Robotic Motion Planning: Roadmap Methods

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RoadMap Definition

- A roadmap, RM, is a union of curves such that for all start and goal points in Q_{free} that can be connected by a path:
  - **Accessibility:** There is a path from $q_{start} \in Q_{free}$ to some $q' \in RM$
  - **Departability:** There is a path from some $q'' \in RM$ to $q_{goal} \in Q_{free}$
  - **Connectivity:** there exists a path in RM between q' and q''
  - **One dimensional**

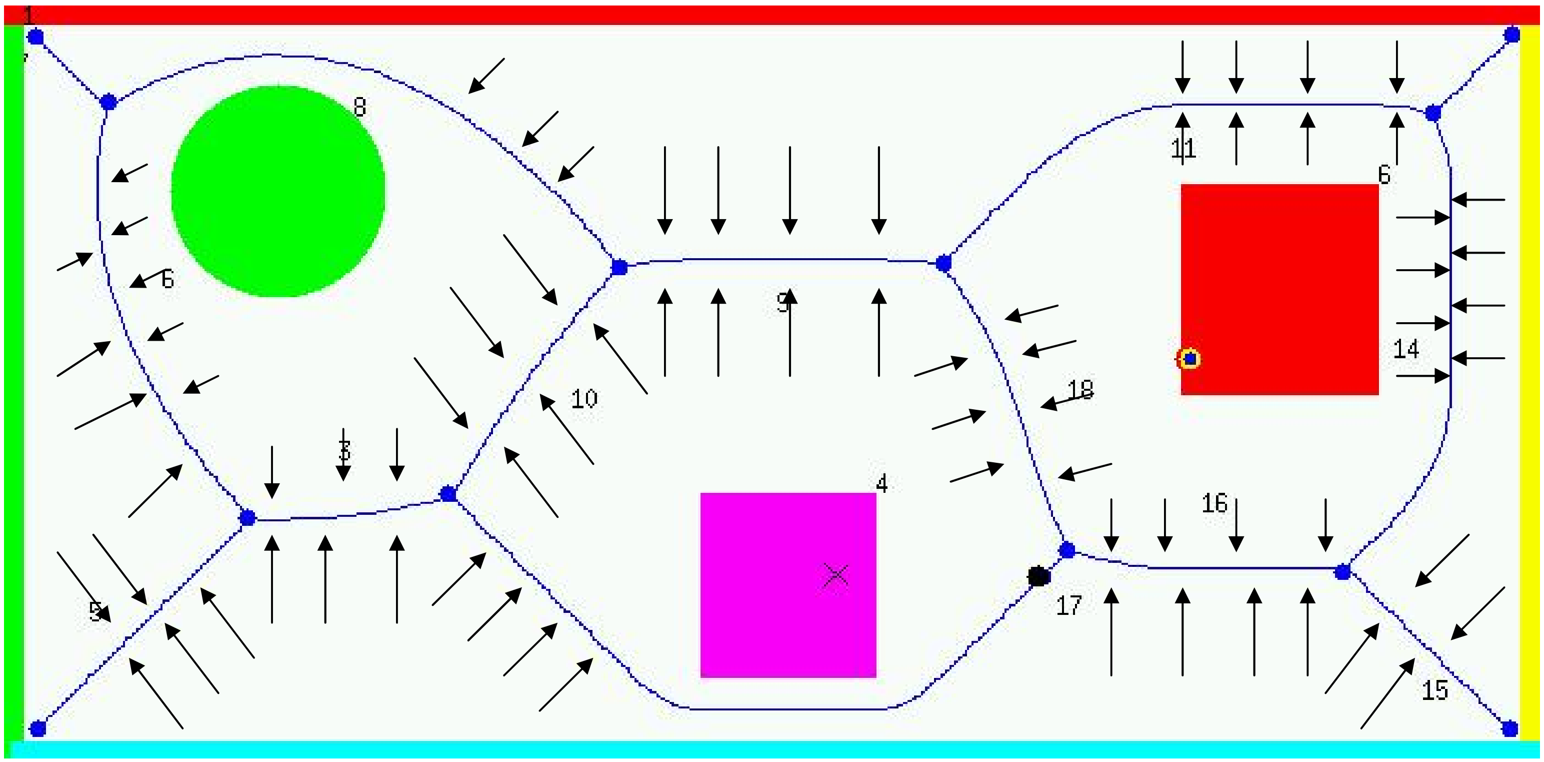
Types of Roadmaps

- Visibility
- Retraction
- Retract-like and Piecewise Retracts
- Silhouette

- **Homework 5**
  16-735 Robotic Motion Planning
  Handout: Mon Oct 8th
  Due: Wed Oct 17th
- Implement a VISIBILITY GRAPH for a two-dimensional polygonal (or three-dimensional polyhedral) robot and work space (Remember to construct the configuration space)
