15-887
Planning, Execution and Learning

Execution II:
Real-time Heuristic Search

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Planning during Execution

- Planning is a repeated process!
  - partially-known environments
  - dynamic environments
  - imperfect execution of plans
  - imprecise localization

- Need to be able to re-plan fast!

- Several methodologies to achieve this:
  - anytime heuristic search: return the best plan possible within T msecs
  - incremental heuristic search: speed up search by reusing previous efforts
  - real-time heuristic search: plan few steps towards the goal and re-plan later

this class

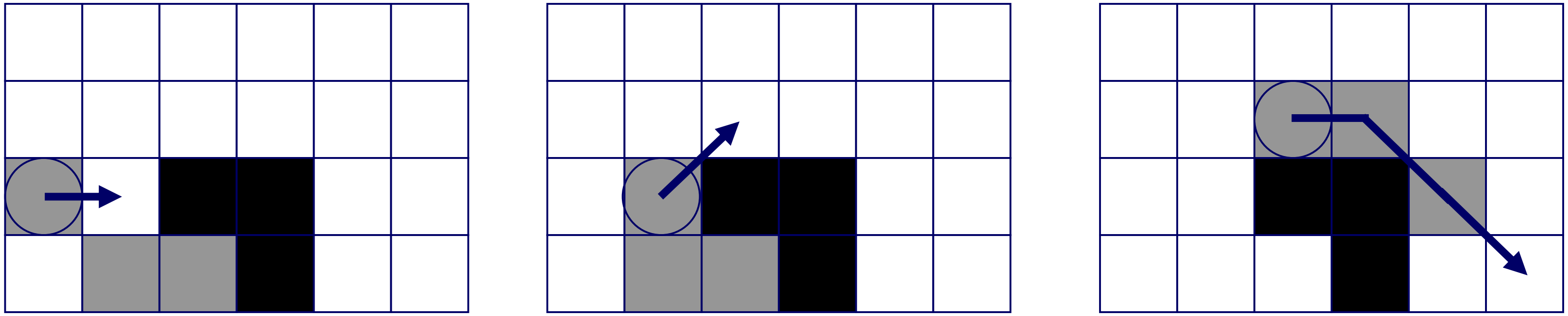
Real-time (Agent-centered) Heuristic Search

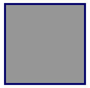
Enforce a strict limit on the amount of computations (no requirement on planning all the way to the goal)

Real-time (Agent-centered) Heuristic Search

1. Compute a partial path by expanding at most N states around the robot
2. Move once, incorporate sensor information, and goto step 1

Example in a fully-known terrain:



 - *expanded*

Real-time (Agent-centered) Heuristic Search

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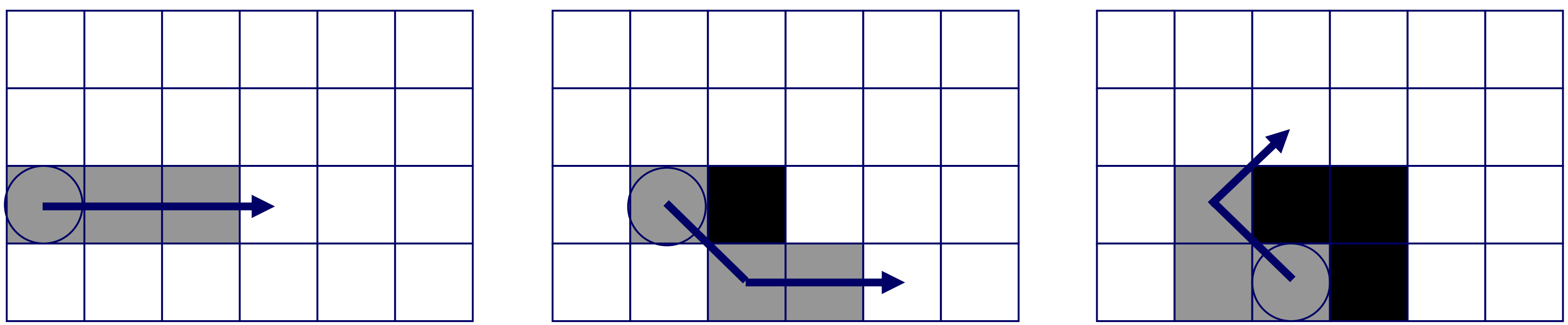
Example in an unknown terrain (planning with Freespace Assumption):

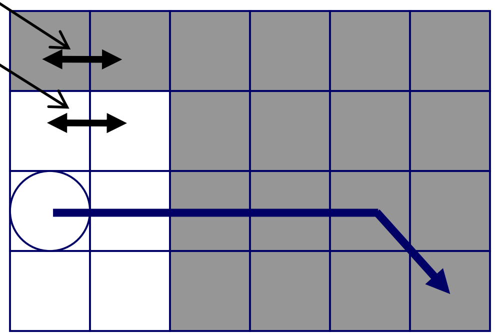


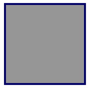
- *expanded*

Planning with Freespace Assumption [Nourbakhsh & Genesereth, ‘96]

