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Improved Extended Kalman Filter Design for Passive Tracking

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***Abstract*—Extended Kalman filters are here modified for coordinate estimation of a stationary object using bearing measurements taken from a moving platform. The modifications improve significantly the coordinate estimation on the initial period of data collection when otherwise the performance is far from optimal. The modifications are to the nonlinearities and could, in some instances, be implemented by the introduction of a time decreasing amplitude dither signal in the extended Kalman filter prior to the output nonlinearity. A bound on a Lyapunov function decay rate is also given which assists in the design of the modified nonlinearities and in the selection of an appropriate coordinate basis to be used in the extended Kalman filter.**

I. INTRODUCTION

An important nonlinear filtering application is passive tracking where, for example, coordinate estimation of a stationary object is calculated on-line using noisy bearing measurements taken from a moving platform. Extended Kalman filters (EKF) are attractive for such applications but their performance can be significantly improved with additional processing. Usually some smoothing of a first batch of the data is required to achieve a good initial estimate. Here we seek to improve the extended Kalman filter performance for the initial processing, without increasing significantly the processing effort and without the need for reprocessing the initial batch of data as when smoothing techniques are employed. We seek modification to the EKF to improve its transient performance. Since the ideas of the paper may have more general application than is validated here, the topic is first reviewed in a general context.

An attractive feature of the class of extended Kalman filters is that when either the estimation errors are small or the nonlinearities are cone bounded with tight cone bounds, the performance is near optimal, and of course, for the linear case, the performance is optimal. However, in highly nonlinear situations, when either the initial estimates are poor or the noise levels are high [1], [2], extended Kalman filters are far from optimal and are subject to divergence.[1] Optimal or near optimal schemes for these situations are usually too costly to implement. There is, therefore, strong motivation to seek simple modifications for the EKF which will improve its performance, even if only for restricted classes of problems. It makes sense to seek simple modifications which in effect broaden the class of filters under consideration.

In optimal linear filtering [1], [2], poor transient performance is usually not a problem, but divergence problems can arise in parameter estimation or when there is inadequate modeling, even though the optimal linear filter for the assumed model is guaranteed to be asymptotically stable. The divergence problems can be overcome by, in effect, assuming additional input noise in the system, or equivalently, giving more weight to the most recent data. As a consequence, the Kalman gain value and the filter degree of stability are increased at the expense of "optimality." Certainly the same divergence avoidance techniques can be applied to an EKF when there is evidence that the calculated Kalman gain can be increased with benefit, as, for example, in parameter estimation problems. However, the situation in the nonlinear filtering case is made more complicated due to the inherent suboptimality of the approach.

Our position in this paper is that for the nonlinear filtering situation, specialized techniques for each type of filter divergence may be required. In this paper we focus on two techniques for improving transient performance of an EKF for which there are no corresponding linear filter techniques. The first concerns coordinate basis selection while the second concerns tightening of the nonlinearity cone bounds during initial transients.

It has been pointed out in [3] that for nonlinear filters, in contrast to the linear filtering case, the selection of an appropriate coordinate basis may be crucial to the achievement of good filter performance. This is an example of how a simple change in the EKF design procedure can significantly affect both transient and steady-state performance. There is no design criteria given in [3] to avoid a poor coordinate basis selection.

In the next section of the paper we first consider a Lyapunov function for an EKF with measurement nonlinearities, and an associated bound on its decay rate thereby giving, indirectly, a stability measure. It is then verified that this measure can be readily employed to avoid a poor coordinate basis selection for the state equations and to guide in the trial and error selection of a suitable coordinate basis. Application of the stability measure is illustrated for the tracking filter of [3] where improved stability of the filter is demonstrated in passing from a Cartesian coordinate basis to a more suitable coordinate basis.

