15-887
Planning, Execution and Learning

Dependent variables, Markov Property, Dominance

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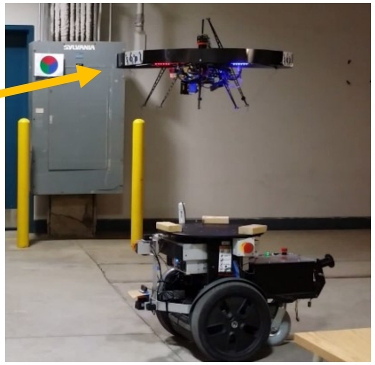
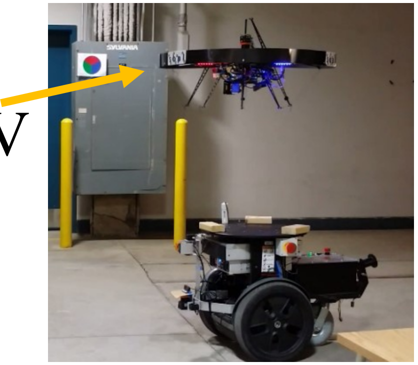
Consider Planning with Battery Constraint



- Suppose we are planning 2D *(x,y)* path for UAV
  - want a collision-free path to $s_{goal} = (x_{goal}, y_{goal})$
  - want to minimize some cost function associated with each transition (for example, minimize the risk of flying close to people)
  - subject to the trajectory being feasible given the UAV battery level L

What should be the variables defining each state (i.e., dimensions of the search)?

Consider Planning with Battery Constraint



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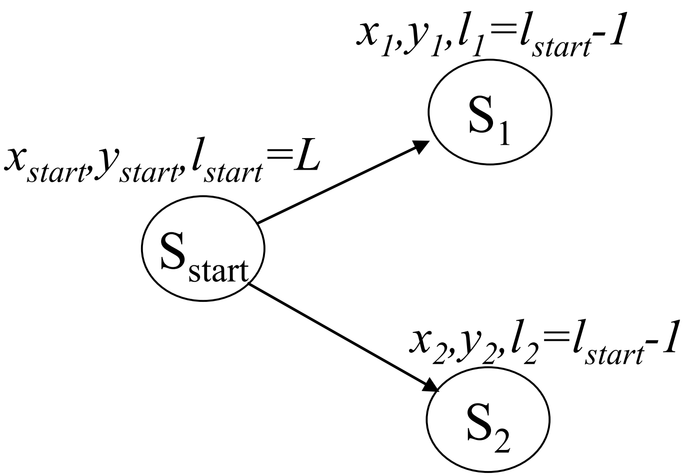
  - **Planning needs to be in (*x,y,l*), where *l* is the remaining battery level**



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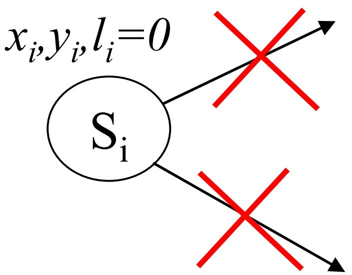
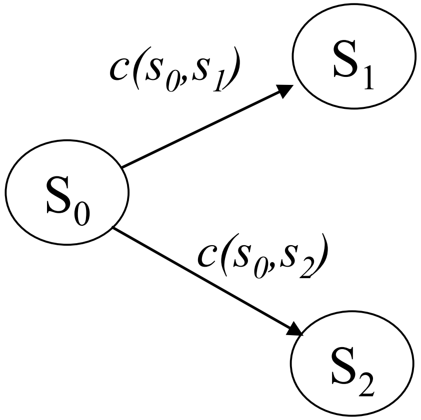


any state with battery level 0 is absorbing

Markov Property

- ***Cost and Set of Successors needs to depend ONLY on the current state (no dependence on the history of the path leading up to it!)***



for all states s: succ(s) = function of s
for all s' in succ(s): c(s,s')= function of s, s'