- On your web page, show at least two images - one that illustrates a successful planning episode, and one that fails due to the start and goal being in different connected portions of the free space. Include on your web page a brief description (pdf file or html ONLY) of your approach. This report should include all relevant equations and algorithm descriptions, along with a description of any parameters used by your algorithm.

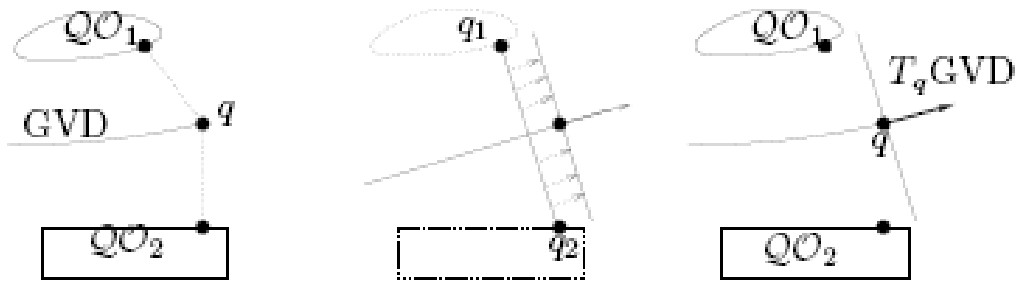
Next homework

Deformation Retraction: GVG in Plane



Traceability in the Plane

Tangent



Pass a line through two closest points on two closest obstacles

Orthogonal is tangent

$$f : R^m \to R^n$$

$$f^{-1}(c) = \{x \in R^m : f(x) = c\}$$

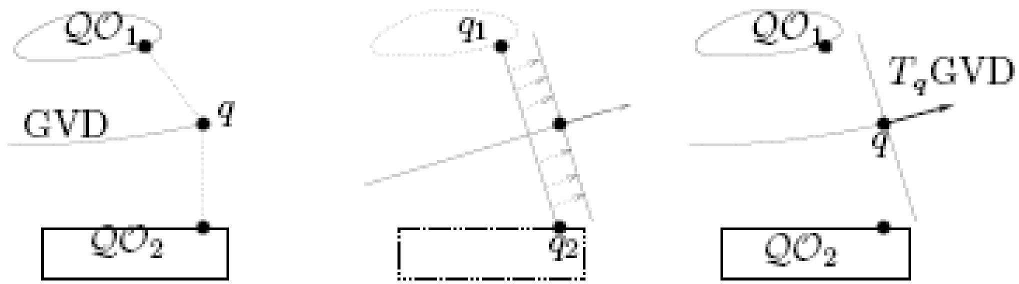
$$\text{if } \forall x \in f^{-1}(c), Df(x) \text{ is full rank,}$$

$$\text{then } f^{-1}(c) \text{ is a manifold of dimension } m - n$$

Go to whiteboard to discuss tangent space is the null space of Df(x)
And then similar triangles!!