The paper moves on in Section III to explore the novel notion of modifying the nonlinearities in an EKF during the initial transient period so as to improve the transient performance. From another viewpoint, the aim of the modification is to avoid filter divergence during the transient period without significantly increasing the on-line calculated error covariance, as happens in applying linear filter divergence avoidance techniques [1], [2]. We seek to achieve a filter performance which is closer to the calculated one and, in turn, is closer to the optimal achievable performance. Motivated as in deterministic feedback systems with cone bounded nonlinearities where dither signals [4]–[6] are used to tighten the cone bounds and to improve stability properties, we use dither-modified nonlinearities in extended Kalman filters with cone-bounded nonlinearities. This techniques gives rise to the term "dithered" EKF. It is important that the class of "dithered" EKF described contains the optimal linear filter.

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[1]Divergence occurs when the calculated error covariance is optimistic and misleading, as in parameter estimation when the calculated error covariance goes to zero, but in the implementation the actual error covariance converges initially and then diverges, or when the actual transient performance is poor compared to what is anticipated from calculations.

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The performance theory in [7] for an EKF with cone-bounded nonlinearities achieves useful performance bounds, and these are also applicable to our "dithered" EKF. However, it turns out that the performance bound theory is unhelpful in explaining why nonlinearity modification yields performance improvement. In fact, for the case study given, simulations demonstrate performance improvements when there is a performance bound deterioration due to the modifications.

The properties of the "dithered" EKF are examined in Section IV for a case study which also illustrates the application of the stability measure in filter design. The application is known as passive tracking and consists of the coordinate estimation of a fixed station from bearing data collected from a moving sensor, as discussed in [8], [9]. We demonstrate that the "dithered" EKF with cone-bounded nonlinearities, when compared to the standard EKF, yields improvements to all state estimates under transient conditions, and as a consequence significantly improves the coordinate estimation.

II. Coordinate Basis Selection via a Stability Measure

Consider a signal model

$$\dot{x}_t = F(t)x_t + \Gamma(t)u_t$$
$$z_t = h(x_t, t).$$

In the stochastic case when there is additive white input noise and independent measurement noise of zero mean, and having covariances Q and R, respectively, then the extended Kalman filter is

$$\dot{\hat{x}}_t = F(t)\hat{x}_t + \Gamma(t)u_t + K(t)[z_t - h(\hat{x}_t, t)]$$

where

$$K(t) = P(t)H_{\hat{x}}'R^{-1}(t) \tag{2.1}$$

$$\frac{d}{dt}[P(t)]^{-1} = -F'(t)P^{-1}(t) - P^{-1}(t)F(t) - P^{-1}(t)Q(t)P^{-1}(t) + H_{\hat{x}}'R^{-1}(t)H_{\hat{x}} \tag{2.2}$$

and $H_{\hat{x}}$ is the Jacobian of $h(\hat{x},\cdot)$. The homogeneous equation of the extended Kalman filter associated with this model is [1]

$$\dot{\xi}_t = F(t)\xi_t - K(t)h(\xi_t, t). \tag{2.3}$$

Motivated as in the linear case [10], [11], let $V(\xi,t) = \xi'P^{-1}(t)\xi$ be a tentative Lyapunov function for (2.3). Then differentiation of $V(\hat{x},t)$ yields

$$\dot{V}(\xi,t) = -\xi'[P^{-1}QP^{-1}]\xi + [H_\xi\xi - h]'R^{-1}[H_\xi\xi - h] - h'R^{-1}h$$
$$< [H_\xi\xi - h]'R^{-1}[H_\xi\xi - h] - h'R^{-1}h \triangleq -\mu_s \tag{2.4}$$

where equality holds when $Q=0$, or equivalently when the input driving noise is zero. Application of Lyapunov stability theory [12], [13] tell us that a sufficient condition for asymptotic stability is that the lower bound on the rate of decay of $\dot{V}(\xi, t)$, viz., $\mu_s(\xi, t)$, be positive for $\xi \neq 0$ and all t. The bound[2] $\mu_s(\xi, t)$ gives some measure of stability and we loosely term it a "stability measure." Of course, when $Q \equiv 0$, the "stability measure" is identical to the rate of decay of "energy" in the system, and the larger its value, the more stable the system.