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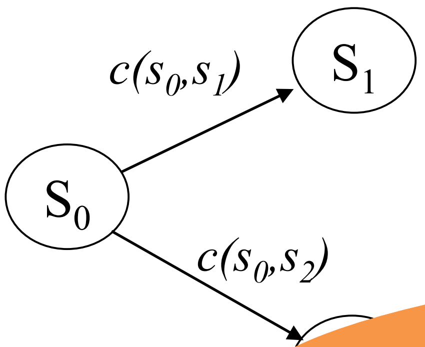

for all states s: succ(s) = function of s
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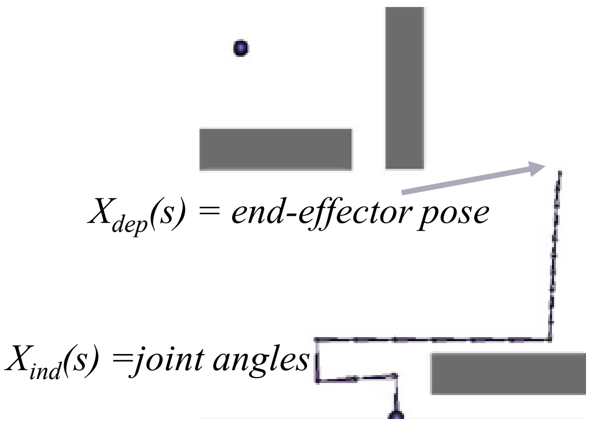
Clearly true in an **explicit** (given) graph

Can be violated in **implicit** (dynamically generated) graphs, where *succ(s)* and *c(s,s')* are computed on-the-fly as a function of *s*,
when using dependent variables

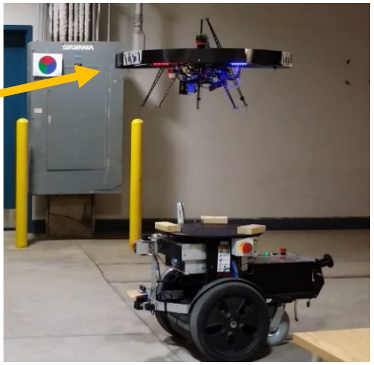
Independent vs. Dependent Variables



- *$X(s)$ – variables associated with s*
- *$X(s) = \{X_{ind}(s), X_{dep}(s)\}$*
- *$X_{ind}(s)$ – independent variables*
- *$X_{dep}(s)$ – dependent variables*

- ***Independent Variables*** are used to define state *s*
  - *two states s and s' are considered to be the same state if and only if $X_{ind}(s) = X_{ind}(s')$*
- ***Dependent Variables*** often used to help with computing cost or list of successor states
  - *if for all s, $X_{dep}(s) = f(X_{ind}(s))$ (that is, only depends on independent variables, then Markov Property holds true)*
  - **Sometimes however, $X_{dep}(s)$ is computed based on the path leading up to $X_{ind}(s)$**

Consider Planning with Battery Constraint



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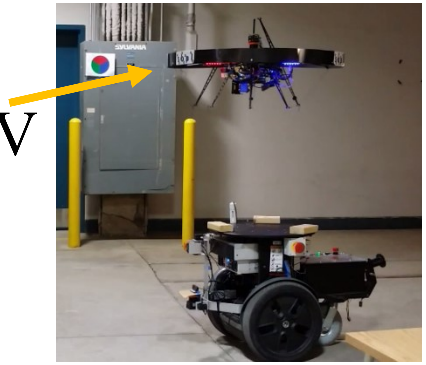
  - **Consider $X_{ind}=(x,y)$, $X_{dep}=(l)$, where *l* is the remaining battery level**

