.

Loosely, we can say, this weighted averaging in the direction of concave curvature.
Why do concavities appear shallow?

\[ L_i = L_s + \rho_i \sum_j L_j K_{ij} \]

Radiance of a facet is given by the linear combination of radiances from other facets.

Loosely, we can say this weighted averaging in the direction of concave curvature.
Matrix Form of Interreflection Equation

\[ L = L_s + P K L \]

where

\[ P = \frac{I}{\pi} \begin{bmatrix} \rho_1 & 0 & \ldots & 0 \\ 0 & \rho_2 & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \rho_m \end{bmatrix} \quad K = \begin{bmatrix} 0 & K_{12} & \ldots & \ldots & \ldots \\ K_{21} & 0 & \ldots & \ldots & \ldots \\ \vdots & \vdots & \ddots & \ddots & \ddots \\ \vdots & \vdots & \ldots & 0 & \ldots \\ \vdots & \vdots & \ldots & \ldots & 0 \end{bmatrix} \]

\[ (I - PK)L = L_s \]
Pseudo Shape and Reflectance

• Apply any shape from intensity algorithm ignoring interreflections!

• KEY IDEA: The pseudo shape and reflectance (albedo) is related to the actual shape and reflectance.
Pseudo Shape/Reflectance from Photometric Stereo

\[ N_i = \rho_i \, n_i \]

Facet Matrix:
\[ F = [N_1, N_2, N_3, N_4 \ldots] \]

\[ L = (I - PK)^{-1} \, L_s \]
\[ \downarrow \]
\[ L = (I - PK)^{-1} \, F \cdot s \]

Three Source Directions:
\[ [L_1, L_2, L_3] = (I - PK)^{-1} \, F \cdot [s_1, s_2, s_3] \]
Pseudo Shape/Reflectance from Photometric Stereo

Three Source Directions:

$$\begin{bmatrix} L_1, L_2, L_3 \end{bmatrix} = (I - PK)^{-1} F \cdot [s_1, s_2, s_3]$$

Pseudo Shape:

$$F_p = [L_1, L_2, L_3] \cdot [s_1, s_2, s_3]^{-1}$$

$$F_p = (I - PK)^{-1} F$$
Key Observations

\[ F_p = (I - P K)^{-1} F \]

- Pseudo Shape and Albedos are independent of source direction! This allows us to reconstruct actual shape.

- Pseudo Facets: Lambertian!
  “Smoothed” versions of actual facets (shallow)
  Pseudo albedos may be greater than 1.
Iterative Refinement of Shape and Albedos

- Start with pseudo shape and albedos as initial guesses.
- Compute Interreflection Kernel $K$, and Albedo matrix $P$.
- Iterate until convergence.

\[
F^{i+1} = (I - P^i K^i)^{-1} F^p
\]

\[
F^0 = F^p
\]
Figure 5: The shape and reflectance recovery algorithm.
Important Assumptions and Observations

• Any shape-from-intensity method can be used.

• Assumes shape is continuous (for integrability).

• All facets contributing to interreflections must be visible to sensor.

• Facets are infinitesimal lambertian patches.

• Complexity: \( O(Mn^2) \) (M Iterations, n facets)

• Convergence shown for 2 facets.

• Does not always converge to the right facet for large tilt angles (> 70).