It is clear from the above analysis that we should avoid a coordinate basis selection such that $H_\xi\xi \equiv 0$, for then $\mu_s = 0$. Moreover, we should select a coordinate basis such that μ_s is "maximized" over the expected range of state estimates. This technique is applied to the following example.

Example: As an example we consider the design of the tracking filter discussed in [3]. In [3] it is reported that in changing from the coordinate basis, in which the equations are easily derived to another one, there is improved performance. No design criteria are given to guide in the selection of a suitable coordinate basis. Here we offer such criteria. The problem is that of estimating the path coordinates $x(t)$, $y(t)$ of a radiant source where a single sensor is available to detect the center frequency of the continuous incoming signal and the direction from which it is arriving. The relative motion of the target and sensor produce Doppler shifts $S(t)$ and source bearing $\theta(t)$ that change through time. The sensor is modeled as observing these quantities in the presence of uncorrelated, zero-mean Gaussian noise. The velocity components v_x, v_y and the center frequency f of the source are assumed to be constant.

From the problem description it is clear that we have a situation where the state equations are noise-free and the measurements are nonlinear and noisy. Using a Cartesian state vector $[x(t)\ y(t)\ v_x\ v_y\ f]'$, the observations are

$$\theta(t) = \tan^{-1}\left[\frac{x(t)}{y(t)}\right] + w_\theta \tag{2.5}$$

$$S(t) = f\left[1 - \frac{|v|}{c}\sin\alpha(t)\right] + w_f \tag{2.6}$$

where c is the velocity of the signal propagation, $\bar{t}$ is the time at which the source is at its closest point to the sensor, and

$$|v| = \left(v_x^2 + v_y^2\right)^{1/2}; \qquad \sin\alpha(t) = \frac{v(t-\bar{t})}{[x^2(t)+y^2(t)]^{1/2}}.$$

With this state selection, the "stability measure" can be calculated after tedious manipulations as

$$\mu_s = R_f^{-1}f^2\left[\left|1 - \frac{|v|}{c}\sin\alpha(t)\right|^2 - \left|\frac{|v|}{c}\sin\alpha(t)\right|^2\right].$$

The use of the "relative" coordinate basis[3] $[-v\bar{t}/r\ \ v/r\ \ \beta\ \ f\ \ -fv/c]'$, as suggested in [3], improves the stability properties of the filter. The observations are then, with the state elements denoted $[x_1\ x_2\ \cdots\ x_5]$

$$\theta(t) = \tan^{-1}[(x_1 + x_2t)] + x_3 + w_\theta \tag{2.5'}$$

$$S(t) = x_4 + x_5\frac{(x_1 + x_2t)}{\left[1 + (x_1 + x_2t)^2\right]^{1/2}} + w_f. \tag{2.6'}$$

For these measurement functions, the stability measure μ_s is increased by the term

$$R_f^{-1}\left(\frac{fv}{c}\right)^2\sin^2\alpha + R_\theta^{-1}\left[(\alpha+\beta) - \left(\alpha - \frac{1}{2}\sin 2\alpha\right)^2\right]$$

for small $|\alpha|$, this term is approximately

$$F_f^{-1}\left(\frac{fv}{c}\right)^2\alpha^2 + R_\theta^{-1}(\alpha+\beta)^2$$

and for $|\alpha| \simeq \pi/2$ it approximates $R_\theta^{-1}[(\alpha+\beta)^2 - \alpha^2]$. It is now clear that this term is positive for a wide range of parameters α, β, etc., and thus that the filter with the "relative" state coordinates has better stability properties than it has with the Cartesian state. This is why it has improved transient performance.

So far we have seen that by a selection of a coordinate basis, so as to increase a stability measure over the expected range of states, the performance of the filter can be improved significantly. The stability measure is simple to use and is readily employed to avoid a poor coordinate basis selection.

