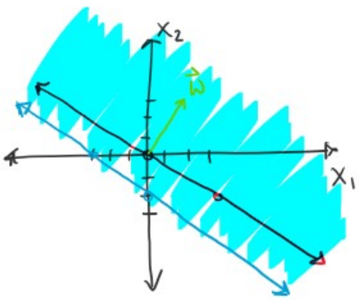
Section B

Friday, February 3, 2023 12:33 PM



$w_1x_1 + w_2x_2 + b > 0$

$w_1 = 2 \quad w_2 = 3 \quad b = 6$

$w_1x_1 + w_2x_2 = 0$

$x_2 = \left(\frac{-w_1}{w_2}\right)x_1$

$w_1x_1 + w_2x_2 + b = 0$

$x_2 = \left(\frac{-w_1}{w_2}\right)x_1 + \left(\frac{-b}{w_2}\right)$

First Perceptron

Linear Classifier

$\hat{y} = h(\vec{x}) = \text{sign}(\vec{w}^T\vec{x} + b)$

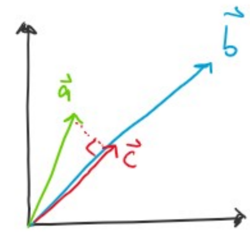
$= \text{sign}(w_1x_1 + w_2x_2 + \dots + w_Mx_M + b)$

Def: the vector projection of $\vec{a}$ onto $\vec{b}$ where $\|b\|_2 = 1$

$\vec{c} = (\vec{a}^T\vec{b})\vec{b}$

Def: the vector projection of $\vec{a}$ onto $\vec{b}$

$$\vec{c} = \frac{(\vec{a}^T\vec{b})}{\|b\|_2}\frac{\vec{b}}{\|b\|_2} = \underbrace{\frac{\vec{a}^T\vec{b}}{(\|\vec{b}\|_2)^2}}_{\text{scalar (length)}}\underbrace{\vec{b}}_{\text{vector (direction)}}$$



(Online) Perceptron Algorithm

Initialize parameters $\vec{w} = [w_1, w_2, \dots, w_M]^T = [0, 0, \dots, 0]^T$

$b = 0$ "intercept term" or "bias term"

for $i = 1, 2, 3, \dots$:

① receive instance $\vec{x}^{(i)}$

② predict $\hat{y} = h(\vec{x}^{(i)}) = \text{sign}(\vec{w}^T\vec{x}^{(i)} + b)$ where $\text{sign}(a) = \begin{cases} +1 & \text{if } a \geq 0 \\ -1 & \text{otherwise} \end{cases}$

③ receive true label $y^{(i)}$

④ if $y^{(i)} \neq \hat{y}$ and $y^{(i)} = +1$: positive mistake

$\vec{w} \leftarrow \vec{w} + \vec{x}^{(i)}$

$b \leftarrow b + 1$

if $y^{(i)} \neq \hat{y}$ and $y^{(i)} = -1$ negative mistake

$\vec{w} \leftarrow \vec{w} - \vec{x}^{(i)}$

$b \leftarrow b - 1$

otherwise :

do nothing

$\vec{x}^{(1)} = \begin{bmatrix} -3 \\ \dots \end{bmatrix}$ $\vec{x}^{(2)} = \begin{bmatrix} 6 \\ \dots \end{bmatrix}$ $\vec{x}^{(3)} = \begin{bmatrix} -10 \\ 2 \end{bmatrix}$ $\vec{x}^{(4)} = \begin{bmatrix} -7 \\ 2 \end{bmatrix}$ $\vec{x}^{(5)} = \begin{bmatrix} -8 \\ -6 \end{bmatrix}$ $\vec{x}^{(6)} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ $\vec{x}^{(7)} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$

do nothing

	$\vec{x}^{(1)} = \begin{bmatrix} -3 \\ -3 \end{bmatrix}$	$\vec{x}^{(2)} = \begin{bmatrix} 6 \\ 7 \end{bmatrix}$	$\vec{x}^{(3)} = \begin{bmatrix} -10 \\ 3 \end{bmatrix}$	$\vec{x}^{(4)} = \begin{bmatrix} -7 \\ 2 \end{bmatrix}$	$\vec{x}^{(5)} = \begin{bmatrix} -8 \\ -6 \end{bmatrix}$	$\vec{x}^{(6)} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$	$\vec{x}^{(7)} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$
	$\hat{y} = +1$	$\hat{y} = +1$	$\hat{y} = -1$	$\hat{y} = +1$	$\hat{y} = +1$	$\hat{y} = +1$	$\hat{y} = +1$
	$y^{(1)} = -1$	$y^{(2)} = +1$	$y^{(3)} = +1$	$y^{(4)} = +1$	$y^{(5)} = -1$	$y^{(6)} = +1$	$y^{(7)} = +1$
$\vec{w}^{(0)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$	$\vec{w}^{(1)} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$	$\vec{w}^{(2)} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$	$\vec{w}^{(3)} = \begin{bmatrix} -7 \\ 6 \end{bmatrix}$	$\vec{w} = ''$	$\vec{w}^{(5)} = \begin{bmatrix} 1 \\ 12 \end{bmatrix}$	$\vec{w}^{(6)} = \begin{bmatrix} 1 \\ 12 \end{bmatrix}$	$w^{(7)} = \begin{bmatrix} 1 \\ 12 \end{bmatrix}$
$b^{(0)} = 0$	$b = -1$	$b = -1$	$b = 0$	$b = ''$	$b = -1$	$b = -1$	$b = -1$