| **Cover** || **Abstract** || **Table of Contents** || **Acknowledgments** |

| Chapter 1 || Chapter 2 || Chapter 3 || Chapter 4 || Chapter 5 || Chapter 6 |

| References || Appendix |

III. KALMAN FILTERING

A. INTRODUCTION

Kalman filtering is a method of recursively updating an estimate of a system state by processing a succession of measurements. The Kalman filter is model-based; each cycle of measured input data is compared with prior (model-based) estimates and are weighted by Kalman gains to obtain updated (output) state estimates. Kalman gains are computed during each cycle and are function's of the filter's covariances and models of the measurement process [GELB88]. In this chapter Kalman filtering will be discussed as implemented in the Phoenix AUV for navigation calculations.

B. PHOENIX IMPLEMENTATION

A discrete asynchronous Kalman Filter was used by the Phoenix navigation module. The use of DiveTracker range data required the addition of an Extended Kalman Filter mode of operation due to the non-linearity of range measurements. The Kalman filter used a non-zero mean movement model, where the input vehicle speed is assumed truth, and results in the filter solving for both an updated position data and estimates of ocean current. This filter also computes a Dimensionless shock quantity based on the received measurements to determine if the filter has possibly lost track or received bad measurements. The state vector ,U, was defined to be [Xpos Ypos Xdrift Ydrift]. The state was processed through the movement and measurement steps based on the previous position, measurements, Kalman gains, and system covariance.

1. Statistical Background

The Kalman computations are manipulations of (multi-variate) normal probability distributions [WASH94]. The computations are conducted in two separate stages consisting of motion and measurement step calculations. The symbol X represents a system state component and is a multi-variate normal with a mean of Mu and a covariance of Sigma , abbreviated as X~N(Mu,Sigma). V is the measurement noise, and is also a multi-variate normal with a mean of Uv and a variance of R abbreviated V~N(Uv,R). W is the movement noise. It too is a multi-variate normal with a mean of Uw and a variance of Q, abbreviated W~N(Uw,Q).

2. Movement Model

The movement model's X and Y position is based on standard dead-reckoning; i.e.,

$$Distance = Rate * Time \qquad (3.1)$$

That is, Distance becomes the new X or Y position. Rates are computed using a rotational transform [CRAI86] of

Phoenix u (longitudinal), v (sway) and w (heave) speeds to arrive with X (north/south), Y (east/west) and Z (up/down) speeds. The earth coordinates were set according to a right hand rule with north, east and down directions being positive. The movement model dead reckons in X and Y positions over a time Del based on the following equations.

$$X_{i+1} = X_i + Xdrift * \Delta + W_x \sim N(Xspeed * \Delta, Q) \quad \textbf{(3.2)}$$

$$Y_{i+1} = Y_i + Ydrift * \Delta + W_y \sim N(Yspeed * \Delta, Q) \quad \textbf{(3.3)}$$

$$Xdrift_{i+1} = Xdrift_i + W_{Xdrift} \sim N(0, Q) \quad \textbf{(3.4)}$$

$$Ydrift_{i+1} = Ydrift_i + W_{Ydrift} \sim N(0, Q) \quad \textbf{(3.5)}$$

That is, The new X and Y positions are the sum of the old position, the distance covered by drift speeds, and an approximately normal non-zero mean random variable W, where W has a mean of Speed*Del and a variance Q. The use of a non-zero mean random variable for the calculations of the X and Y positions is the primary driver for the solution of X and Y drift speeds. The X and Y drift calculations use a zero mean random variable W, with variance Q.

C. KALMAN FILTER FORMULAS

The Kalman filter uses Equations (3.6) and (3.7) for the motion modeling described by Equations (3.2-3.5). Equations (3.8-3.11) are used in the calculation of the measurement step. All operations are matrix operations. With the addition of Phi and H as movement and measurement matrices, Equations 3.2 and 3.5 are transformed to the Kalman filter formulas. For example if X_{i+1}= Phi*X_i + W_x (simplification of Eq. 3.2) and X~N(Mu,Sigma), then, Mu_{i+1} =Phi*Mu_i+Uw as demonstrated in Equation (3.6).

