Lecture 24: 4/17/17

Background: the marginal prob. of a for the joint $p(t,h,a,c)$ is

$$p(a) = \sum_t \sum_h \sum_c p(t,h,a,c)$$

1st Order Markov Assumption

Let y_t state of system at time t.

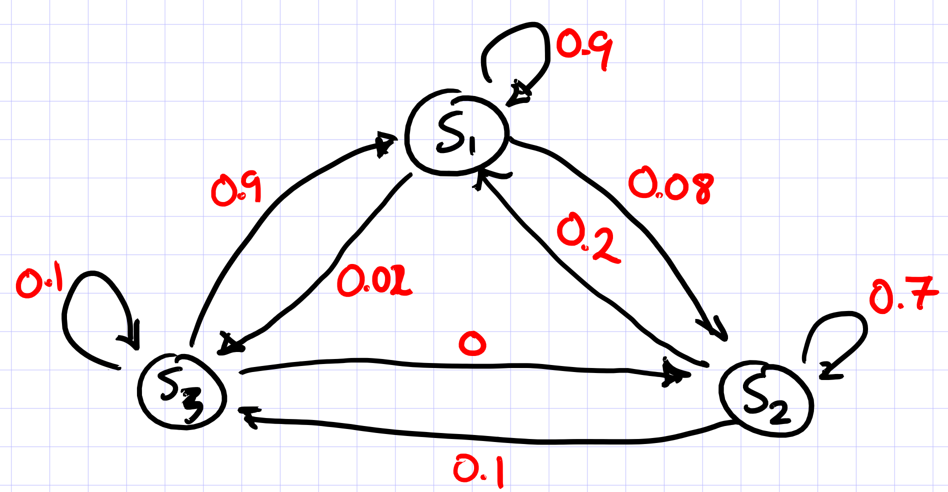
$$p(y_t | y_1, \ldots, y_{t-1}) = p(y_t | y_{t-1})$$

$$\Rightarrow y_t \perp\!\!\!\perp y_j \mid y_{t-1} \quad \forall j < t-1$$

$$p(y_1, \ldots, y_T) = \prod_{t=1}^{T} p(y_t | y_1, \ldots, y_{t-1})$$

$$= \prod_{t=1}^{T} p(y_t | y_{t-1})$$

1st Order M.M. as Finite State Machine



3 Problems for HMM

How many $\vec{y}$? K^T

assume y_0 = START

Evaluation: $p(\vec{x}) = \sum_{\vec{y}} p(\vec{x}, \vec{y}) = \sum_{\vec{y}} \prod_{t=1}^{T} p(y_t | y_{t-1}) \, p(x_t | y_t)$

Decoding: $\hat{y} = \operatorname{argmax}_{\vec{y}} \, p(\vec{y} | \vec{x})$

Marginal: $p(y_t = k \mid \vec{x}) = \sum_{\vec{y} \text{ s.t. } y_t = k} p(\vec{y} | \vec{x})$

$= \dfrac{p(\vec{x}, \vec{y})}{p(\vec{x})}$

Forward-Backward Algo.

Define: $\alpha_t(k) \triangleq p(x_1, \ldots, x_t, y_t = k)$

$\beta_t(k) \triangleq p(x_{t+1}, \ldots, x_T \mid y_t = k)$

Assume: y_0 = START

y_{T+1} = END

① Initialize $\alpha_0(\text{START}) = 1 \qquad \alpha_0(k) = 0 \quad \forall k \neq \text{START}$

$\beta_{T+1}(\text{END}) = 1 \qquad \beta_{T+1}(k) = 0 \quad \forall k \neq \text{END}$

forward algo

② For $t = 1, \ldots, T$:

For $k = 1, \ldots, K$:

include emission prob.

$$\alpha_t(k) = \underbrace{p(x_t | y_t = k)}_{} \sum_{j=1}^{K} \alpha_{t-1}(j) \, p(y_t = k \mid y_{t-1} = j)$$

backward algo

③ For $t = T, \ldots, 1$:

For $k = 1, \ldots, K$:

$$\beta_t(k) = \sum_{j=1}^{K} p(x_{t+1} | y_{t+1} = j) \, \beta_{t+1}(j) \, p(y_{t+1} = j \mid y_t = k)$$