Section A: HMMs

Friday, November 4, 2022 10:07 AM

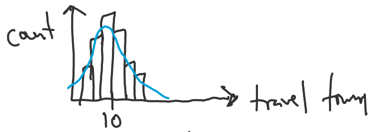
Ex: Tunnel Closures

x_t = Totoro's travel time on day t

y_t = state of the tunnel on day t

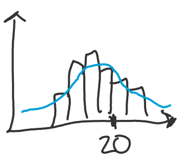
T = 365 days of data

y_t = OPEN



y_t = SEMI-CLOSED

y_t = CLOSED



$\underbrace{\qquad\qquad}_{p(x_t \mid y_t)}$

MLE of y_t

$$\hat{p}_{OPEN} = \frac{\#(y_t = OPEN)}{T}$$

$$\hat{p}_{SC} = \frac{\#(y_t = SC)}{T}$$

$$\hat{p}_{C} = \frac{(\# y_t = C)}{T}$$

$\underbrace{\qquad\qquad}_{p(y_t)}$ ← Gaussian Multiclass N.B.

Model: $p(x_t, y_t) = p(x_t \mid y_t)\, p(y_t)$

1st Order Markov Assumption

Let y_t be the state of system at time t

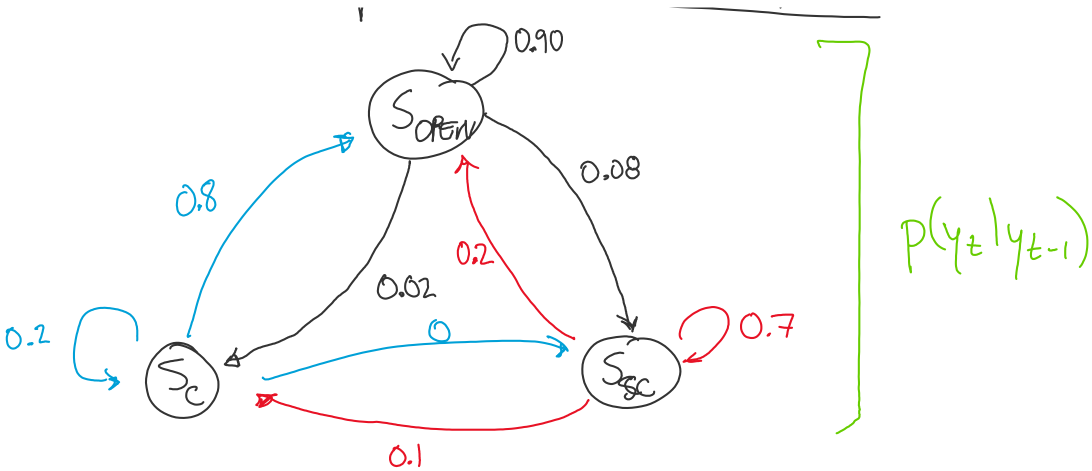
$$p(y_t \mid y_{t-1}, y_{t-2}, \dots, y_1) = p(y_t \mid y_{t-1}) \quad \leftarrow \text{by assumption}$$

$$\Rightarrow y_t \perp\!\!\!\perp y_j \mid y_{t-1},\ \forall j \quad t-1$$

$$p(y_1, \dots, y_T) = \prod_{t=1}^{T} p(y_t \mid y_{t-1}, y_{t-2}, \dots, y_1) \quad \text{by Chain Rule}$$

$$= \prod_{t=1}^{T} p(y_t \mid y_{t-1}) \quad \text{by 1st Order Markov assumption}$$

1st Order Markov Assumption as a Finite State Machine


0.90
S_{OPEN}
0.08
0.8
0.2
0.02
0.2
S_C
0
S_{SC}
0.7
0.1
$p(y_t | y_{t-1})$