Rao-Blackwellised Particle Filtering

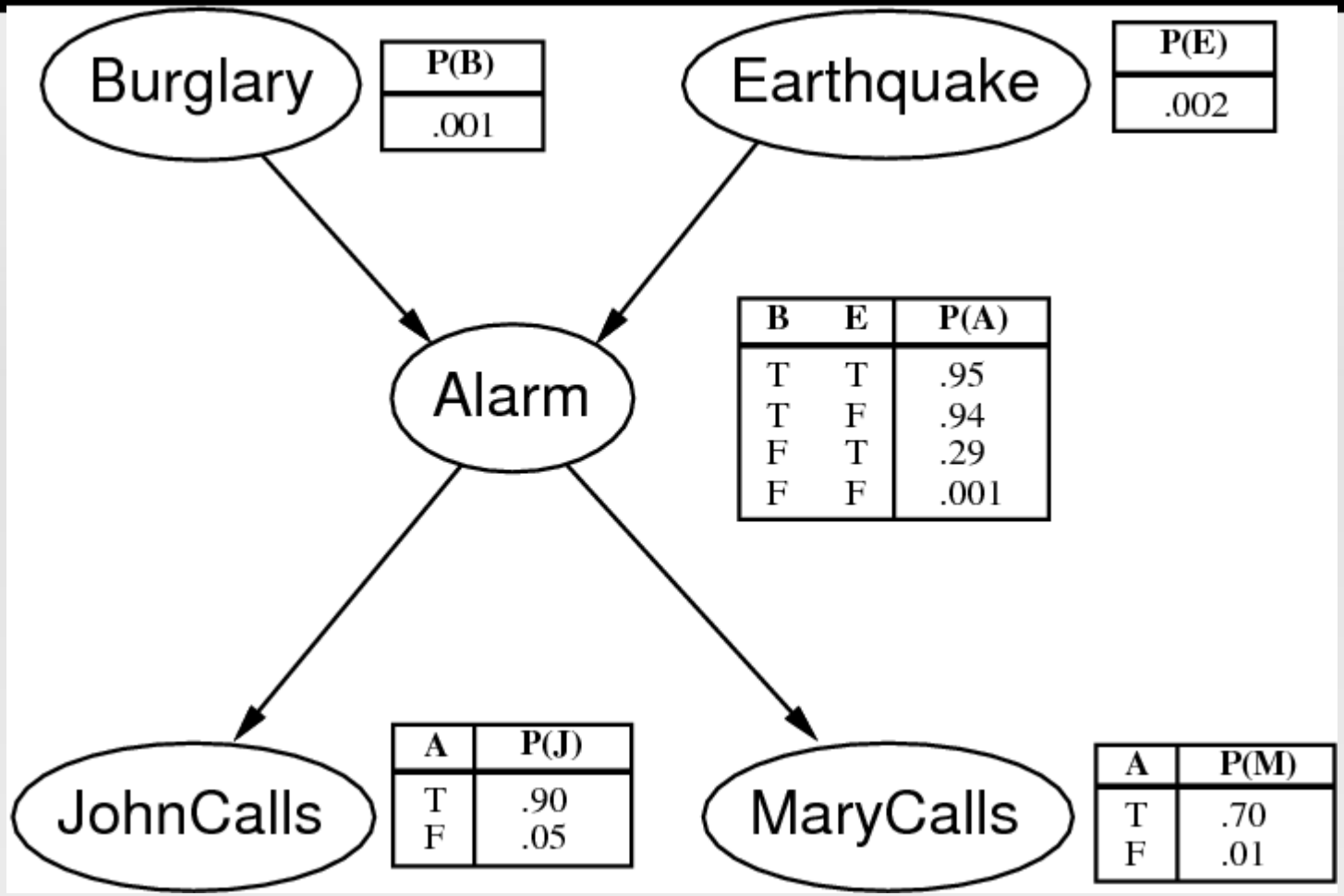
- Based on *Rao-Blackwellised Particle Filtering for Dynamic Bayesian Networks* by Arnaud Doucet, Nando de Freitas, Kevin Murphy, and Stuart Russel
- Other sources
  - Artificial Intelligence: A Modern Approach by Stuart Russel and Peter Norvig

