10-301/601: Introduction to Machine Learning Lecture 3 – Decision Trees

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9/7/22

Front Matter

- Announcements:
  - HW1 released 8/29, due 9/7 (today!) at 11:59 PM
    - Keep an eye out on Piazza for the HW1 exit poll, which will count towards your participation grade
      - You must complete this poll within one week of its release to receive full credit
  - HW2 released 9/7 (today!), due 9/19 at 11:59 PM
    - Unlike HW1, you will only have:
      - 1 submission for the written portion
      - 10 submissions of the programming portion to our autograder

Q & A:

Do I have to use LaTex to complete the HWs?

- Technically no… but we do **strongly** encourage it! So much so that we are offering **one bonus point** for each assignment that you complete in LaTex.
- Follow-up: but what if I don't know LaTex?
  - Don't worry, we have you covered! Over on Piazza, we've pinned some resources on how to use LaTex and our TAs have recorded an awesome tutorial which you can find in our course Panopto folder.
- Regardless of whether or not you use LaTex, you are responsible for making sure that your submission is aligned with our PDF template; **do not omit any pages!**

Q & A:

How do these in-class polls work?

- Open the poll, either by clicking the [Poll] link on the schedule page of our course website or going to http://poll.mlcourse.org
- Sign into Google Forms using your **Andrew email**
- Answer all poll questions **during lecture for full credit** or **within 24 hours for half credit**
- Avoid the **toxic option** (will be clearly specified in lecture) which gives **negative poll points**
- You have 8 free "poll points" for the semester that will excuse you from all polls from a single lecture; you cannot use more than 3 poll points consecutively.

Poll Question 1:

Which of the following did you bring to class today? Select all that apply

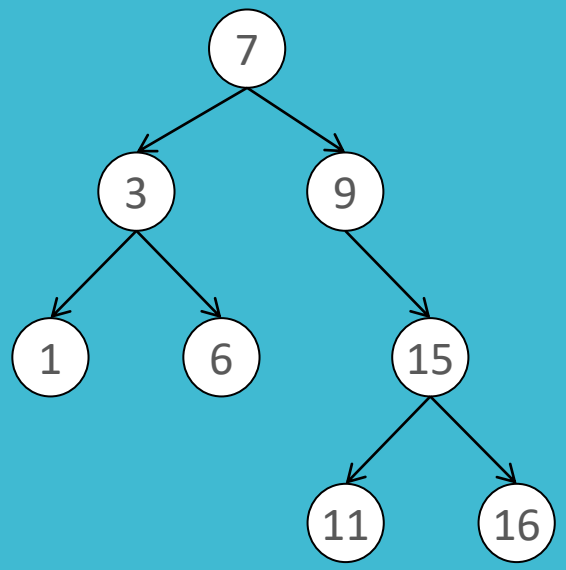
A. A smartphone

B. A flip phone

C. A payphone **(TOXIC)**

D. No phone

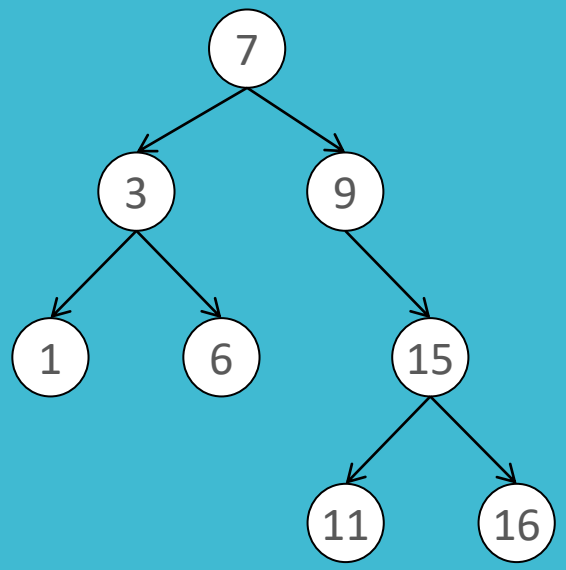
Background: Recursion



- A **binary search tree** (BST) consists of nodes, where each node:
  - has a value, v
  - up to 2 children, a left descendant and a right descendant
  - all its left descendants have values less than v and its right descendants have values greater than v
- We like BSTs because they permit search in O(log(n)) time, assuming n nodes in the tree

```
def contains_iterative(node, key):
    cur = node
    while true:
        if key < cur.value & cur.left != null:
            cur = cur.left
        else if cur.value < key & cur.right != null:
            cur = cur.right
        else:
            break
    return key == cur.value
```