- Planning with the Freespace Assumption always moves the robot on a shortest potentially unblocked path in a partially-known terrain to the goal state
- Replan the path whenever a new sensor information received

costs between unknown states is the same as the costs in between states known to be free





- unknown

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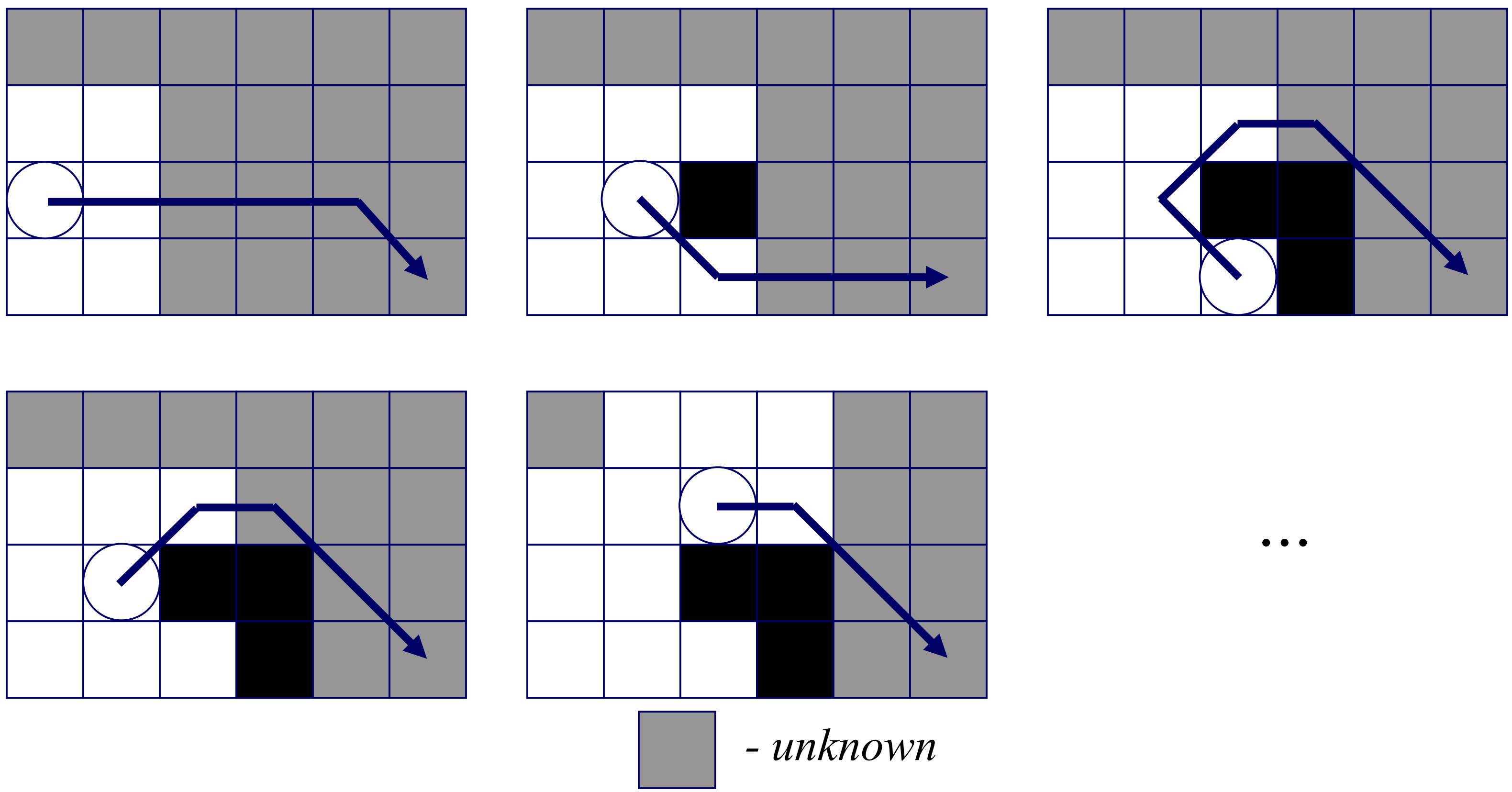

Real-time (Agent-centered) Heuristic Search

1. Compute a partial path by expanding at most N states around the robot
2. Move once, incorporate sensor information, and goto step 1

Research issues:

- how to compute partial path

- how to guarantee complete behavior (guarantee to reach the goal)

- provide bounds on the number of steps before reaching the goal

Real-time (Agent-centered) Heuristic Search

1. Compute a partial path by expanding at most N states around the robot
2. Move once, incorporate sensor information, and goto step 1

Research issues:

- how to compute partial path

Any ideas?