III. Tightening the Output Nonlinearity Cone Bounds

In this section we restrict our attention to an EKF with cone-bounded nonlinearities. As an autonomous system, the EKF then belongs to the class of systems studied in the Popov and subsequent stability theories [14]. These theories tell us that classes of systems exist which, when the cone bounds are tightened, exhibit an improvement in stability properties. As noted in [5], experience has shown that the class of such systems

[2]A reviewer has pointed out that $E[\dot{V}(\xi,t)] < -\operatorname{tr}[P^{-1}A] - \mu_s$ which may give an alternative useful "stability measure."

[3]The pair (r, β) denotes the polar coordinates of the target when it is at its closest point of approach to the sensor.

is surprisingly large. This suggests that for some classes of EKF, stability properties are improved if the cone bounds are tightened. We cannot, of course, claim that all EKF with cone bounded nonlinearities have improved stability properties as the cone bounds are tightened. Rather, we claim that the class of EKF with cone-bounded nonlinearities for which there is improvement in stability properties as the cone bounds are tightened is large, and for this class there is a potential for finding an improved EKF modified by tightening the cone bounds during transients. At this stage there appears to be no simple and useful classification of such filters.

The above remarks suggest that there is a good *a priori* chance that an improvement in filter stability properties can be achieved by modifying the filter nonlinearities so as to tighten the cone bounds. However, at least in the low noise case after initial transients, it is clear that such an action will degrade the near optimal performance of an EKF. We are led to the notion of modifying the output nonlinearities so as to tighten the cone bounds only during the initial transient when the filter is far from optimal and with poor stability properties. Denoting the modified nonlinearities with tightened cone bounds as $h^*(x,t)$, then for such a case, $h^*(x,t)$ can be selected so that $h^*(x,t)\to h(x,t)$, say, exponentially over the transient period.

Of course, it is desirable to have a theory for telling us when and how to modify the nonlinearities of an extended Kalman filter, and to indicate how significant the improvement is from such modifications. Such a theory is elusive at this time other than one based on the performance bounds of [7]. For our application the bounds turned out to be too loose to be helpful. We now move on to explore a convenient way of implementing the concept.

A possibly convenient method to tighten the cone bounds is to introduce dither signals [4]–[6], or to use a nonlinearity equivalent to that obtained by injecting dither signals prior to the unmodified nonlinearity. A dither is simply a high frequency signal which, when injected prior to a nonlinearity in a system with low pass filtering characteristics, modifies the nonlinearity. By sweeping back and forth quickly across the domain of a nonlinear element and then low-pass filtering, dither has the effect of averaging the nonlinearity, making it smoother and in some sense more linear. When the nonlinearity $h(x,t)$ is cone bounded and lies in the incremental sector $\{\alpha,\beta\}$, then the introduction of an appropriate dither signal with amplitude d yields a modified nonlinearity $h_d^*(x,t)$ which lies in the incremental sector $\{\alpha^*(d),\beta^*(d)\}$ where $\alpha<\alpha^*(d)<\beta^*(d)<\beta$ [5]. As the magnitude of the dither signal increases, we can loosely say that $h_d^*(x,t)$ becomes closer to a linear function. Analytically it is a function of the dither waveform and its amplitude. It is defined by [4], [5]

$$h_d^*(x,t)=\int_{-d}^{d}h(\eta+x,t)p(\eta)\,d\eta \tag{3.1}$$

where d is the dither amplitude and $p(\eta)$ is its amplitude probability density function.

In the design of an EKF with tightened cone bounds we will use the analytical expression of (3.1). The thrust of this technique is that by an adjustment of only one parameter, d, we can control the tightness of the cone bounds. Note that with $d=d_0e^{-t/\tau}$, then $h_d^*(x,t)\to h(x,t)$ exponentially as $t\to\infty$. Of course, this same end could be achieved in any of a number of ways.

IV. Coordinate Estimation from Bearing Data

Consider the specific problem of estimating the coordinates of a fixed station from bearing observations taken by a moving sensor, such as when an aircraft senses the direction of a fixed radar or radio transmitter with coordinates $[X, Y]$ as in Fig. 1.