Presented by Boris Lipchin

Introduction

- PF applications (localization, SLAM, etc)
- Draw-backs/Benefits
- Bayes Nets
- Dynamic Bayes Nets
- Particle Filtering
- Rao-Blackwell PF

Bayes Net Example



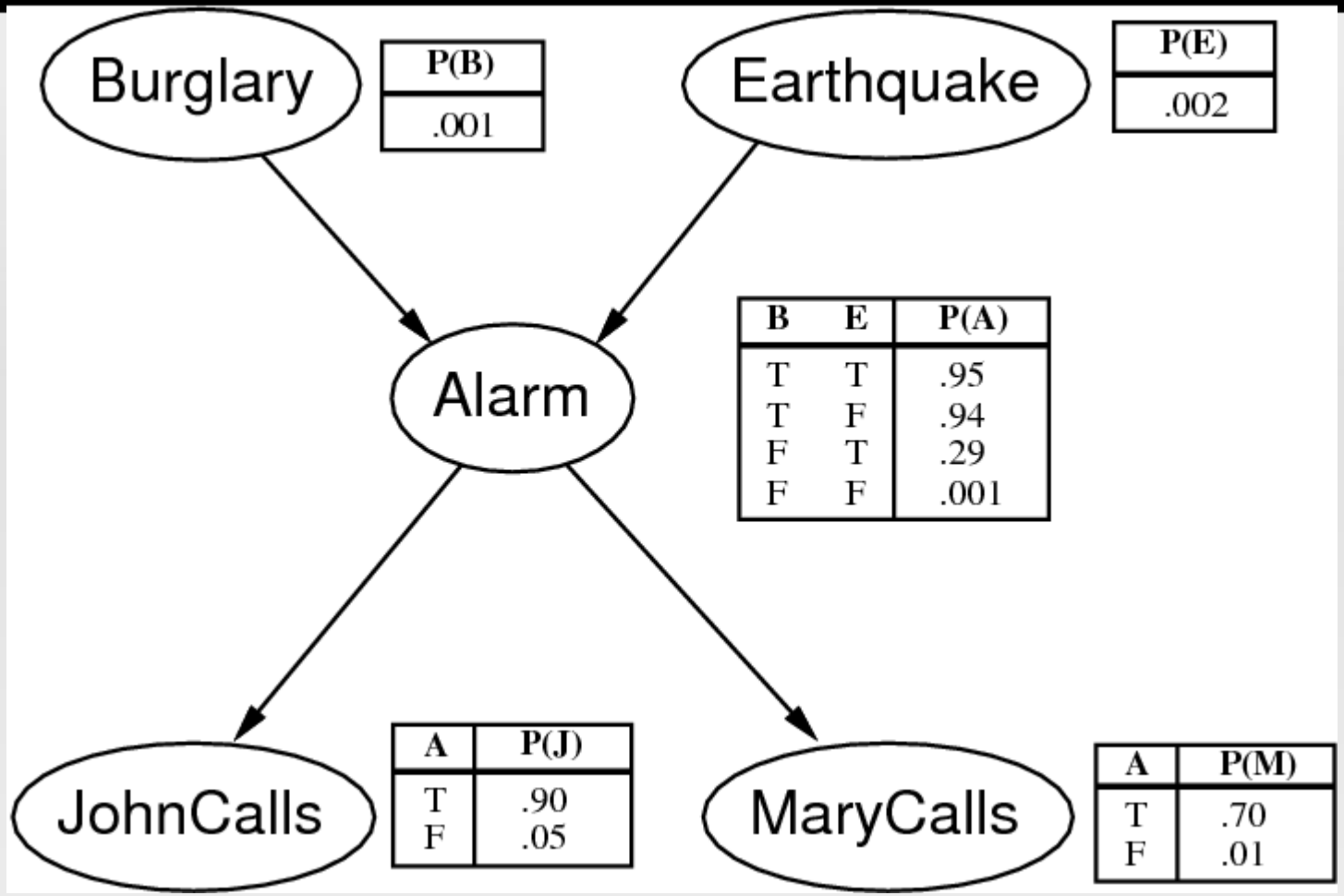
What is the probability of Burgalry given that John calls but mary doesn't call?