Traceability in the Plane

Tangent



Pass a line through two closest points on two closest obstacles

Orthogonal is tangent

Correction

$$y^{k+1} = y^k - (\nabla_y G)^{-1} G(y^k, \lambda^k)$$

Control Laws

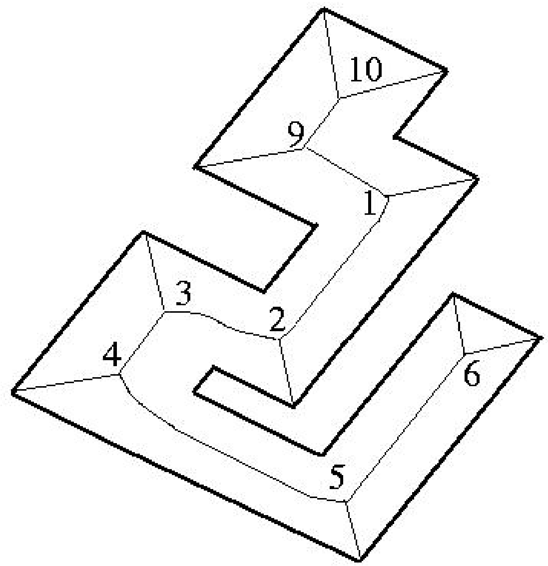
Edge : $G(x) = 0 = d_i(x) - d_j(x)$

$$\dot{x} = \alpha\, Null(\nabla G(x)) + \beta (\nabla G(x))^{\dagger} G(x)$$

where

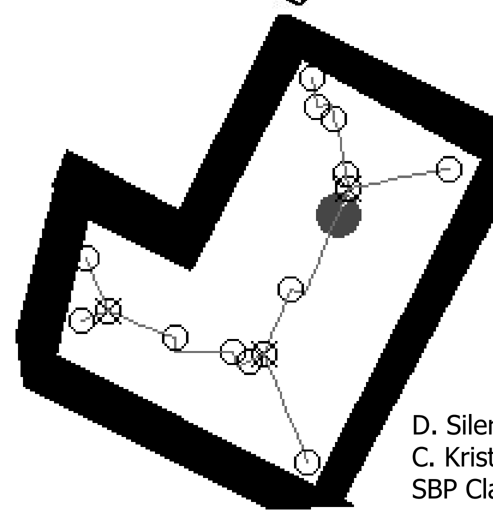
- α and β are scalar gains
- $Null(\nabla G(x))$ is the null space of $\nabla G(x)$
- $(\nabla G(x))^{\dagger}$ is the Penrose pseudo inverse of $\nabla G(x)$, i.e.,

$$(\nabla G(x))^{\dagger} = (\nabla G(x))^{T} (\nabla G(x) (\nabla G(x))^{T})^{-1}$$



Meet Point : $G(x) = 0 = \begin{Bmatrix} d_i(x) - d_j(x) \\ d_i(x) - d_k(x) \end{Bmatrix}$