Consider Planning with Battery Constraint



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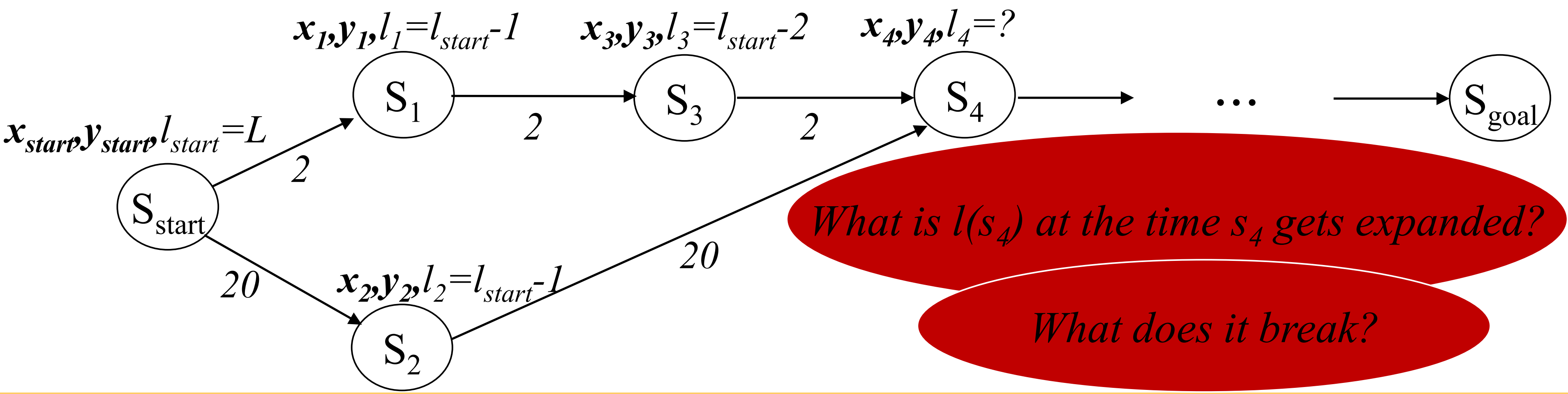

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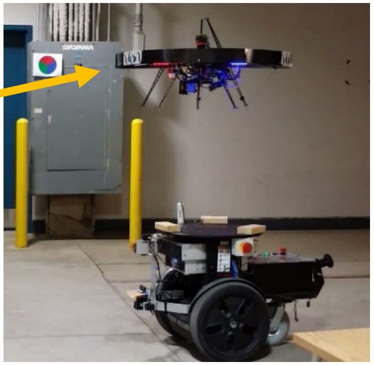

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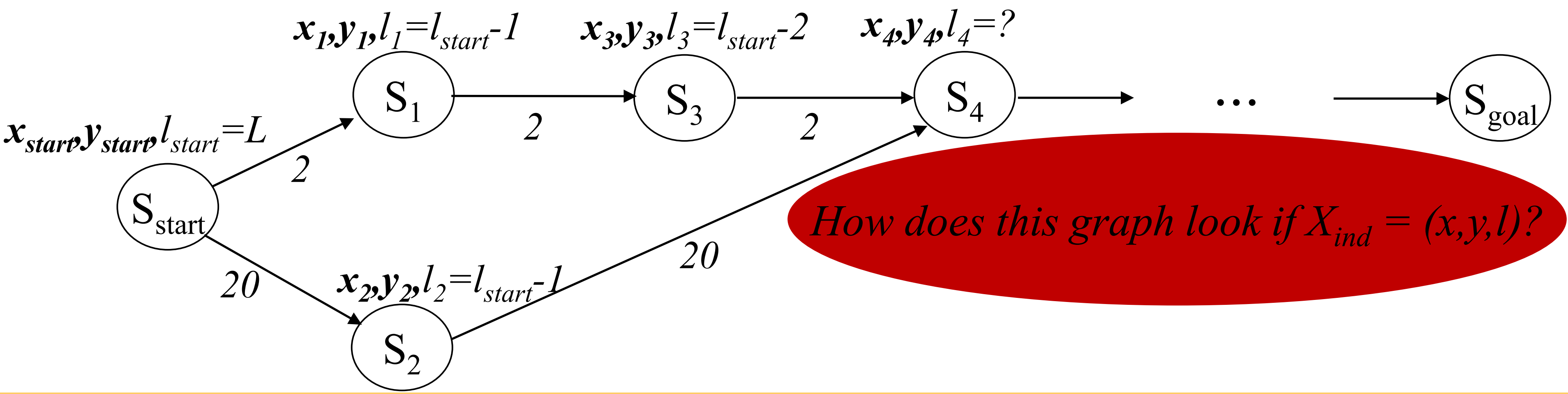


Consider Planning with Constraints on Rate of Turning

- Suppose we are planning 2D *(x,y)* path for a ground robot and constraining its heading to change by at most 45 degrees at each timestep

  - **Consider X_{ind}=(x,y), X_{dep}=(Θ), where Θ is robot's heading**

Example of incompleteness?

