Section A: HMMs

3 Problems for an HMM

Evaluation:
\[ p(x) = \sum_{y \in \mathcal{Y}_x} p(x, y) \]

Viterbi:
Decoding:
\[ \hat{y} = \arg \max_{y \in \mathcal{Y}_x} p(y|x) \]

Marginals:
\[ p(y_k = k | x) = \sum_{y \in \mathcal{Y}_x} p(y | x) \]

Example:

1. Evaluation
\[ p(x_1, x_2 | x_3) = \sum_{y_1} \sum_{y_2} \sum_{y_3} p(x, y) \]

\[ \hat{y} = \arg \max_{y_1, y_2, y_3} p(y_1, y_2, y_3, x_1, x_2, x_3) \]

3. Marginals
\[ p(y_k = k | x_1, x_2, x_3) = \sum_{y_1} \sum_{y_2} p(y_1, y_2, y_3 | x_1, x_2, x_3) \]

Public Service Announcement

BEFORE

FIRST

def eval(x):
  D[x] = 0

impliment the E-B algo

implement the
Bote From
algorithm

Terrible Idea
```
def evl(x):
    p_x = 0
    for y in all_y(x):
        p_x += joint(x, y)
    return p_x
```

**Terrible Idea**

**Good Idea**

START find preferred tags END

**Key idea:** the correct tag holds all the information for the scoring of the next tag (Markov assumption)

**F-B Alg**

forward (edge weights):

   " Start "  Δ
   ___
   Δ
   ___
   T-1  T
   ___
   " End "

Trellis

\[
\alpha_t^k(v) = \alpha_{t-1}(v) s_t^1 + \alpha_{t-1}^k(v) s_t^2 + \alpha_{t-1}^k(v) s_t^3
\]

= total weight of path prefixes ending node \((t,v)\)

For HMM case:

\[
s_{kjt} = p(y_t = k | y_{t-1} = j) p(x_t = w | y_t = k)
\]
backward (erased weights):

same also, but on the mirror image of the trellis

\[
p(x_1, y, x_2) = \frac{p(x_1, y, x_2)}{p(y, x_2)}
\]

\[
= \frac{p(x_1, y) p(x_2 | y) p(y)}{p(y, x_2)}
\]

\[
= \frac{p(x_1, y) p(x_2 | y)}{p(x_2, y)}
\]

\[
= \frac{p(x_1 | y)}{p(x_2 | y)} \Rightarrow x_1 \perp \perp x_2 \mid y
\]

\[
p(A \mid B, C) = p(A \mid C) \Rightarrow A \perp \perp B \mid C
\]

\[
p(A, B \mid C) = p(A \mid C) p(B \mid C)
\]

\[
p(x_1, y, x_2) = p(x_1 | y) \quad \text{by N.B. assumption}
\]

\[
\forall m, n \quad .
\]
MAP Estimation

\[ J(\theta) = \left[ \sum_{i=1}^{N} \log p(y^{(i)} | x^{(i)}, \theta) \right] + \log p(\theta) \]

\[ = \text{Likelihood} + \text{Prior} \]

\[ L_2 \text{ reg} \iff \text{Gaussian prior} \]

\[ L_1 \text{ reg} \iff \text{Laplace prior} \]