- how to guarantee complete behavior (guarantee to reach the goal)

- provide bounds on the number of steps before reaching the goal

Learning Real-Time A* (LRTA*) [Korf, '90]

- Repeatedly move the robot to the most promising adjacent state, using heuristics

1. *always move as follows:* $s_{start} = \mathrm{argmin}_{s \in succ(sstart)} c(s_{start}, s) + h(s)$

$$h(x,y) = \max(\mathrm{abs}(x-x_{goal}), \mathrm{abs}(y-y_{goal})) + 0.4 * \min(\mathrm{abs}(x-x_{goal}), \mathrm{abs}(y-y_{goal}))$$

6.2	5.2	4.2	3.8	3.4	3
5.8	4.8	3.8	2.8	2.4	2
5.4	4.4	3.4	2.4	1.4	1
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Any problems?

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…

Local minima problem (myopic or incomplete behavior)

Any solutions?

Learning Real-Time A* (LRTA*) [Korf, ‘90]

- Repeatedly move the robot to the most promising adjacent state, using **and updating** heuristics

makes h-values more informed

1. *update* $h(s_{start}) = \min_{s \in succ(sstart)} c(s_{start}, s) + h(s)$
2. *always move as follows:* $s_{start} = \text{argmin}_{s \in succ(sstart)} c(s_{start}, s) + h(s)$

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5	4	5		1	0

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…

h-values remain admissible and consistent

proof?

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robot is guaranteed to reach goal in finite number of steps if:

- all costs are bounded from below with $\Delta > 0$
- graph is of finite size and there exists a finite-cost path to the goal
- all actions are reversible

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- all costs are bounded from below with $\Delta > 0$
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Why conditions?

Learning Real-Time A* (LRTA*)

- LRTA* with $N \geq 1$ expands [Koenig, '04]

necessary for the guarantee to reach the goal

1. *expand N states*
2. *update h-values of expanded states by Dynamic Programming (DP)*
3. *move on the path to state s = argmin* $_{s' \in OPEN}$ *g(s') + h(s')*

- *expanded*

Learning Real-Time A* (LRTA*)

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state *s*:

- the state that minimizes cost to it plus heuristic estimate of the remaining distance

- the state that looks most promising in terms of the whole path from current robot state to goal

- *expanded*

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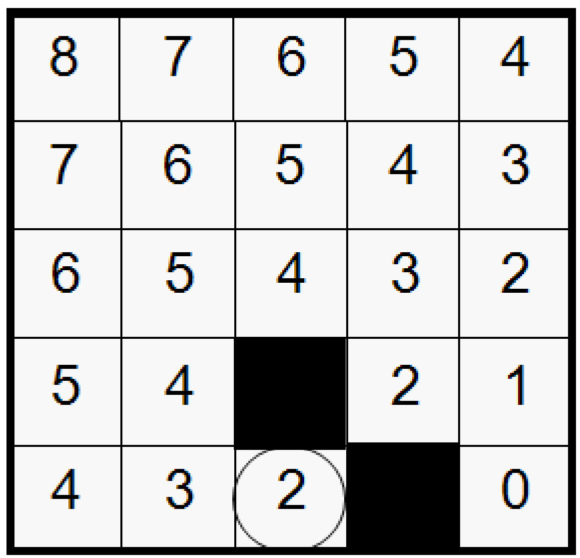

4-connected grid (robot moves in 4 directions)

example borrowed from ICAPS'06 planning summer school lecture (Koenig & Likhachev)

- expanded

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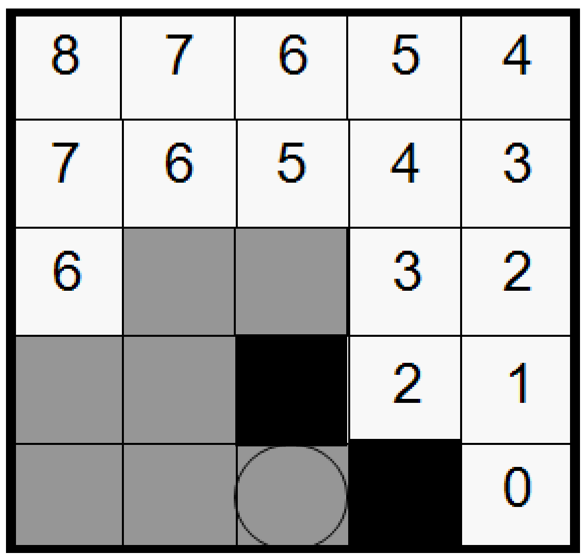