In the case of discrete-time bearing and velocity measurements θ_i and v_i at time instants t_i, recursive estimates of X and Y can be obtained. In the simple noise free case when the velocity is constant ($v_i=v$), then $Y=v(t_2-t_1)/(\tan\theta_1-\tan\theta_2)$, $X=Y\tan\theta_1+vt_1$. In the case of noisy measurements, the following model is assumed:

$$\dot{x}=v,\qquad \dot{v}=\delta_a,\qquad \dot{X}=0,\qquad \dot{Y}=0 \tag{4.1}$$

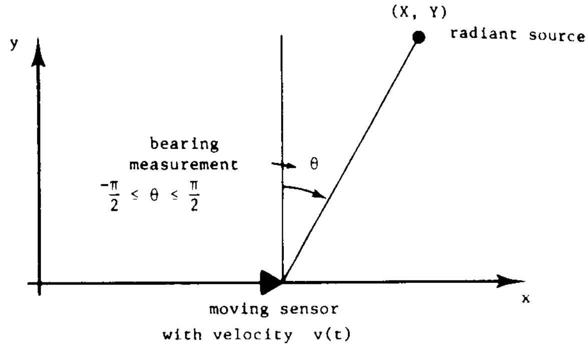

Fig. 1. Problem illustration.

where δ_a is an acceleration uncertainty considered as white noise with distribution $N[0,\sigma_a^2]$. The sampled measurements are modeled as

$$\theta_i=\tan^{-1}\left[\frac{X}{Y}-\frac{x_i}{Y}\right]+\delta_\theta$$

where δ_θ is the uncertainty in angle measurement considered as white noise with distribution $N(0,\sigma_\theta^2)$. Likewise, velocity is measured as

$$V_i=v_i+\delta_v,\qquad \delta_v\sim N(0,\sigma_v^2).$$

It turns out that the filter performance can be improved significantly if the Cartesian coordinate basis is modified. Since the input noise covariance in this problem is known to be small we can use the results of Section II. Following the example described there, we select a new coordinate basis $\xi'=[\xi_1,\xi_2,\xi_3,\xi_4]$ as

$$\xi'(t)=\left[\frac{x(t)}{Y}\;\;\frac{v(t)}{Y}\;\;v(t)\;\;\frac{X}{Y}\right].$$

The coordinates X and Y of the radiant source are estimated as $\hat{Y}=\hat{\xi}_3/\hat{\xi}_2$ and $\hat{X}=\hat{\xi}_4\hat{Y}$.

The state equations are

$$\begin{bmatrix}\dot{\xi}_1\\ \dot{\xi}_2\\ \dot{\xi}_3\\ \dot{\xi}_4\end{bmatrix}=\begin{bmatrix}0&1&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}\begin{bmatrix}\xi_1\\ \xi_2\\ \xi_3\\ \xi_4\end{bmatrix}+\begin{bmatrix}0\\ \delta_a/Y\\ \delta_a\\ 0\end{bmatrix} \tag{4.3}$$

$$\begin{bmatrix}\theta_i\\ V_i\end{bmatrix}=\begin{bmatrix}\tan^{-1}[\xi_4-\xi_1(t_i)]\\ v_i\end{bmatrix}+\begin{bmatrix}\delta_v\\ \delta_v\end{bmatrix}. \tag{4.4}$$

For our simulations, we work with discrete-time versions of the above equations and introduce a simplification that v_i is a known constant with δ_v very small. This means that the state dimension can be reduced thereby simplifying the filter.

Notice that the change of the coordinate basis has an additional effect. The nonlinearity of (4.2) involving $\tan^{-1}(\cdot)$ and division of one state variable by another has been simplified in (4.4). The division amongst state variables in both the state model and the state estimation process is avoided. This is a useful simplification when working with dither-modified nonlinearities since dither has no effect on either a multiplication or a division process.

Notice also that in (4.3), the input noise term δ_a/Y is, in effect, state dependent and so for estimation purposes δ_a/Y will be replaced by $\delta_a/\hat{Y}$. This approximation turns out to be a good one since simulations show that the state estimation process has low sensitivity to this term.