$$\dot{x} = \alpha\, Null(\nabla G(x)) + \beta (\nabla G(x))^{\dagger} G(x)$$



D. Siler
C. Kristoff
SBP Class

Algorithm for exploration

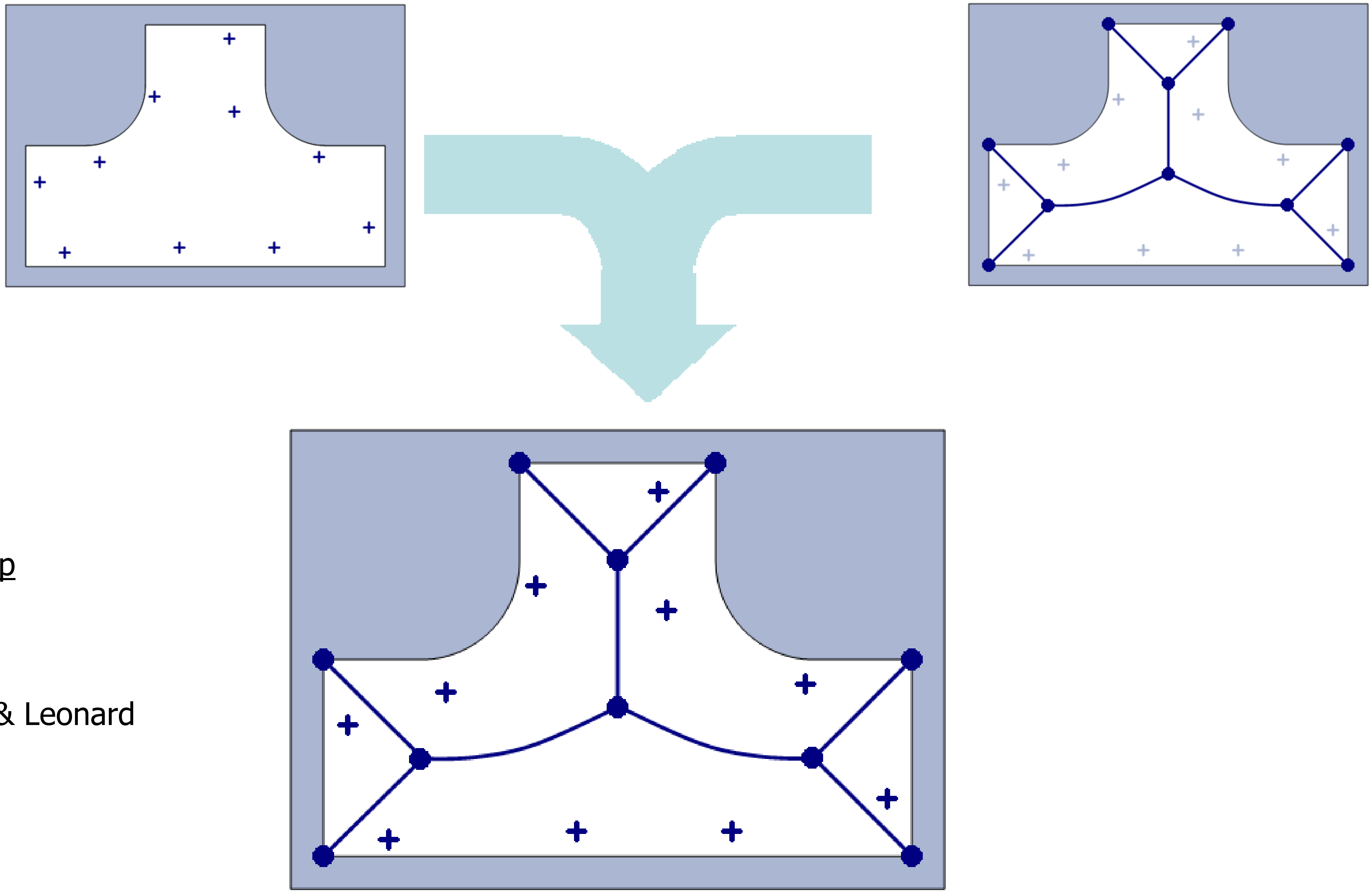
- Trace an edge until reach a meet point or a boundary point
- If a boundary point, return to the previous meet point, otherwise pick a new edge to trace
- If all edges from meet point are already traced, search the graph for a meet point with untraced edges
- When all meet points have no untraced edges, complete.

Demo

Are those meet points really necessary?