Background: Recursion



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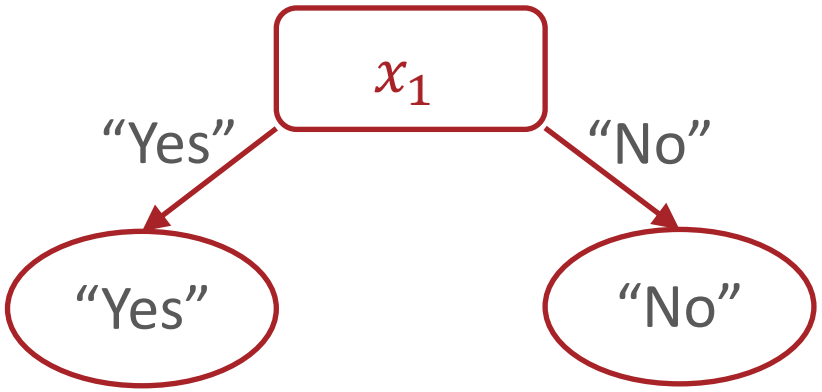
```
def contains_recursive(node, key):
    if key < node.value & node.left != null:
        return contains(node.left, key)
    else if node.value < key & node.right != null:
        return contains(node.right, key)
    else:
        return key == node.value
```

Recall: Decision Stumps

- Alright, let's actually (try to) extract a pattern from the data

x_1 Family History	x_2 Resting Blood Pressure	x_3 Cholesterol	y Heart Disease?	$\hat{y}$ Predictions
Yes	Low	Normal	No	Yes
No	Medium	Normal	No	No
No	Low	Abnormal	Yes	No
Yes	Medium	Normal	Yes	Yes
Yes	High	Abnormal	Yes	Yes



Decision Stumps: Questions

1. Why stop at just one feature?

2. How can we pick which feature to split on?

From Decision Stump …

x_1 Family History	x_2 Resting Blood Pressure	x_3 Cholesterol	y Heart Disease?
Yes	Low	Normal	No
No	Medium	Normal	No
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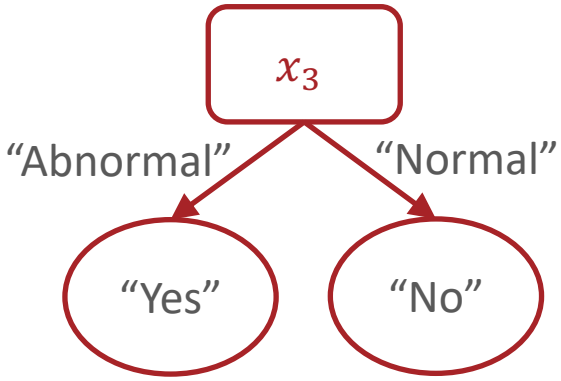

From Decision Stump to Decision Tree

x_1 Family History	x_2 Resting Blood Pressure	x_3 Cholesterol	y Heart Disease?
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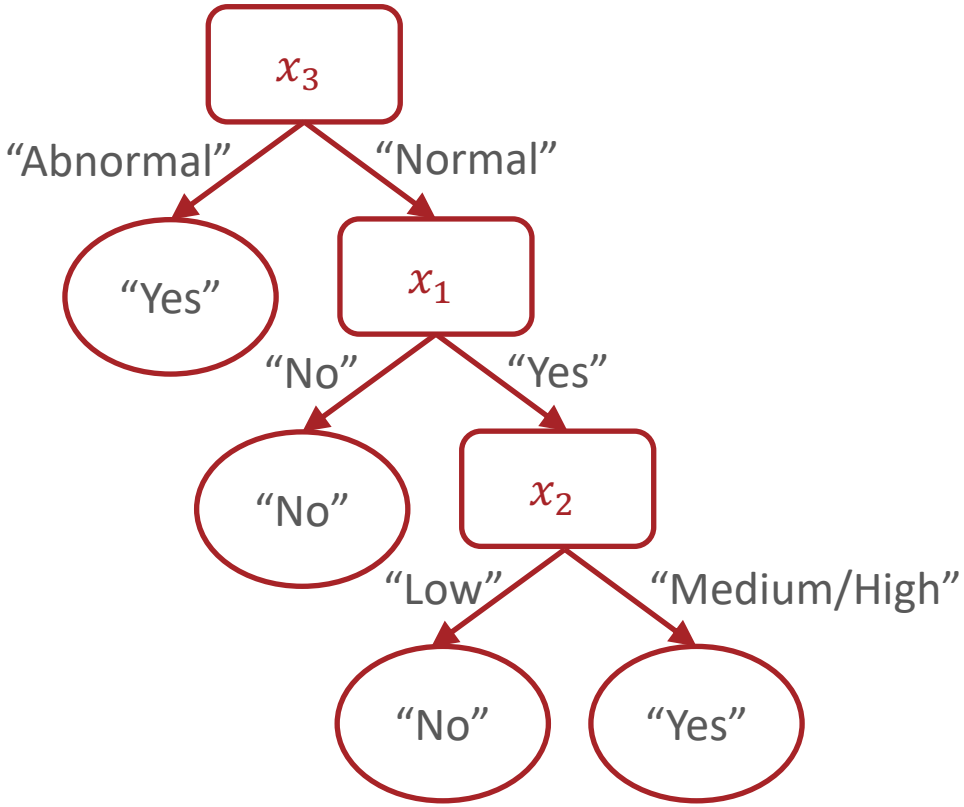

