

# 15-103 HOMEWORK 5 - Spring 2008

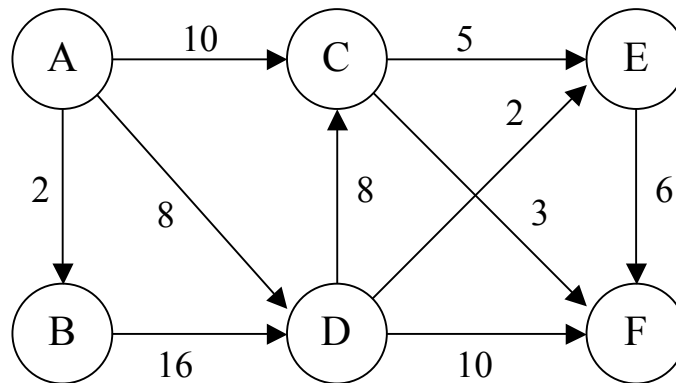
due in class on Sunday, February 17

1. Consider the Towers of Hanoi problem with  $N$  disks.

(a) How many times does the largest disk move from one peg to another peg? Explain your answer?

(b) How many times (as a function of  $N$ ) does the smallest disk move from one peg to another peg? Show your work to receive full credit.

2. Using the graph shown below:



(a) Trace the shortest path algorithm given in class, showing the final values stored in the L table as illustrated in lecture. What is the length of the shortest path from node A to node F?

(b) If we used a greedy algorithm to find the shortest path from node A to node F, starting with node A, what path would we pick? What is its total length? Does the greedy algorithm find the shortest path as you found in part (a)? Why or why not?

3. The following algorithm computes  $f = n^2$  for  $n > 0$ :

1. Input  $n$ , an integer  $> 0$
2. Set  $i = 1$ .
3. Set  $f = n$ .
4. While  $i \neq n$  do the following:
  - a. Add 1 to  $i$ .
  - b. Set  $f$  equal to  $f + n$ .
5. Output  $f$ .

Using  $f = i * n$  as your loop invariant, show that this algorithm performs the correct computation:

- (a) Show the invariant is true immediately before the loop begins
- (b) Show that if it is true at the start of each iteration, it must also be true at the end of the same loop iteration.
- (c) Show that after the loop terminates,  $f$  is equal to  $n^2$ .
- (d) Give a brief logical argument that the loop does, indeed, terminate.

4. Show that the sum of the first  $N$  odd numbers is equal to  $N^2$ . That is:  
$$1 + 3 + 5 + \dots + (2n - 1) = n^2$$

(a) [Base Case] Show that this is true for  $n = 1$ .

(b) [Inductive Step] Assume that it is true for the first  $N$  odd numbers; prove that it holds for the next odd number.

5. Based on the book you are reading for the term paper, answer the following:

(a) Before Google, what was the first major search engine developed at Digital Equipment Corporation? Why did it not do well?

(b) Where was Google born? How was its search algorithm different than the algorithm used by the engine in part (a)?