Adapted from Artificial Intelligence: A Modern Approach (Norvig and Russell)

Bayesian Network

- Digraph where edges represent conditional probabilities
- If A is the parent of B, B is said to be conditioned on A
- More compact representation than writing down full joint distribution table
- People rarely know absolute probability, but can predict conditional probabilities with great accuracy (i.e. doctors and symptoms)

Bayes Net Example



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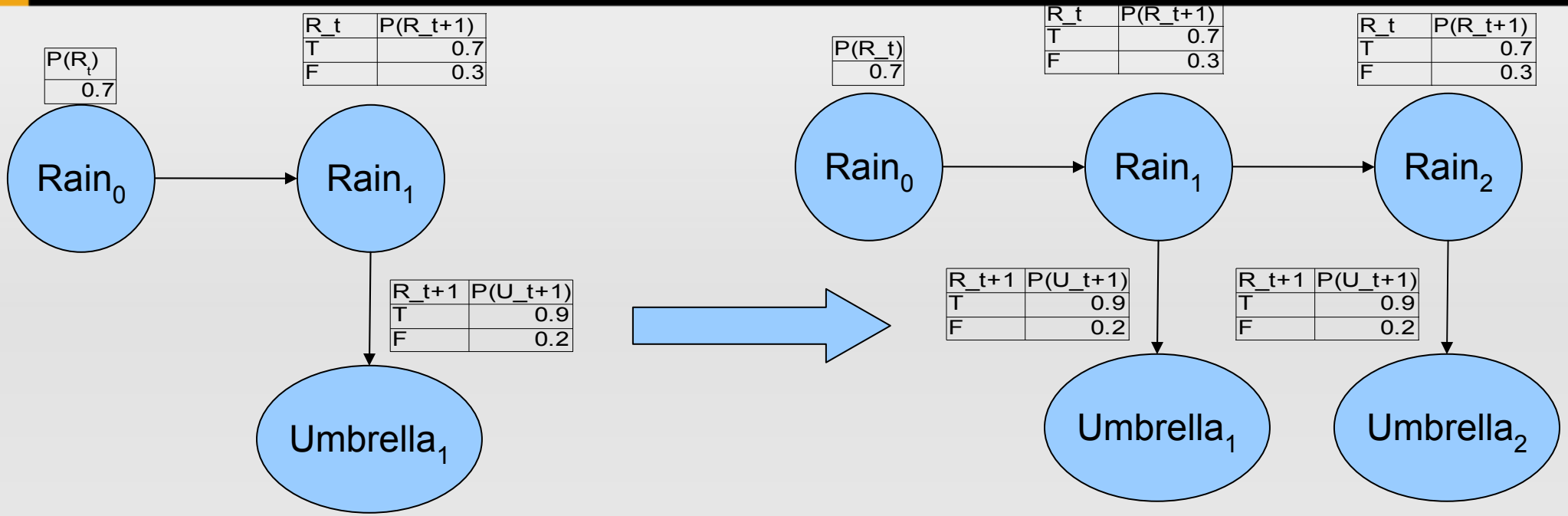
Dynamic Bayesian Networks

- Represent progress of a system over time
- 1st Order Markov DBN: state variables can only depend on current and previous state
- DBNs represent temporal probability distributions
- Kalman Filter is a special case of a DBN
- Can model non-linearities (Kalman produces single multivariate Guassian)
- Untractable to analyze

Basic DBN Example



Adapted from Artificial Intelligence: A Modern Approach (Norvig and Russell)

DBN Analysis

- Unrolling makes DBNs just like Bayesian Network
- Online filtering algorithm: variable elimination
- As state grows, complexity of analysis per slice becomes exponential: $O(d^{n+1})$
- Necessity for approximate inference
  - Particle Filtering