The dither-modified nonlinearity for the $\tan^{-1}$ function is calculated for a triangular dither signal. Using the fact that a triangular dither signal has a uniform amplitude distribution function,[4]

$$p(\eta)=\begin{cases}\dfrac{1}{2d} & |\eta|<d\\ 0 & |\eta|>d\end{cases}$$

[4]Actually, other density functions for dither signals can be employed but we avoid those which could lead to a modified nonlinearity with dead zones.

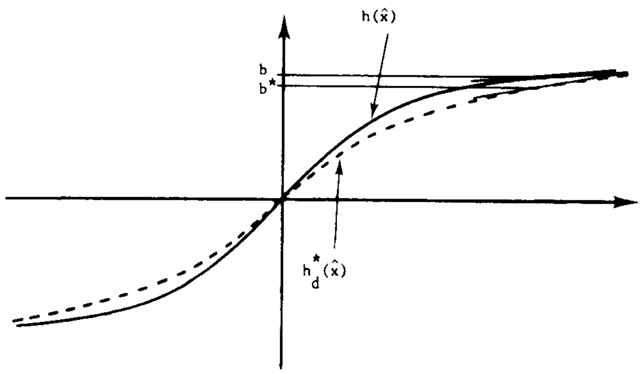

Fig. 2. Effect of nonlinearity dither on the cone bounded.

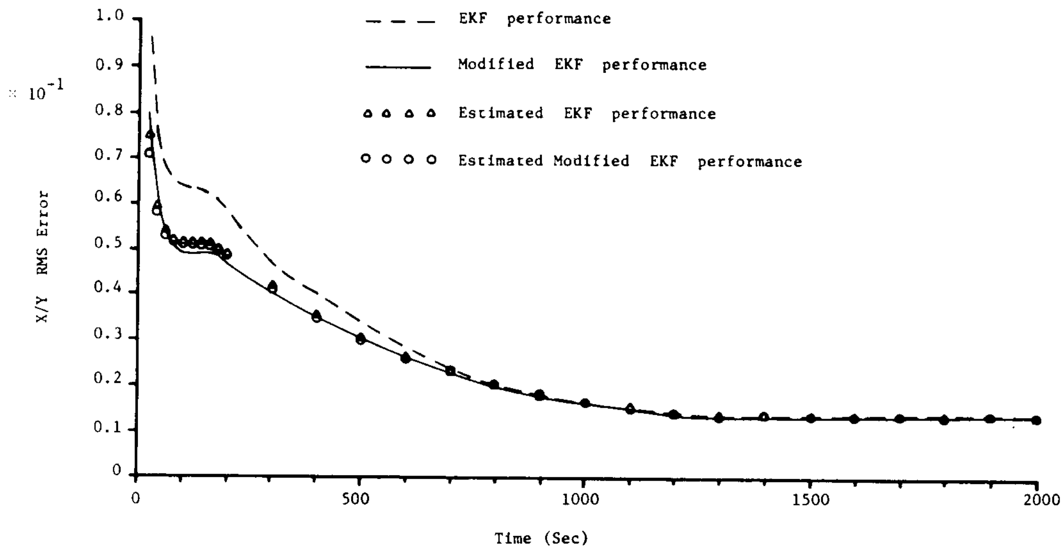

Fig. 3. Error in estimate of (X/Y).

