15-103 HOMEWORK 4 - Spring 2008

due in class on Sunday, February 10

1. Here is a set of Prolog rules:

parent(homer, bart). male(bart).
parent(homer, lisa). male(homer).
parent(homer, maggie). male(abe).
parent(marge, bart). female(marge).
parent(marge, lisa). female(lisa).
parent(marge, maggie). female(maggie).
parent(abe, homer). female(jacqueline).
parent(jacqueline, marge). female(patty).
parent(jacqueline, patty). female(selma).
parent(jacqueline, selma).

(a) What is each of the following Prolog queries asking (in English)? What are all of the results of these queries based on the Prolog rules above?

?- parent(homer, marge).

?- parent(X, patty).

?- male(Y).

(b) Add the following Prolog rules to the rules above:

father(X,Y) :- parent(X,Y), male(X).

mother(X,Y) :- parent(X,Y), female(X).

grandparent(X,Y) :- parent(X,Z), parent(Z,Y).

What is each of the following Prolog queries asking (in English)? What are all of the results of these queries based on the Prolog rules above?

?- father(X,maggie).

?- grandparent(abe,Y).

?- mother(X,Y).

(c) Write a Prolog rule for the relationship **daughter** using the **parent** relationship.

daughter(X,Y) :- ____

(Read: *X is the daughter of Y if....*)

2. Assume you have n left parentheses and n right parentheses in an arithmetic expression. For example, if $n = 3$,

some arrangements are valid: (( ) ( )) (( )) ( )

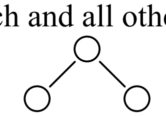
while others are not: ( )) (( )))) (((

Let $P(n)$ = the number of proper arrangements of n left parentheses and n right parentheses in an arithmetic expression. $P(n)$ is given by the recursive formula:

$$P(n) = \frac{4n-2}{n+1} P(n-1), \quad n > 1$$
$$P(1) = 1$$

(a) Show that $P(3) = 5$ using the recursive formula above. Write out all 5 proper arrangements of parentheses when there are 3 left and 3 right parentheses.

(b) $P(n)$ is also the number of binary trees containing n nodes with exactly 2 children each and all other nodes as leaves. For example, $P(1) = 1$, so there is only one binary tree that has 1 node with exactly 2 children each and all other nodes as leaves:



Since we know $P(3) = 5$, this means there must be 5 binary trees that have 3 nodes with exactly 2 children each and all other nodes as leaves. Draw them. (HINT: All of these trees have 7 nodes.)

3. Here is a recursive function:

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f(n) = n + f(n – 1), n >= 1
f(0) = 0
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(a) Compute f(5). Show your work.

(b) What does this recursive function compute in general in terms of n?

4. A DNA sequence consists of only the letters A, C, T and G. Suppose we examine many sequences and find the relatively frequencies (in percentages) for each letter:

LETTER	FREQUENCY
T	63
A	20
G	10
C	7

(a) Derive a Huffman tree for this 4-letter alphabet, and assign a binary Huffman code for each letter based on your tree.

(b) Using your Huffman codes from part (a), how many bits would be required to encode the DNA sequence below? Show your work.

ATTGCTATTTAGTT

(c) Assign a binary code for each letter using the smallest fixed-width code possible. How many bits would be required to encode the DNA sequence in part (b) now? Show your work.