Implication for SLAM

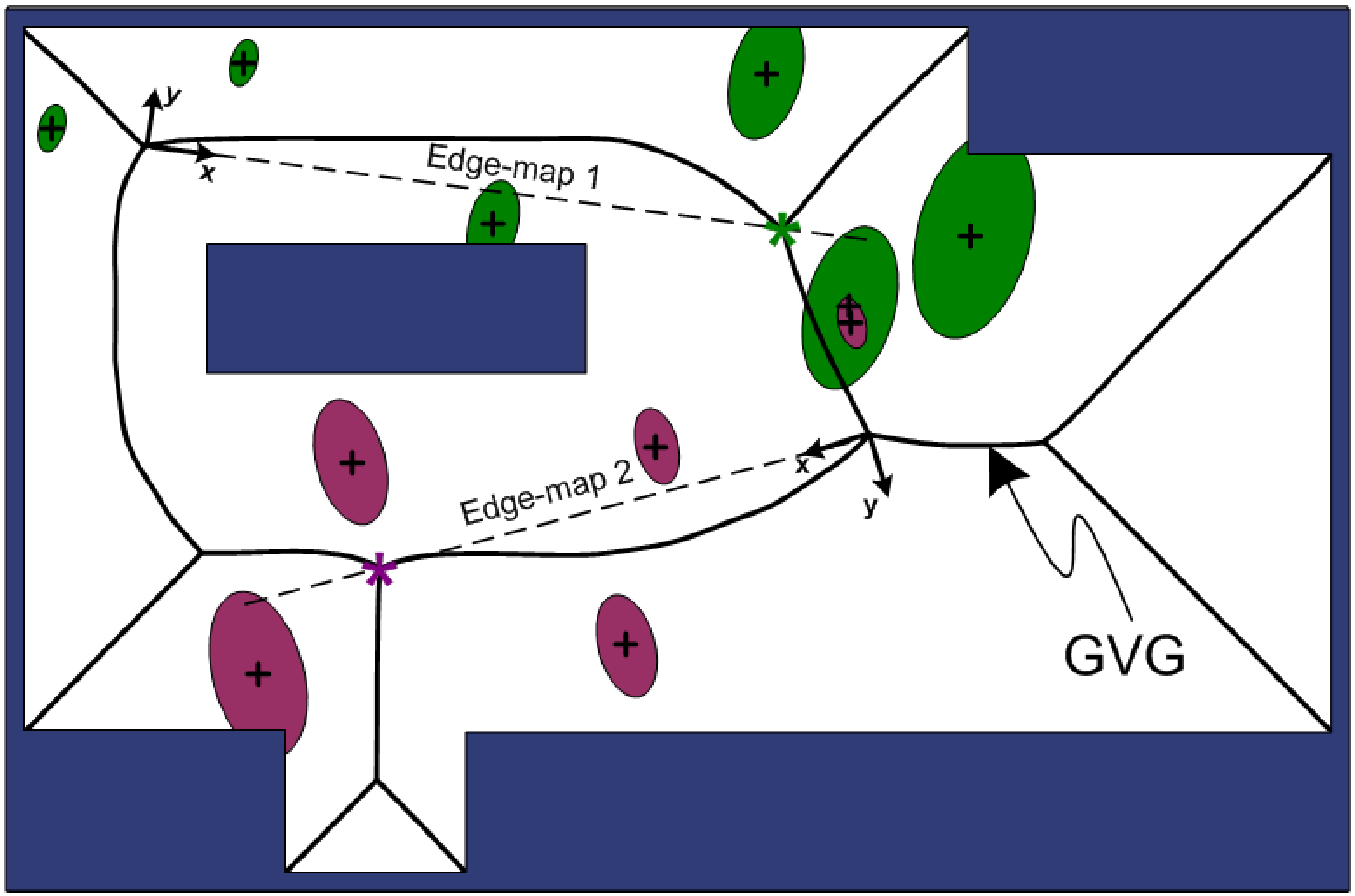
Implement a feature-based technique in a topological framework



<u>Submap</u>
Chong

<u>Atlas</u>
Bosse & Leonard

Embedded H-SLAM Map



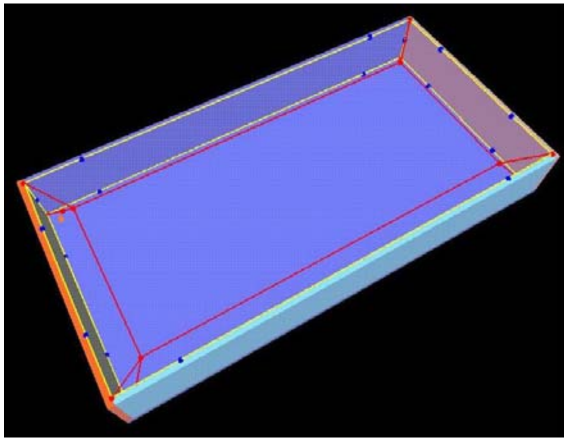
Meet points help: Submaps, close the loop