expand N=7 states

- expanded

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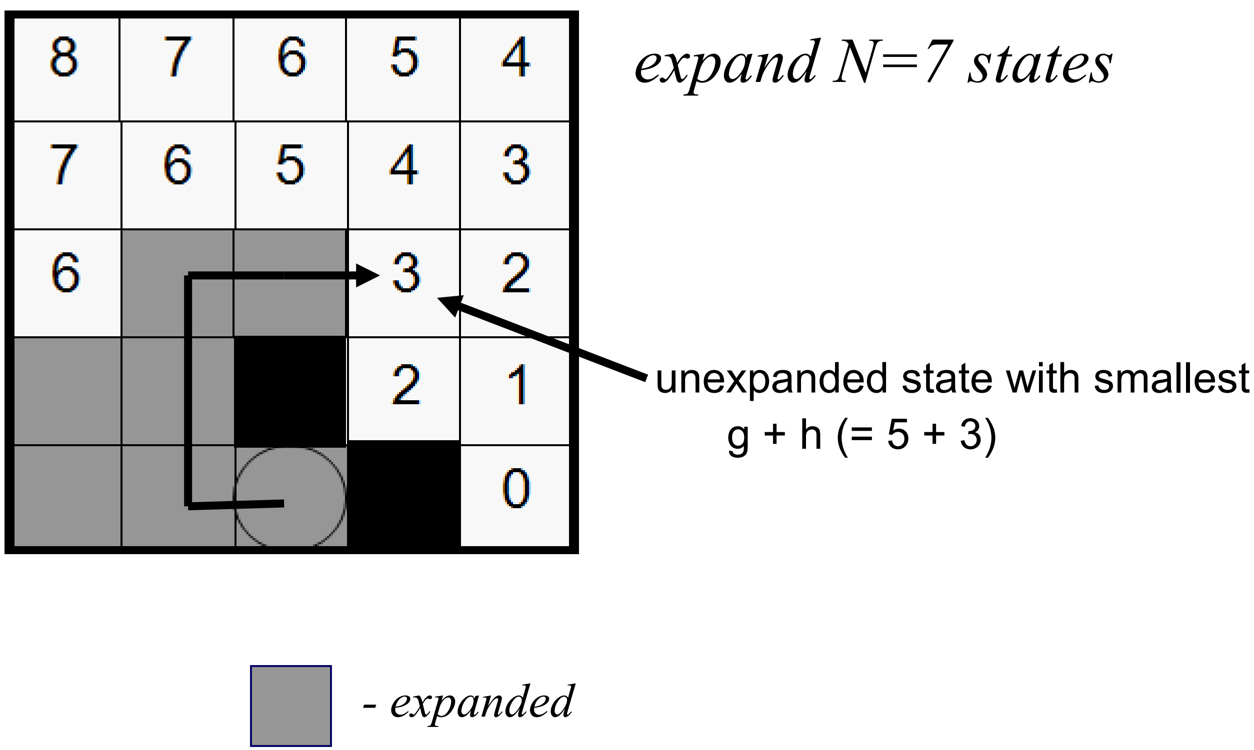

Learning Real-Time A* (LRTA*)

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How path is found?

1. *expand N states*
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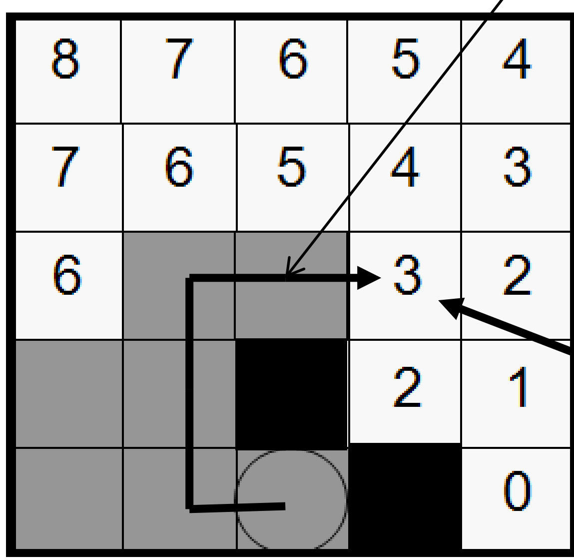

expand N=7 states

unexpanded state with smallest g + h (= 5 + 3)

- *expanded*

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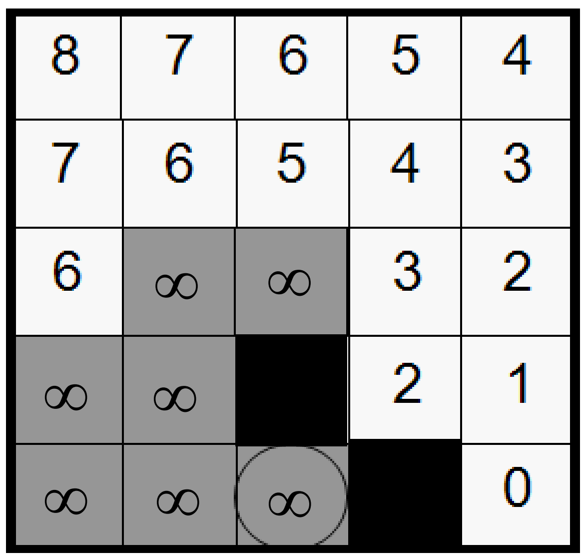

update h-values of expanded states via DP:
compute $h(s) = \min_{s' \in succ(s)} (c(s,s')+h(s'))$
until convergence