Consider Planning with Continuous (x,y,Θ)

- Suppose we are planning 3D *(x,y,Θ)* path for a ground robot but we don't have motion primitives (lattice) that move the robot exactly between centers of 3D cells

  - **Consider $X_{ind}=(x_{disc},y_{disc},\Theta_{disc})$, $X_{dep}=(x_{cont},y_{cont},\Theta_{cont})$, where X_{dep} keeps track of the continuous robot pose along its path** [Barraquand, J. & Latombe, '93]

Example of "incompleteness"?

Consider Planning in Dynamic Environments

- Suppose we are planning a path among moving obstacles

  - want a collision-free path to s_{goal}
  - want to minimize some cost function associated with each transition
  - **Consider X_{ind}=(*robot pose*), X_{dep}=(*t*), where *t* is time**

Example of incompleteness?

Consider Planning in Dynamic Environments

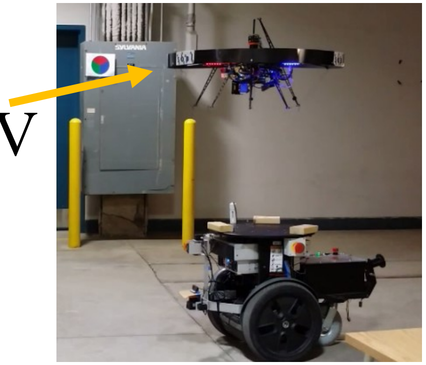
- Suppose we are planning a path among moving obstacles

- want a collision-free path to s_{goal}
- <u>assume cost function is time</u>

- **Consider X_{ind}=(*robot pose*), X_{dep}=(*t*), where *t* is time**

Is it incomplete?

Back to Planning with Battery Constraint



- Suppose we are planning 2D *(x,y)* path for UAV
  - want a collision-free path to $s_{goal} = (x_{goal}, y_{goal})$
  - <u>assume cost function is battery consumption</u>
  - subject to the trajectory being feasible given the UAV battery level L

  - **Consider $\mathbf{X_{ind}=(}x,y\mathbf{)}$, $\mathbf{X_{dep}=(}l\mathbf{)}$, where l is the remaining battery level**

Is it incomplete?

Back to Planning with Battery Constraint

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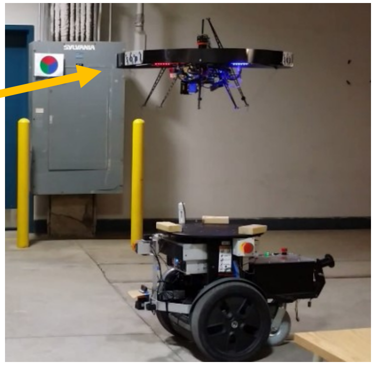