1. Motion and Measurement Models

The motion formulas are:

$$U(-)_{i+1} = \Phi_i * U(+)_i + Uw_i \quad \textbf{(3.6)}$$

$$\Sigma(-)_{i+1} = \Phi_i * \Sigma(+)_i * \Phi^T{}_i + Q_i \quad \textbf{(3.7)}$$

The measurement and update formulas are:

$$K_{i+1} = \Sigma(-)_{i+1} * H_{i+1} / (H_{i+1} * \Sigma(-)_{i+1} * H_{i+1}{}^T + R) \quad \textbf{(3.8)}$$

$$U(+)_{i+1} = U(-)_{i+1} + K_{i+1} * Shock_{i+1} \quad \textbf{(3.9)}$$

$$Shock_{i+1} = Z_{i+1} - Uv - H_{i+1} * U(-)_{i+1} \quad \textbf{(3.10)}$$

$$\Sigma(+)_{i+1} = (I - K_{i+1} * H_{i+1}) * \Sigma(-)_i \quad \textbf{(3.11)}$$

where:

U,Sigma= The mean and covariance of the System State.

Phi= The movement Matrix, which describes how the state changes.

Uw,Q = The mean and covariance of the movement noise.

H = The measurement matrix (how the measurement depends on the state).

Uv,R = The mean and covariance of the measurement noise.

Z = The measurements (GPS/DGPS or DiveTracker).

K = Kalman Gains (a ratio of the filter Covariances)

I = Identity Matrix

and '+' indicates a measurement step while '-' indicates a movement step calculation.

2. Movement Step

The new movement step position given by Equation (3.6) is the sum of the product of the movement matrix Phi and state vector U(+), as shown in Equation (3.12).

$$NewPosition = \begin{bmatrix} 1 & 0 & \Delta & 0 \\ 0 & 1 & 0 & \Delta \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Xdft \\ Ydft \end{bmatrix} \qquad \textbf{(3.12)}$$

The addition of Uw results in Equation (3.6). The new value of given by Equation (3.7) also depends on the movement matrix and the addition of the covariance of the movement noise, and results in a new covariance matrix for the system state U.

3. Measurement Step

The measurement step computes a new state vector U based upon measurements and Kalman gains. Kalman gains given by Equation (3.8) are computed as a ratio of the state covariance, as it depends upon the measurement vector and the sum of the state covariance and the measurement Equation (3.11). The gains indicate how much the state vector U values depend upon the measurements Z1 and Z2. Specifically,

$$K = \begin{bmatrix} [X\,Z1gain & X\,Z2gain \\ Y\,Z1gain & Y\,Z2gain \\ Xdft\,Z1gain & Xdft\,Z2gain \\ Ydft\,Z1gain & Ydft\,Z2gain \end{bmatrix} \qquad \textbf{(3.13)}$$

The computed gains are used as weights on the amount of change in the system based on the measurements. The difference between the estimated position based on the movement model and the measured position Z is denoted as "Shock" [WASH94], or as equivalently to as "innovation" and is given by Equation (3.9). Where a measured position is from GPS/DGPS or is a position derived from DiveTracker ranges. The new system state U is a sum of the previous state and a gain weighted shock given by Equation (3.10). The new covariance, Equation (3.11), is the product of the "complement" of the state dependent gain (a measure of truth) and the old covariance. The complement is derived by subtracting the state dependent gain from an Identity matrix.

D. DIMENSIONLESS SHOCK

In a perfect system, the value of the shock would be zero. As the shock increases and becomes large, then the probability that the system has lost track also increases. A problem develops in determining what value of shock should be considered "large". Dimensionless shock (Eq. 3.14) is used to determine what value of shock relates to "large".

$$DimensionlessShock = Shock^{T} * (H * \Sigma(-) + R)^{-1} * Shock \qquad \textbf{(3.14)}$$

A large value of DimensionlessShock indicates a possible measurement problem or that the filter has lost track. DimensionlessShock can be gauged against the degrees of freedom of the shock [WASH94]. However, it has been found in the research of this thesis that an order of magnitude increase over the degrees of freedom provides better results.

An order of magnitude increase was determined to be required due to the shift in measurement methods. When using a consistent measurement method, a large shift in the DimensionlessShock value as gauged against the degrees of freedom of the shock does indicated a possible loss of track. However, when shifting measurement methods it is possible to get a change in position that results in a higher value than expected of DimensionlessShock. To ensure that the new measurement is not ignored, an order of magnitude increase in the DimensionlessShock threshold level is used. This enables the filter to use the new measurement and maintain track.