- expanded

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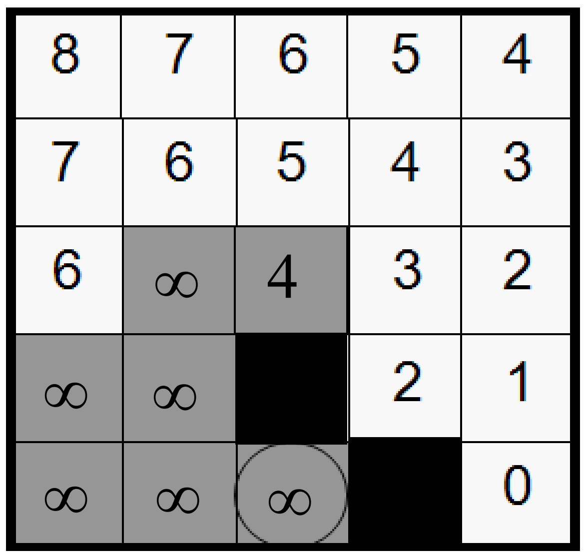

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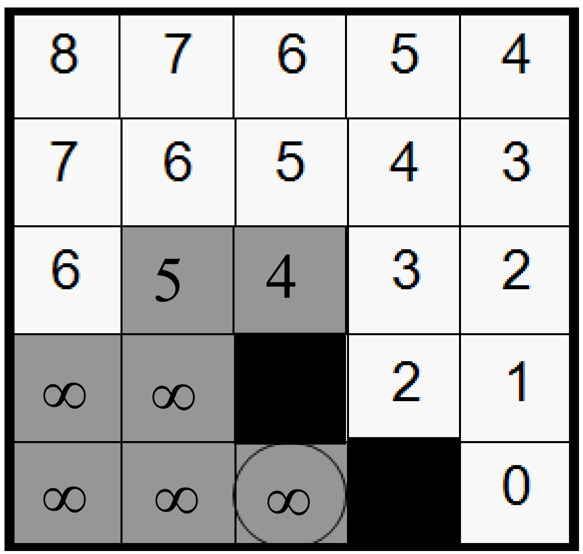

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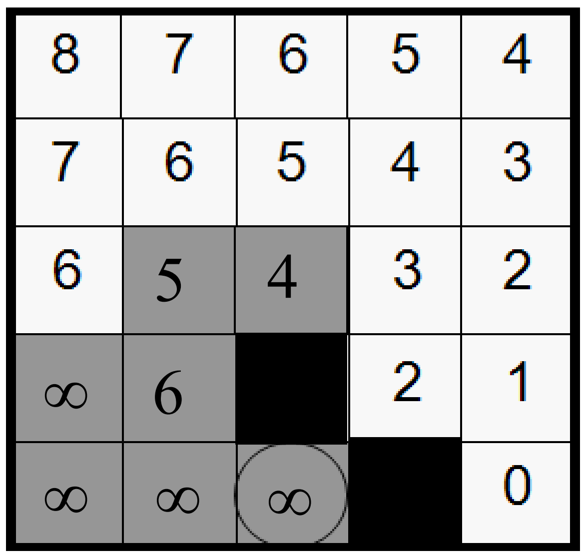

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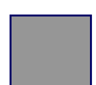
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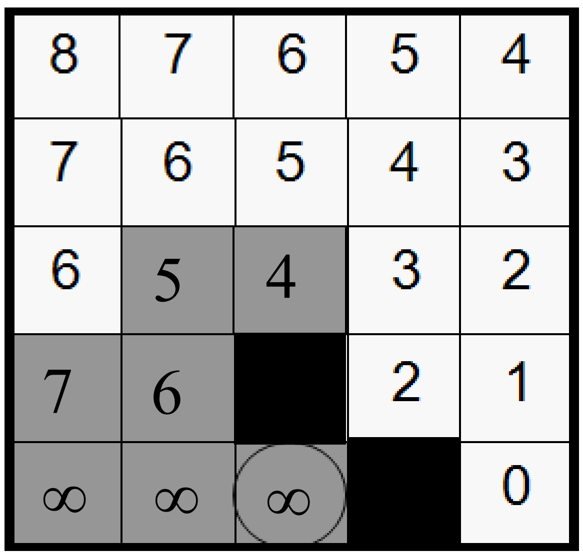

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Does it matter in what order?