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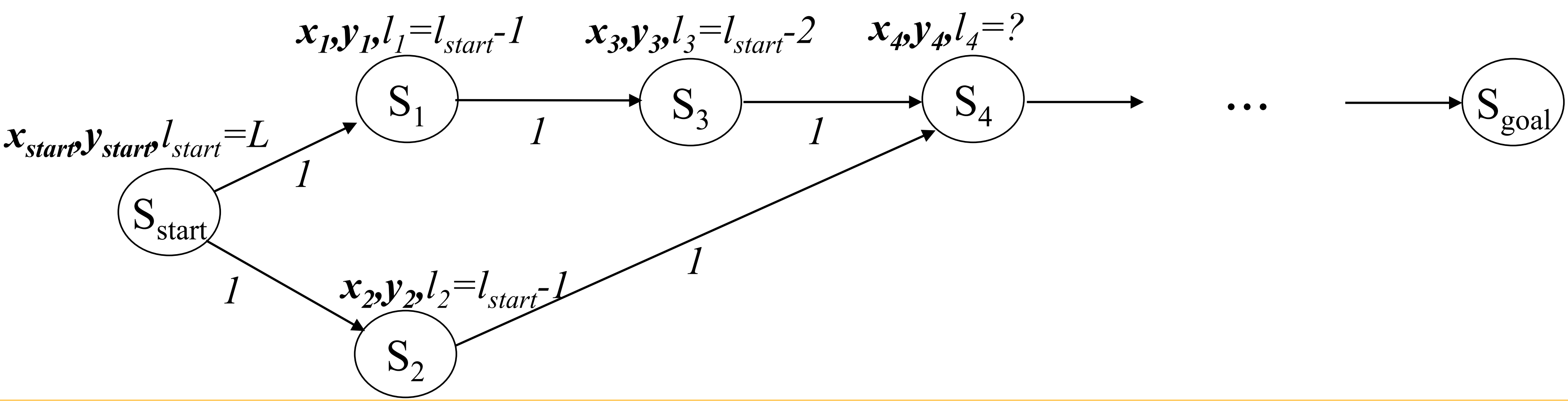

Back to Planning with Battery Constraint

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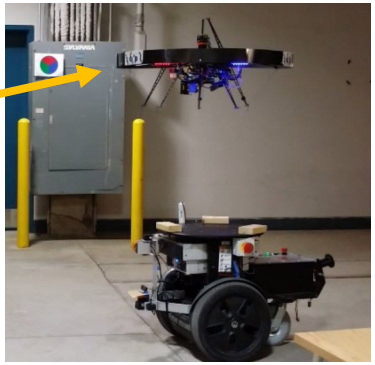

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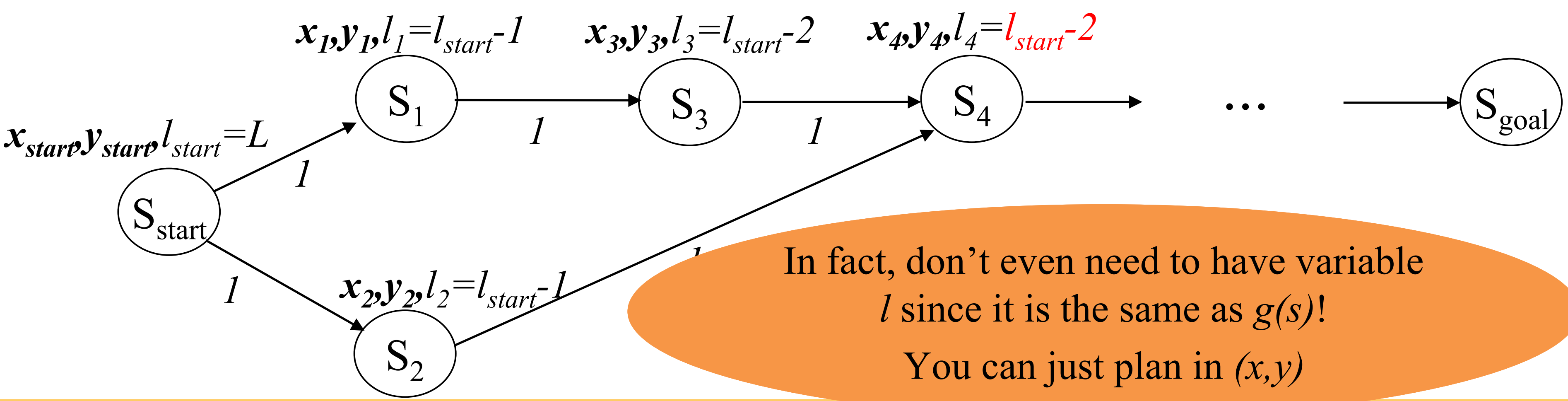

In fact, don't even need to have variable *l* since it is the same as *g(s)*!