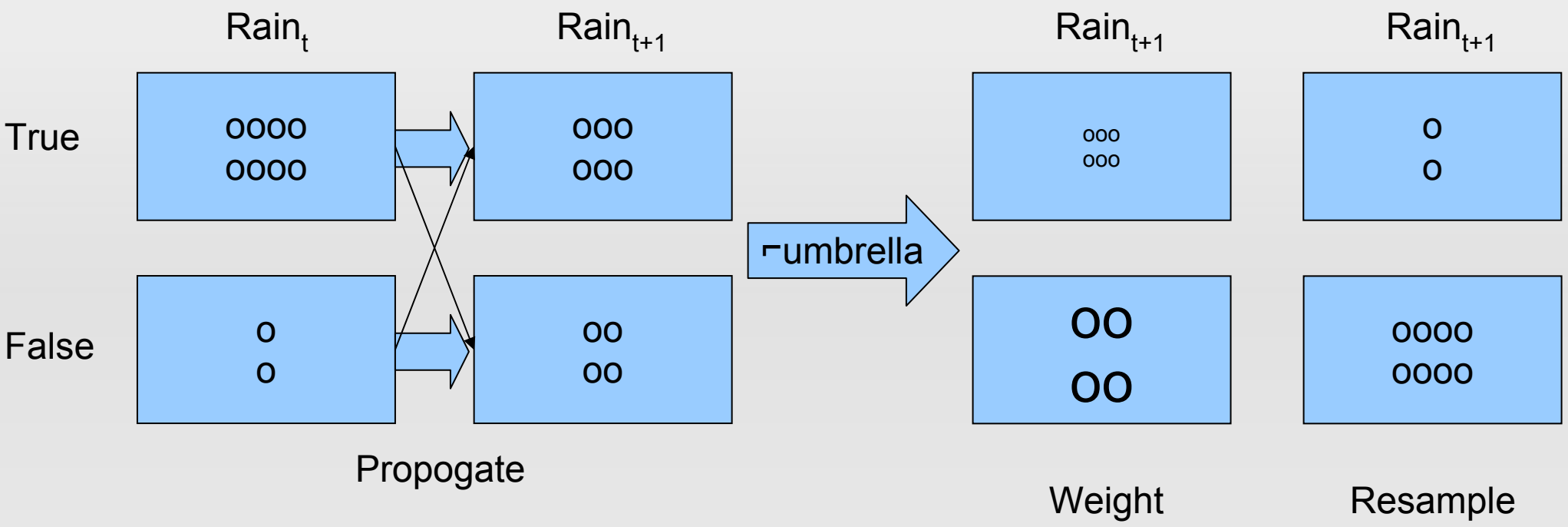
Particle Filtering

- Constant sample count per slice achieves constant processing time
- Samples represent state distribution
- But evidence variable (umbrella) never conditions future state (rain)!
- Weigh future population by evidence likelihood
- Applications: localization, SLAM

Particle Filtering: Basic Algorithm

- Create initial population:
  - Based on $P(X_0)$
- Update phase: propogate samples
  - Transition model: $P(x_{t+1} | x_t)$
- Weigh distribution with evidence likelihood
  - $W(x_{t+1} | e_{1:t+1}) = P(e_{t+1} | x_{t+1}) * N(x_{t+1} | e_{1:t})$
- Resample to re-create unweighted population of N samples based on created weighted distribution

Visual example



This method converges assymptotically to the real distribution as N → ∞

Adapted from Artificial Intelligence: A Modern Approach (Norvig and Russell)