- *expanded*

- LRTA* with $N \geq 1$ expands

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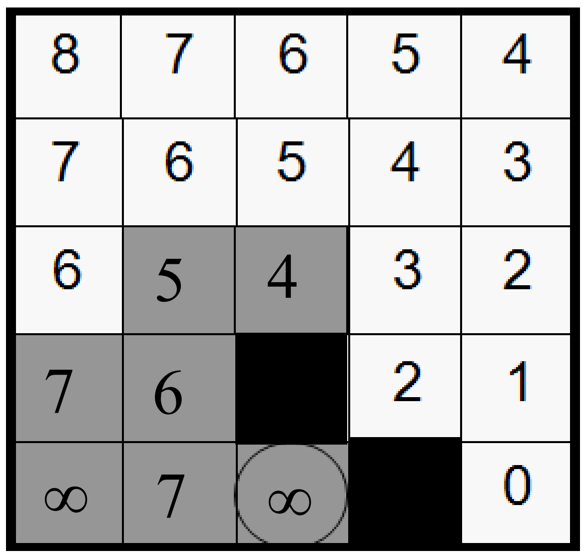

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- expanded

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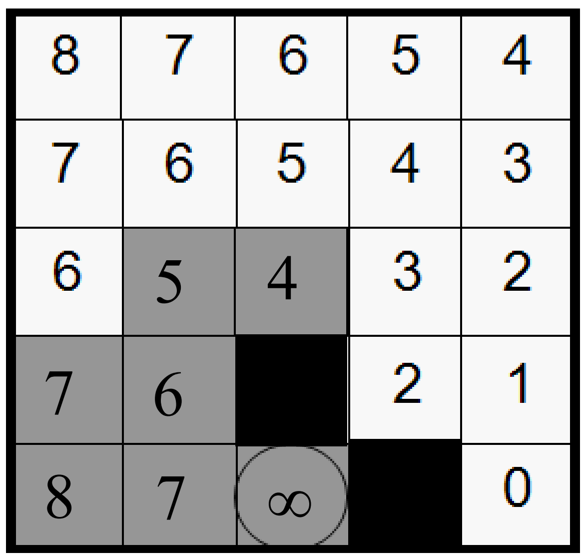

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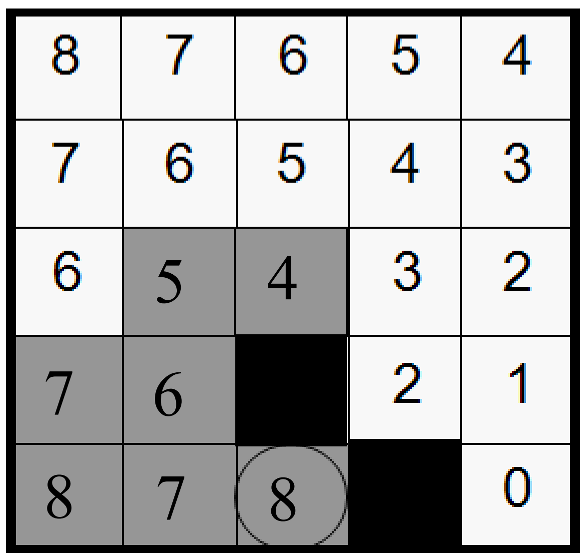

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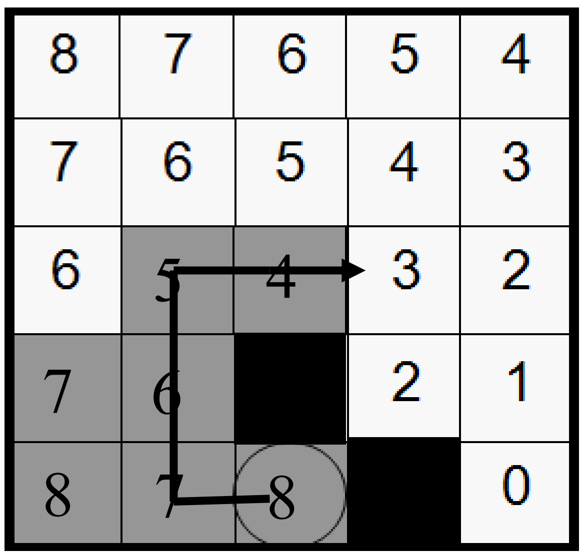