You can just plan in *(x,y)*

Back to Planning with Battery Constraint

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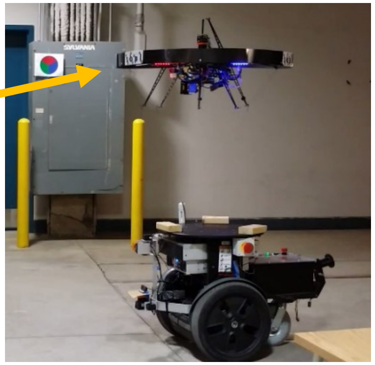

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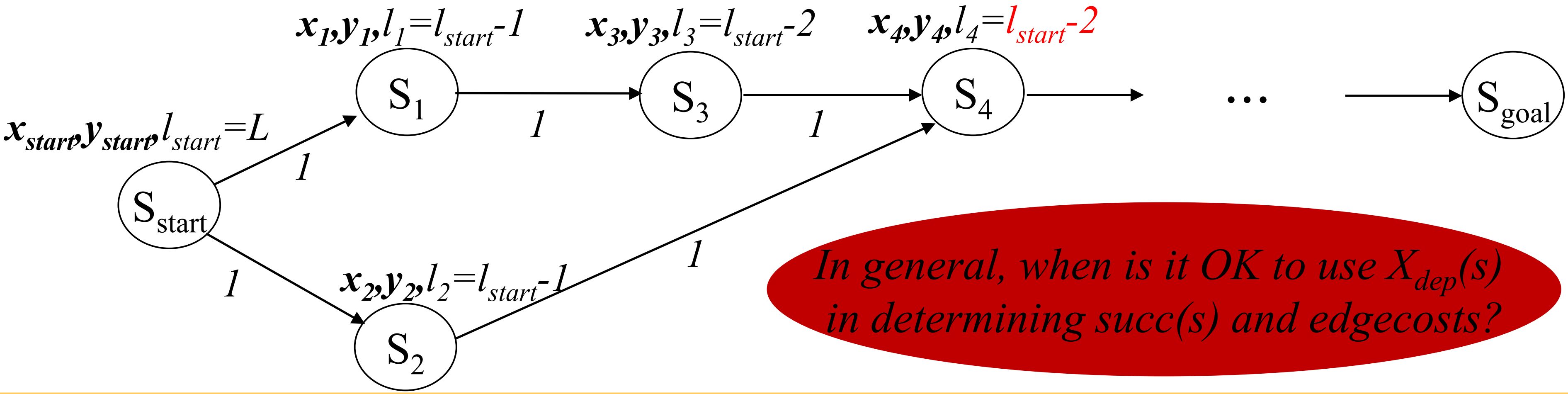

Back to Planning with Battery Constraint

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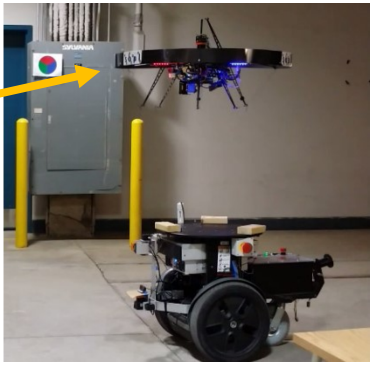
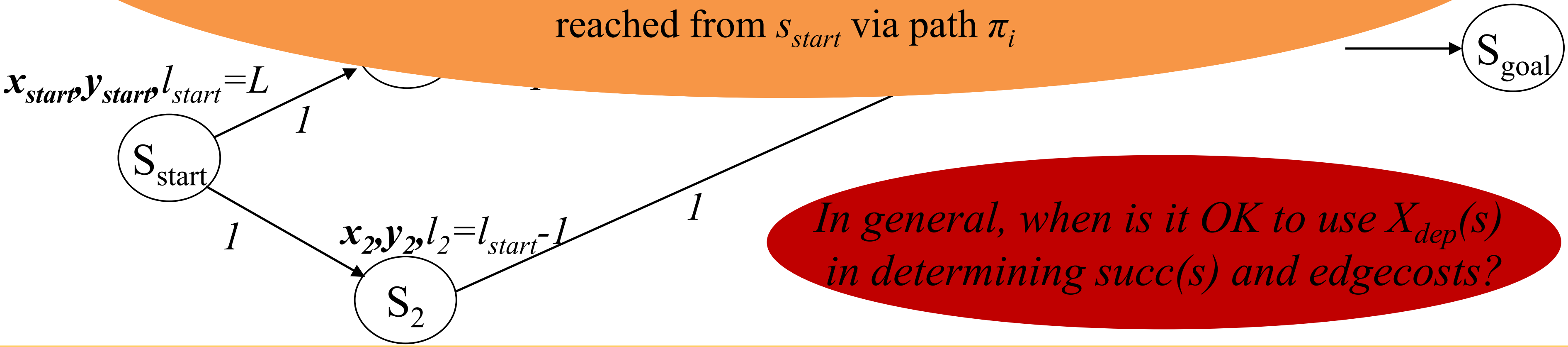