General Voronoi Graph

- In 3-Dimensions

$$F_{ijk} = F_{ij} \cap F_{ik} \cap F_{jk}$$

- In *m*-Dimensions



$$F_{ijk\ldots m} = F_{ij} \cap F_{ik} \ldots \cap F_{im}$$

$$= F_{ij\ldots m-1} \cap F_{im}$$

GVD vs. GVG

	Equidistant (#obs)	Dim	Codim
GVD	2	*m-1*	1
GVG	*m*	1	*m-1*

Proofs by Pre-Image Theorem to come

Proof for GVG Dimension

- For 3-Dimensions

$$f = \begin{pmatrix} d_i - d_j \\ d_i - d_k \end{pmatrix}, \quad f : R^3 \to R^2$$

$$f^{-1}\begin{pmatrix} 0 \\ 0 \end{pmatrix} = f^{-1}(0)$$

$$D\begin{pmatrix} d_i - d_j \\ d_i - d_k \end{pmatrix} \neq 0, \ \text{since} \, \nabla d_i \neq \nabla d_j, \nabla d_i \neq \nabla d_k$$

Proof for GVG (cont.)

- For *m*-Dimensions

$$f : \begin{pmatrix} d_{i_1} - d_{i_2} \\ \cdot \\ \cdot \\ \cdot \\ d_{i_1} - d_{i_m} \end{pmatrix}, \text{where } f : R^m \rightarrow R^{m-1}$$

$$\text{By Pre-Image Theorem, } \dim(f^{-1}) = m - (m-1) = 1$$

Traceability in m dimensions

- *x* is a point on the GVG
  - *normal slice plane*
  - "*sweep" coordinate*
- Define

Pass a hyperplane through the m closest points on the m closest obstacles

$$y = (z_2, \ldots z_m)$$

$$\lambda = z_1$$

Tangent is orthogonal to this hyperplane

$$G(y,\lambda) = \begin{bmatrix} (d_1 - d_2)(y,\lambda) \\ (d_1 - d_3)(y,\lambda) \\ . \\ . \\ . \\ (d_1 - d_m)(y,\lambda) \end{bmatrix}$$

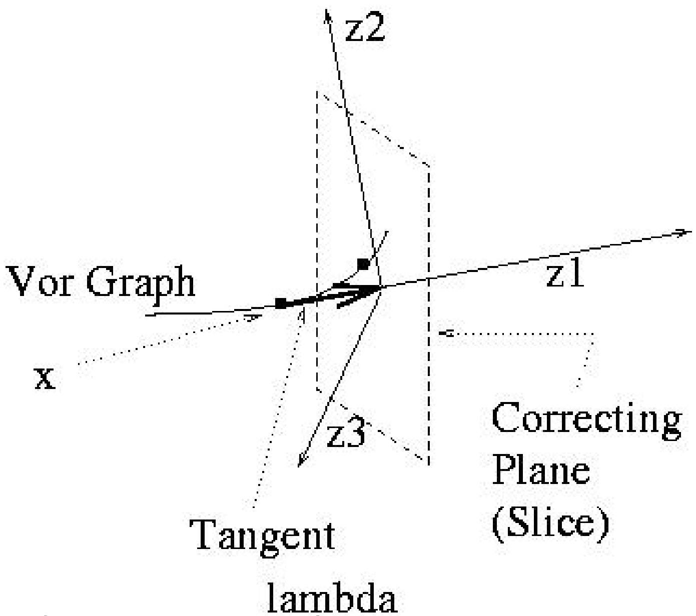
where $G : R^{m-1} \times R \rightarrow R^{m-1}$

Traceability (cont.)

Predictor-corrector scheme

- Take small step, $\Delta\lambda$ in z_1 direction (tangent).
- Correct using iterative Newton's Method

$$y^{k+1} = y^k - (\nabla_y G)^{-1} G(y^k, \lambda^k)$$



Accessibility

Gradient Ascent: Cascading Sequence of Gradient Ascent Operations

– Move until F_{ij}

– Maintain 2-way equidistant while $\uparrow D$

$$\prod_{T_x F_{ij}} \nabla D = \prod_{T_x F_{ij}} \nabla d_i = \prod_{T_x F_{ij}} \nabla d_j$$

GVG Connected?

$$F_{ijk} \subseteq \partial F_{ij}$$

$$F_{ijk} \subseteq \partial F_{ik}$$

$$F_{ijk} \subseteq \partial F_{jk}$$

Assuming ∂F_{ij} is connected $\forall F_{ij}$

Is GVG connected?

GVG Connected?

∂F_{ij} is not connected