make a move along the found path and repeat steps 1-3

Drawbacks compared to A?*

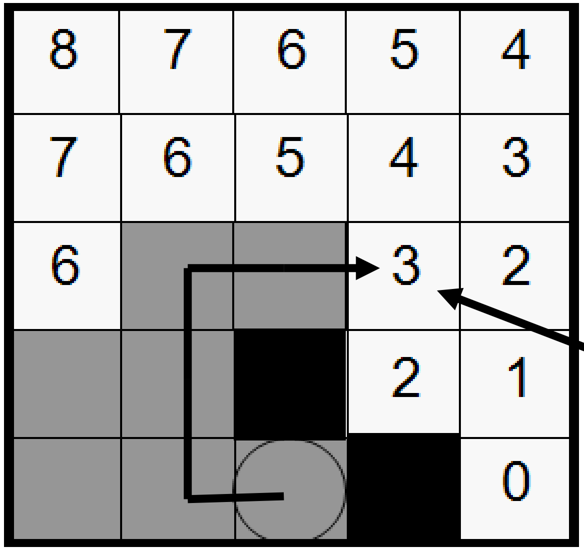
- expanded

Real-time Adaptive A* (RTAA*) [Koenig & Likhachev, '06]

- RTAA* with $N \geq 1$ expands

one linear pass, and even that can be lazy(postponed)

1. *expand N states*
2. *update h-values of expanded states u by $h(u) = f(s) - g(u)$,*
   where $s = \mathrm{argmin}_{s' \in OPEN}\, g(s') + h(s')$
3. *move on the path to state $s = \mathrm{argmin}_{s' \in OPEN}\, g(s') + h(s')$*



expand N=7 states

unexpanded state *s* with smallest *g* + *h* (= 5 + 3)

- expanded

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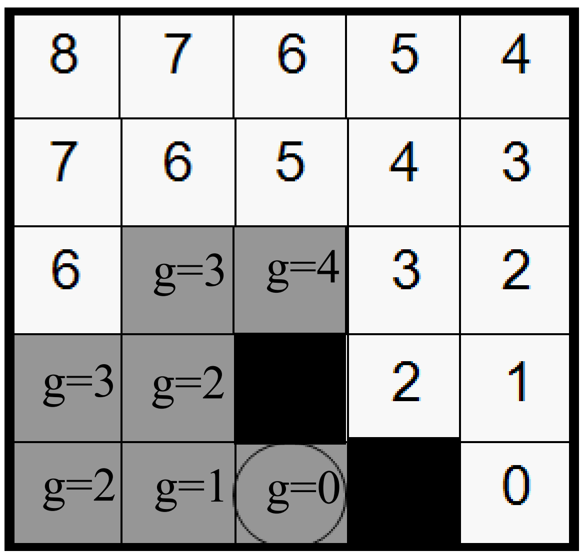

update all expanded states u:

$h(u) = f(s) - g(u)$

unexpanded state *s* with smallest

f(s) = 8

- expanded

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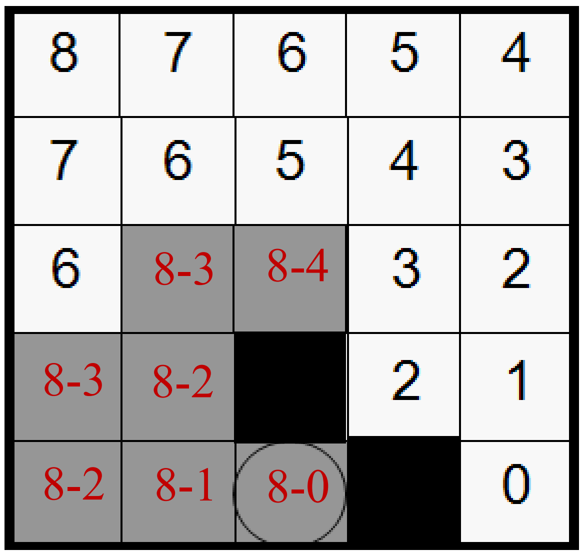
   where $s = \operatorname{argmin}_{s' \in OPEN} g(s') + h(s')$
3. *move on the path to state* $s = \operatorname{argmin}_{s' \in OPEN} g(s') + h(s')$



update all expanded states u:

$h(u) = f(s) – g(u)$

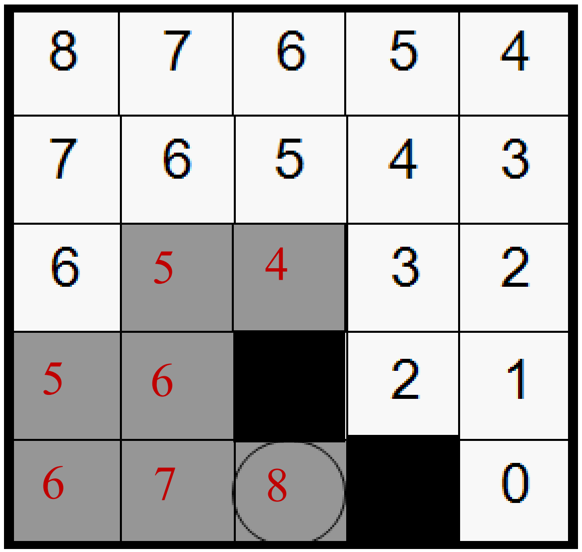
unexpanded state *s* with smallest *f(s)* = 8