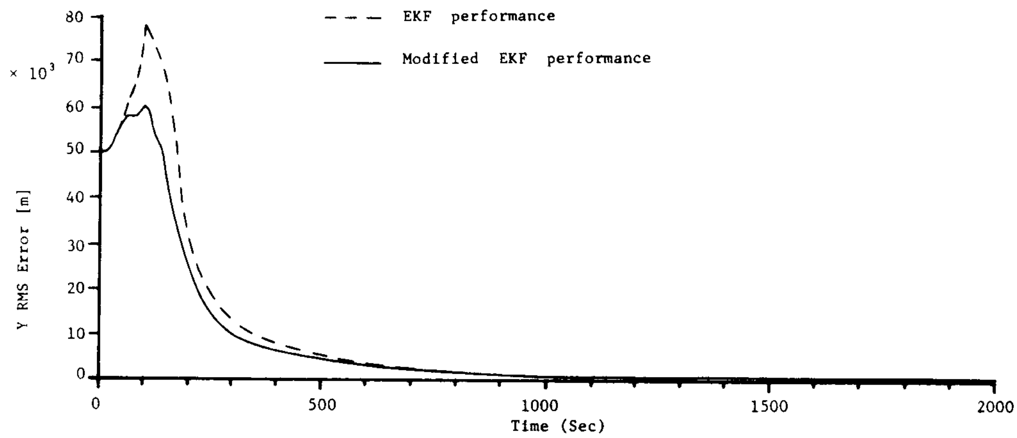

Fig. 4. Error in estimate of Y.

the modified $\tan^{-1}$ function is calculated via (3.1) as

$$h_d^*(x)=\frac{1}{2d}\Bigg\{(x+d)\tan^{-1}(x+d)-(x-d)\tan^{-1}(x-d)+\frac{1}{2}\ln\left[\frac{1+(x-d)^2}{1+(x+d)^2}\right]\Bigg\}. \quad (4.5)$$

Note that $h_d^*(0)=0$ and $h_d^*(\pm\infty)=\pm\pi/2$. Fig. 2 gives a sketch of $h(\hat{x})$ and $h_d^*(x)$ for this case with $d=0.5$. It is not difficult to see that the "stability measure" associated with $h^*(\hat{x})$ is greater than that the $h(\hat{x})$ for all $\hat{x}$, and thus as d increases, the filter becomes more stable.

The measurement nonlinearity used in the modified filter is $h_d^*(x)$ with $h_d^*(x)\to h(x)$ exponentially as $t\to\infty$ with time constant τ. One way to achieve this is for d to be set as $d=d_0e^{-t/\tau}$ for some d_0.

The value of τ is selected by a comparison of two Monte Carlo runs, one for $\tau=0$ (the unmodified filter) and one for $\tau=\infty$. The value of τ is then selected as, say, $1/4$ the period for which there is an improvement in the performance of the second run over the first run.

The selection of the initial dither amplitude d_0 is best guided by a trial and error procedure. Since with a zero $d_0, h_d^* = h$, it makes sense to start the runs characterized by $\tau = \infty$ with a small d_0 and then to increase its value until performance begins to deteriorate.

In order to eliminate numerical problems in the calculation of the gain and the covariance matrix using discrete-time verions not spelled out in detail here, we use U-D factorization [15], [16].

Figs. 3 and 4 illustrate typical performance of the state estimate and the coordinate estimate. The errors are obtained by averaging the estimated and actual square errors over 400 runs. The results shown correspond to the parameters

$$\sigma_a = 0.1 \text{ m/s}^2, \quad \sigma_\theta = 2 \text{ mrad}, \quad X = 200 \text{ km}, \quad Y = 100 \text{ km}$$

$$x(t_0) = 0.0 \text{ m}, \quad v(t_0) = 200 \text{ m/s}, \quad 0 < t < 2000 \text{ s}$$

$$\xi_0 \sim N(\bar{\xi}_0, P_0), \quad \bar{\xi}_0' = [0, 1.33 \times 10^{-3}, 200, 1.67]$$

$$P_0 = \text{diag}[4.44 \times 10^{-7}, 0.5 \times 10^{-6}, 1, 1].$$

Observations are taken every 20 s. The inital error in the estimate of X and Y is 50 km. The time constant τ used is 12.5 s, and $d_0 = 0.55$.

Simulations show that the modified EKF yields improvements to the state estimates under transient conditions, and significant performance improvement in the coordinate estimation. Also, the modified EKF has no increase in the on-line calculated error covariance in comparison to the unmodified EKF.

V. Conclusions

We have seen that by modifying an EKF with relatively simple modifications, the transient performance of the filter may be improved significantly. Specific modification techniques have been derived and studied in the important application of passive tracking. The restriction of the techniques to limited classes of EKF suggests that there could be other techniques which also broaden the range of useful applications of the EKF concept.

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