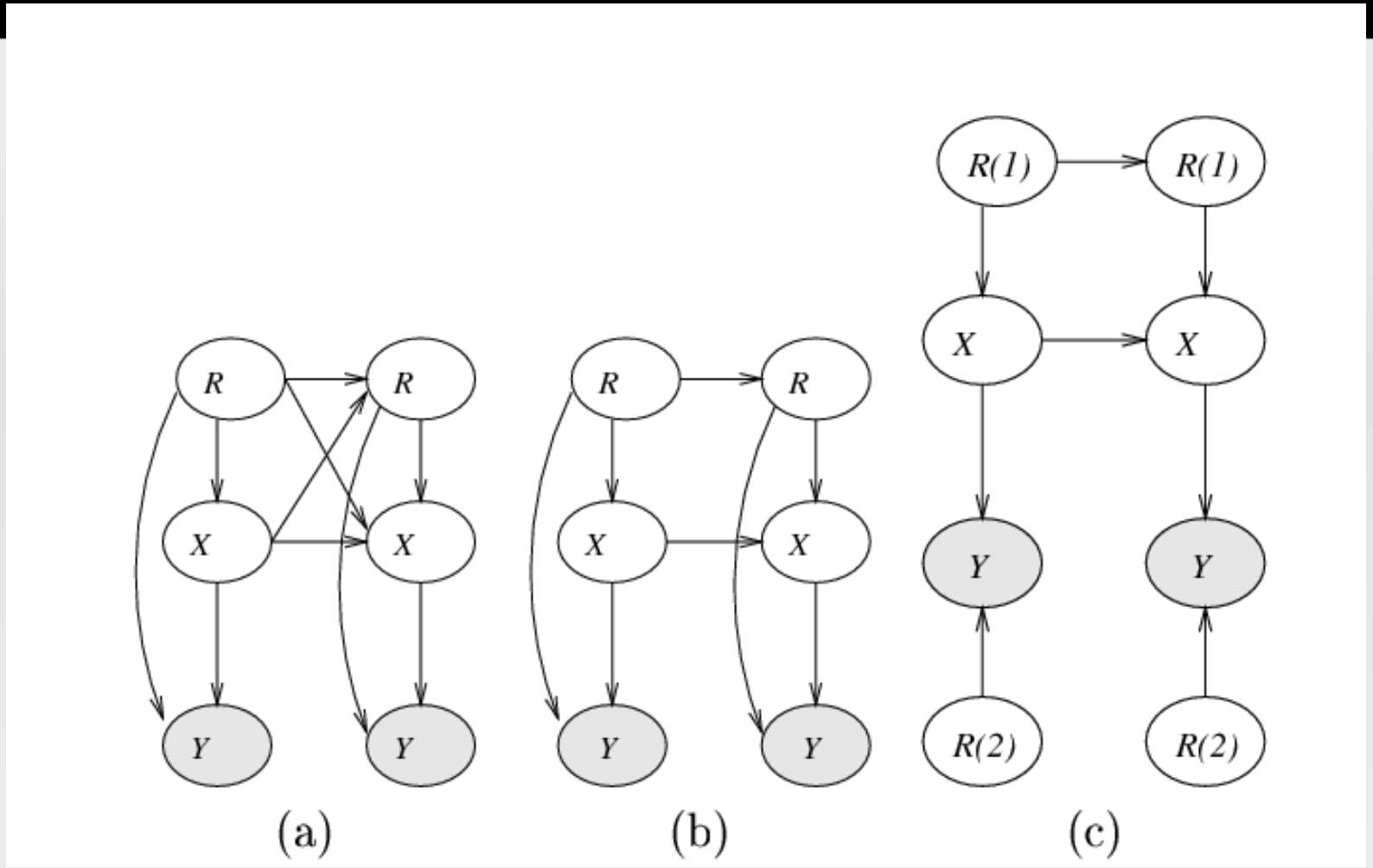
RBPF

- Key concept: decrease number of particles neccessary to achieve same accuracy with regular PF
- Requirement: Partition state nodes Z(t) into R(t) and X(t) *s.t.*:
  - $P(R_{1:t} | Y_{1:t})$ can be predicted with PF
  - $P(X_t | R_{1:t}, Y_{1:t})$ can be updated analytically/filtered
- Paper does not describe partitioning methods, efficient partitioning algorithms are assumed

RBPF: Concept Proof

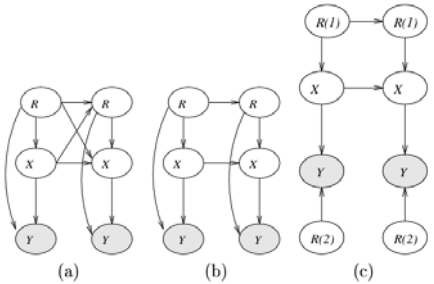
- PF approximates $P(Z_{1:t} | Y_{1:t}) = P(R_{1:t} , X_{1:t} | Y_{1:t})$
  - Remember state space Z partitioned into R and X
- $P(R_{1:t} , X_{1:t} | Y_{1:t}) = P(R_{1:t} | Y_{1:t}) * P(X_t | R_{1:t}, Y_{1:t})$
  - By chain rule property of probability
- Sampling just R requires fewer particles, decreasing complexity
- Sampling X becomes amortized constant time