E. EXTENDED KALMAN FILTERING

In the previous discussion of the Kalman Filter, the measurement was always a linear function of the system state. In the non-linear case, the relationship between the system state and the measurements must be linearized. In the Phoenix Kalman filter, the DiveTracker ranges are a non-linear function of the state. The DiveTracker ranges are two independent ranges from base station transducers to the Phoenix. In this case a non-linear filter (Extended Kalman Filter) must be used [WASH94]. This linearization is performed by taking the derivative of a calculated range, f(U), given by Equation (3.15). Where f(U) is a function of the X and Y components of the system state vector U. If Dx and Dy are distances between the Phoenix state position U and the DiveTracker base transponder positions, then

$$f(U) = CalcRange = \sqrt{((Dx)^2 + (Dy)^2)} \qquad \textbf{(3.15)}$$

Since the values of the measurements are non-linear with respect to the state, the development of a new H (how the measurement depends upon the state) matrix is required. This new H (Equation 3.16) is now composed of the first partial derivatives of calculated measurements f(U), based upon the current values of the state, to form a Jacobian.

$$H = \begin{bmatrix} \partial Range1/\partial x & \partial Rang1/\partial y \\ \partial Range2/\partial x & \partial Range2/\partial y \end{bmatrix} \quad (3.16)$$

This H matrix represents a linearized relationship between the state and the measured ranges. The new H is used by Equations (3.8) and (3.9) to calculate Kalman gains and covariances as they relate to the measurements. The shock calculations must also change to reflect the amount of state change required. The new shock (Equation 3.17) is the difference between the actual measurements Z and the calculated measurements f(U) as based on the system state. Where Z holds the received ranges from the DiveTracker system.

$$Shock = Z - Fu - Uv \quad (3.17)$$

F. SPEED/CURRENT ERROR MODEL

If a measured Phoenix position does not agree with the motion model's position, then as the filter updates the system state the X and Y ocean current speed components will be increased to explain the difference. The ocean current speed components of the system state are actually a combination of ocean current and navigation errors caused by inaccurate vehicle speed and heading inputs. In the absence of measurements, the speed variances will slowly increase. In the long run, according to the movement model, vehicle speeds in excess of 1000 knots are not only possible but likely [WASH94]. Modeling these speeds as a discrete Ornstein-Uhlenbeck process (O-U) will correct this problem by exponentially decreasing the value of the ocean current speeds over time. This is useful for long term modeling of ocean or tidal currents. With this approach a value of C, where (0 C 1), is used to decrease the value of the drift speed exponentially (Eq 3.18). That is,

$$C = \exp(-\Delta/T) \quad (3.18)$$

In this equation, Del is the time step between cycles and is the drift relaxation time. As an example for the case of Xdrift, the state component update equation changes to;

$$X_{i+1} = C^* X_i + W_x \sim N(0,Q) \quad (3.19)$$

Consequently, the Xdrift variance changes to;

$$Var(Xdrift) = C^2 * Var(Xdrift) + Q \quad (3.20)$$

The limit of Var(Xdrift) as time approaches infinity is the average of $Xdrift^2$, so Q reduces to,

$$Q = Xdrift^2 * (1 - C^2) \quad (3.21)$$

The final modification in the O-U process involves the Del used in the Phi matrix. Now, the drift speeds not only fluctuate about zero, but they also decay toward zero at the rate specified by C. This results in a new term Delta = *(1-C), where Delta always smaller than , although there is very little difference when Del is small compared to Tau. The final result is a modified Phi matrix given by,

$$\Phi = \begin{bmatrix} 1 & 0 & \delta & 0 \\ 0 & 1 & 0 & \delta \\ 0 & 0 & C & 0 \\ 0 & 0 & 0 & C \end{bmatrix} \qquad (3.22)$$

G. SUMMARY

Discrete Kalman filtering is a statistical method of calculating a new system state based on a series of measurements. The Phoenix navigation module uses a system state of $[\text{Xpos Ypos Xdrift Ydrift}]^T$, and measurements of GPS position and DiveTracker ranges. The use of DiveTracker ranges requires an Extended Kalman filter due to non-lineararity of the measured ranges. Drift speeds are modeled as a Ornstein-Uhlenbeck process to keep the calculated speeds in bounds.

Next page (Chapter 4)

Back to top of page.