- *expanded*

Real-time Adaptive A* (RTAA*)

- RTAA* with $N \geq 1$ expands

1. *expand N states*
2. *update h-values of expanded states u by* $h(u) = f(s) - g(u)$,
   where $s = \arg\min_{s' \in OPEN} g(s') + h(s')$
3. *move on the path to state* $s = \arg\min_{s' \in OPEN} g(s') + h(s')$



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proof of admissibility:

h() – true cost-to-goal*

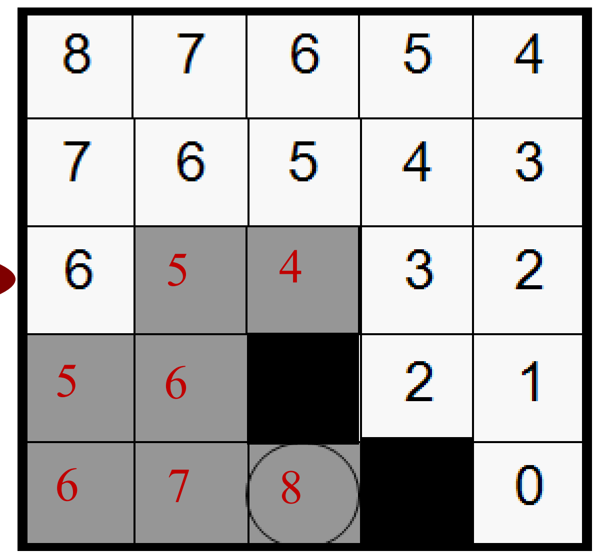
$$g(u) + h^*(u) \geq h^*(s_{start})$$

$$h^*(u) \geq h^*(s_{start}) - g(u)$$

because $f(s) \leq h^*(s_{start})$

$$h^*(u) \geq f(s) - g(u)$$

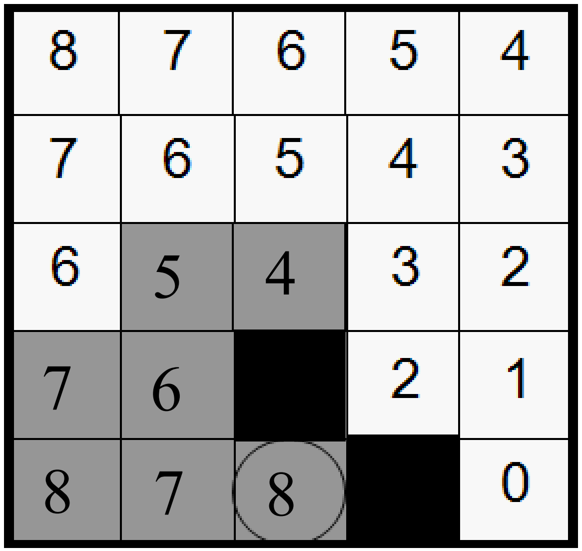
$$h^*(u) \geq h_{updated}(u)$$



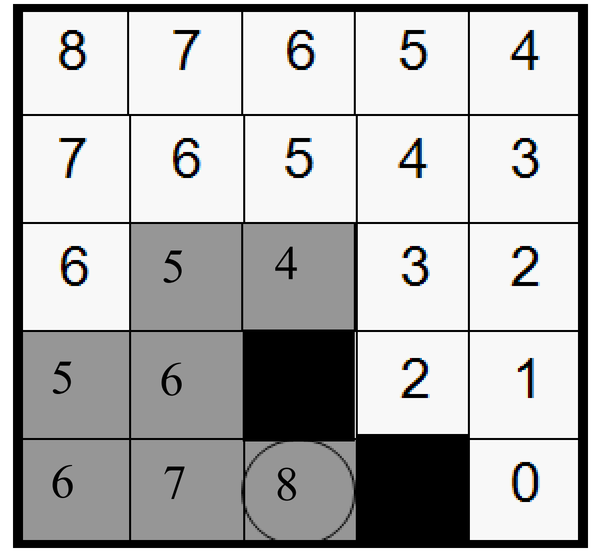
- *expanded*

LRTA* vs. RTAA*

LRTA*



RTAA*



- Update of *h*-values in RTAA* is much faster but not as informed
- Both guarantee adimssibility and consistency of heuristics
- For both, heuristics are monotonically increasing
- Both guarantee to reach the goal in a finite number of steps (given the conditions listed previously)

Summary

- Real-time Heuristic Search puts a hard constraint on planning time (usually, a smaller planning time than what is required to plan a path all the way to the goal)
- Computing a partial path to the goal may result in highly sub-optimal behavior
- It is important to think how to avoid infinite oscillations
  - Updating heuristics is a popular way for doing it
  - Mostly applicable to low-dimensional planning
  - How to extend it to high-dimensional planning is a research question