Whenever you can guarantee that: for any state s and s', given any two paths $\pi_1(s_{start},s)$ and $\pi_2(s_{start},s)$ s.t. $c(\pi_1) \geq c(\pi_2)$,

$c_1(s,s') \geq c_1(s,s')$,

where $c_i(s,s')$ – cost of least-cost path from s to s' <u>after</u> s is reached from s_{start} via path π_i



In general, when is it OK to use $X_{dep}(s)$ in determining succ(s) and edgecosts?

Back to Planning with Battery Constraint

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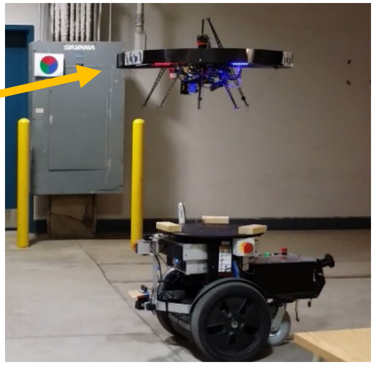


  - want a …
  - assume …
  - subject to the tra… the UAV battery level *L*

What does this assume about the search itself?

Whenever you can guarantee that: for any state *s* and *s'*,
given any two paths $\pi_1(s_{start},s)$ and $\pi_2(s_{start},s)$ s.t. $c(\pi_1) \geq c(\pi_2)$,
$c_1(s,s') \geq c_1(s,s')$,
where $c_i(s,s')$ – cost of least-cost path from *s* to *s'* after *s* is reached from s_{start} via path π_i

S_{goal}

$x_{start}, y_{start}, l_{start}=L$

1

S_{start}

1

$x_2, y_2, l_2=l_{start}-1$

1

S_2

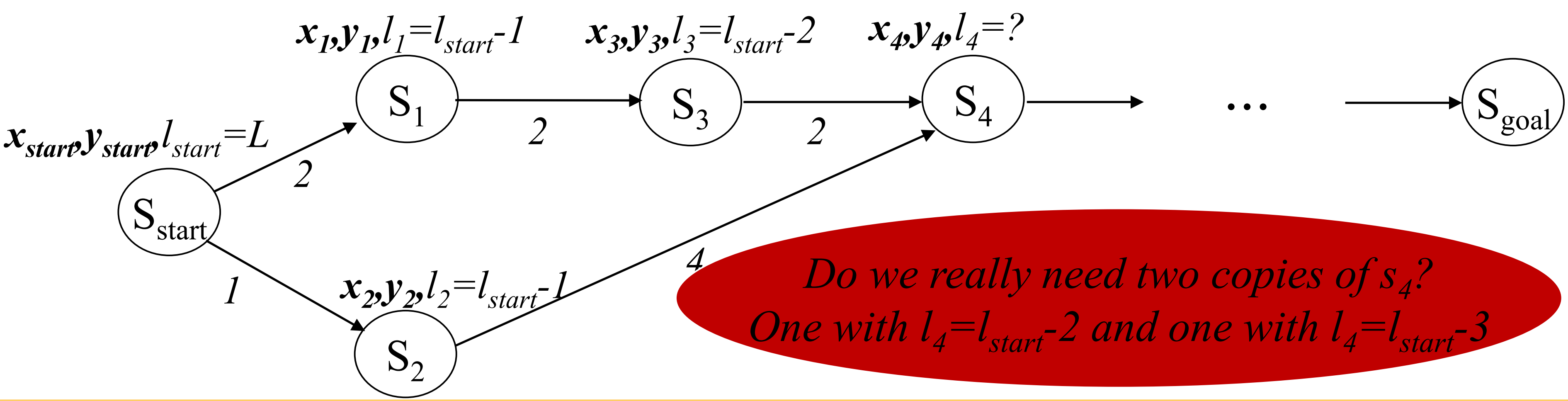
In general, when is it OK to use $X_{dep}(s)$ in determining succ(s) and edgecosts?