RBPF DBNs – remember arrows



Adapted from *Rao-Blackwellised Particle Filtering for Dynamic Bayesian Networks* (Murphy and Russell 2001)

RBPF DBNs



- R(t) is called a root, and X(t) a leaf of the DBN
- (a) Is a canonical DBN to which RBPF can be applied
- (b) R(t) is a more common partitioning as it simplifies the Particle Filtering of the root in the RBPF
- (c) Is a convenient partitioning when some root nodes model discontinuous state changes, and others some are the parent of the observation, and model observed outliers

RBPF: Algorithm

- Root marginal distribution: $P(r_{1:t}|y_{1:t}) \approx \sum_{i=1}^{N} w_t^i \delta(r_{1:t}^{(i)}, r_{1:t})$
  - δ is the Dirac delta function
  - w is the weight of the i-th particle at slice t and is computed by

$$w_t^{(i)} \propto \frac{p\left(y_t \middle| y_{1:t-1}, r_{1:t}^{(i)}\right) p\left(r_t^{(i)} \middle| r_{1:t-1}^{(i)}, y_{1:t-1}\right)}{q\left(r_t; r_{1:t-1}^{(i)}, y_{1:t}\right)}$$

    and then normalized.

- Leaf marginal:

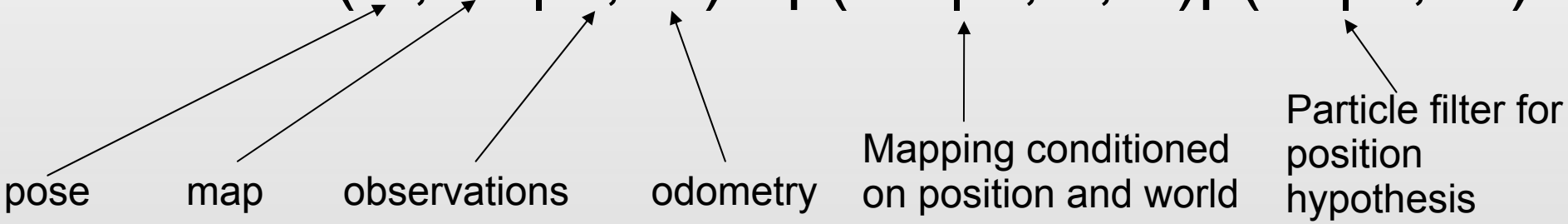
$$\begin{aligned} P(X_t|y_{1:t}) &= \sum_{r_{1:t}} P(X_t|r_{1:t}, y_{1:t}) P(r_{1:t}|y_{1:t}) \\ &\approx \sum_{r_{1:t}} P(X_t|r_{1:t}, y_{1:t}) \sum_{i=1}^{N} w_t^i \delta(r_{1:t}^{(i)}, r_{1:t}) \\ &= \sum_{i=1}^{N} w_t^i P(X_t|r_{1:t}^{(i)}, y_{1:t}) \end{aligned}$$

RBPF: Update, Propogate, Weigh

- The root particles in RBPF are propogated much like PF particles
- The leaf marginal propogation is computed with an optimal filter (Rao-Blackwellisation step)
- The leaf and root nodes together compose the entire state of the system, and thus can be weighted and resampled for the next slice.