Dominance Relationship

- Suppose we are planning 2D *(x,y)* path for UAV
  - want a collision-free path to $s_{goal} = (x_{goal}, y_{goal})$
  - want to minimize some cost function associated with each transition (for example, minimize
  - subje

What are the general conditions for pruning "dominated" states?

  - **Consider $\mathbf{X_{ind}=(x,y,l)}$**



Do we really need two copies of s_4?
One with $l_4=l_{start}-2$ and one with $l_4=l_{start}-3$

Dominance Relationship

if $(g(s) \geq g(s'))$ and s **dominates** s'), then s' can be pruned by search

s dominates s' implies s cannot be part of a solution that is better than the solution from s' [Horowitz and Sahni, '78]

- want to … with each transition (for example, minimize th…
- subje…

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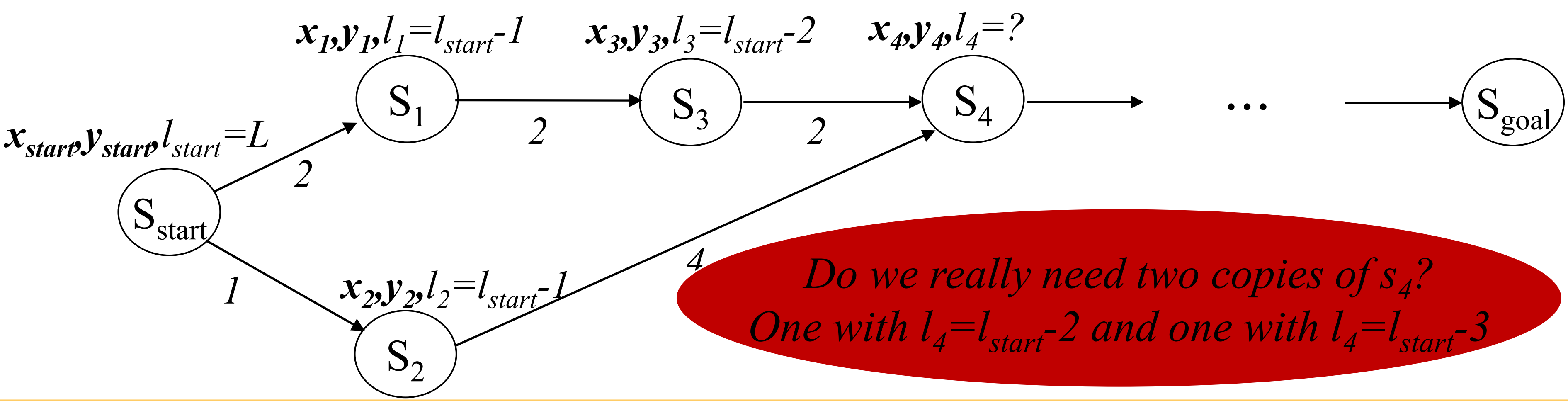

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Summary

- Dependent vs. Independent variables important to understand

- Markov Property = dependence of the cost and successor functions ONLY on the current state

- Dominance relationship can be used to speedup search if present