Example: Localization

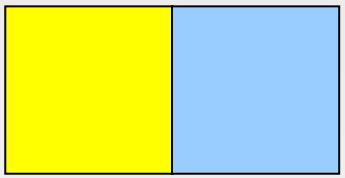
- SLAM: $P(x, m \mid z, u) = p(m \mid x, z, u)\, p(x \mid z, u)$



- m is the leaf, x is the root in the RBPF
- Particle updates based on input, expensive, we keep number of particles down

The Scenario

- Assume a world of two blue and yellow states labeled a and b (left to right)
- A robot can successfuly move between adjacent states with a probabilty P_m=.5 (transition model)
- The robot is equipped with a color sensor that correctly identifies color with probability P_c=.6



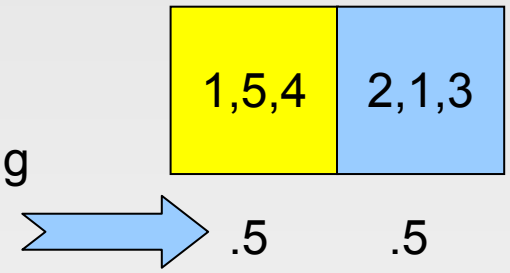
RBPF SLAM

- Using N = 5 particles
- P(X) represents state distribution (localization)
- P(M) represents color distribution (mapping)
- Prior for colors is an even distribution (unmapped)
- For simplicity, P(X=a) = 1, P(X=b) = 0

RBPF SLAM

- Randomly select particles according to prior distribution (labeled by number)
- arrow represents real robot position/detected color
- Create particles based on color

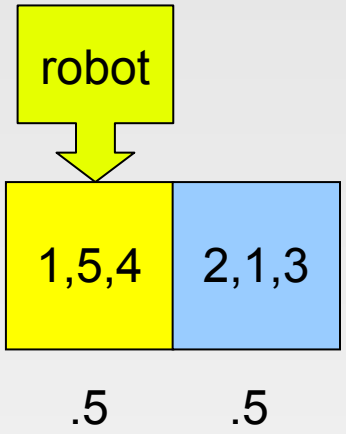
Represents mapping based on particle count



Remember: This means particle 1 hallucinates yellow in both boxes, particle 2 hallucinatees yellow only in right box and blue in left, so on and so forth.

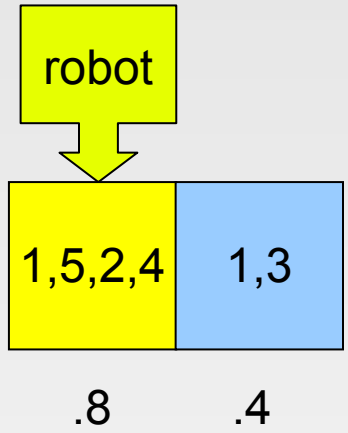
RBPF SLAM

- P(X(t)=a) = 1
- Calculate weights:
  - W1 -> P(E(a)=y | M(a)=y,M(b)=y)=
    P(E(a)=y)*P(M(a)=y | E(a)=y)*P(M(b)=y | E(a)=y,M(a)=y)/
    (P(M(a)=y)*P(M(b)=y | M(a)=y)) =
    .5 * .6 * .5 / (.5 * .5) = .6
  - W2 -> P(E(a)=y | M(a)=b,M(b)=y)=
    P(E(a)=y)*P(M(a)=b | E(a)=y)*P(M(b)=y | E(a)=y, M(a)=b)/
    (P(M(a)=b)*P(M(b)=y | M(a)=b))=
    .5 * .4 * .5 / (.5 * .5) = .4
  - You can calculate these guys ad nauseum



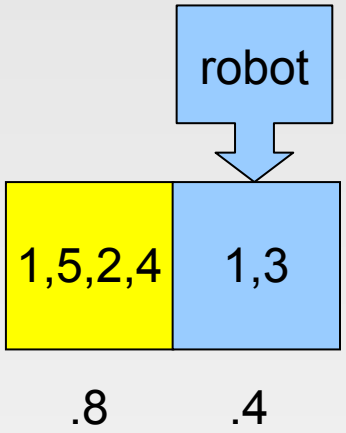
RBPF SLAM

- P(X(t)=a) = 1
- Next step is to resample based on weights (shown below)
- Find P(X(t)) distribution given previous state and current map



RBPF SLAM

- Calculate new weights
- Weigh samples and resample to obtain updated distribution for the particles
- estimate X(t) using optimal filter, evidence, and previous location



RBPF SLAM: Key Ideas

- Imagine if X(t) was part of state space
  - Calculations increase with number of states
  - Number of particles
- RBPF simplifies calculations by giving one a ”free” localization with an optimal filter

Questions?