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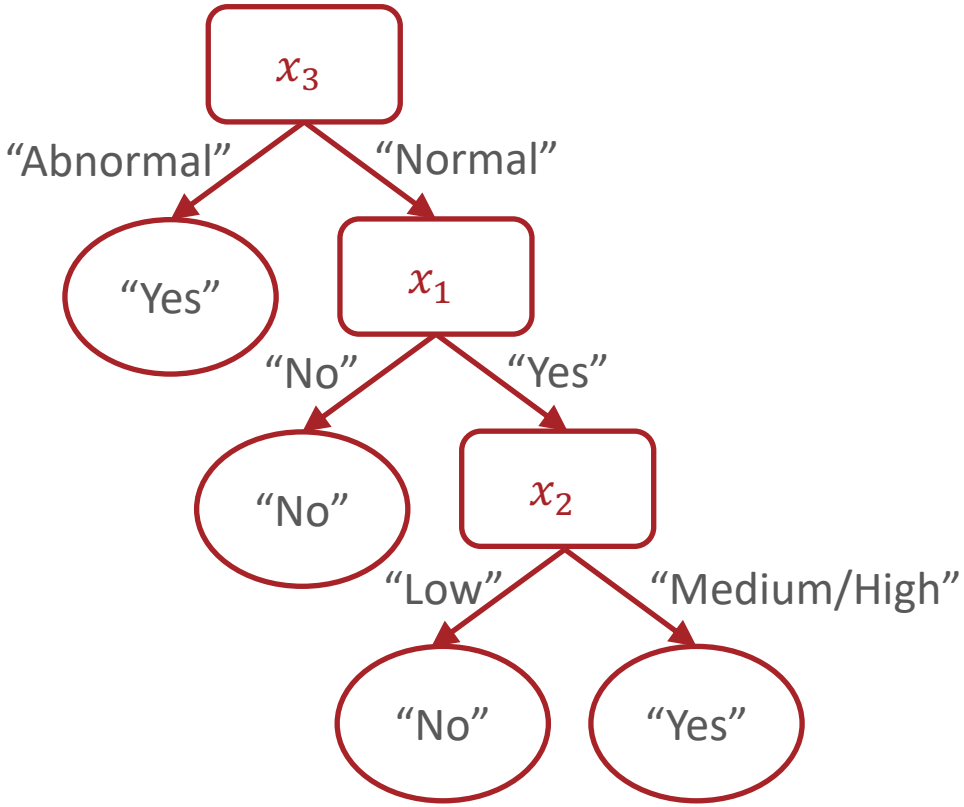

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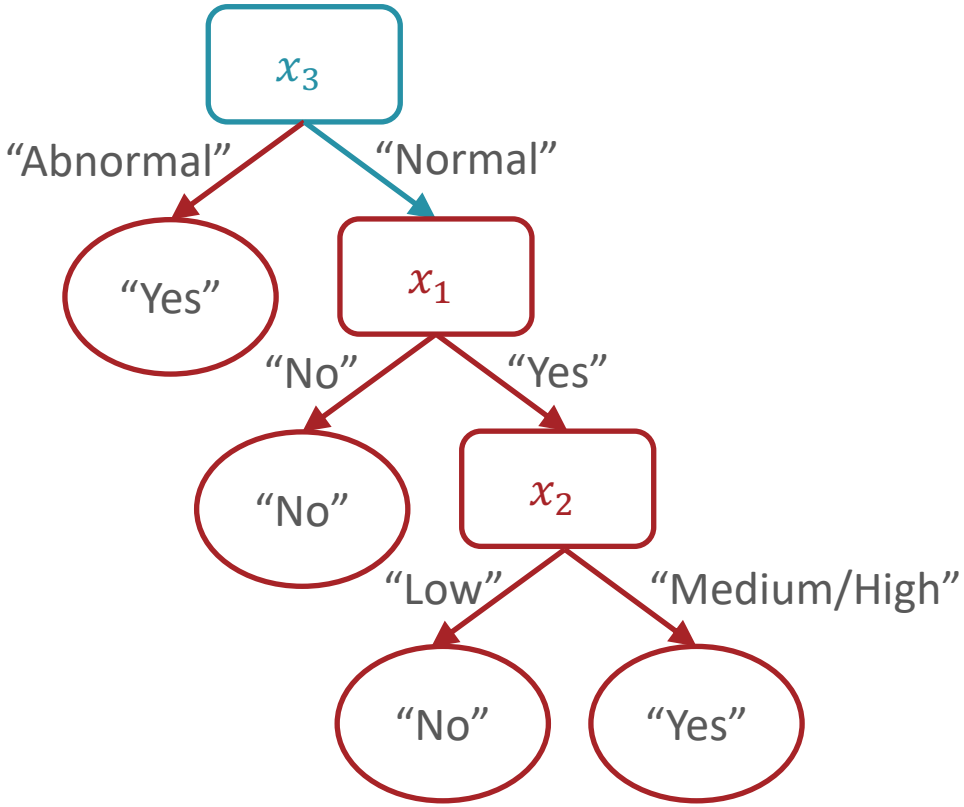

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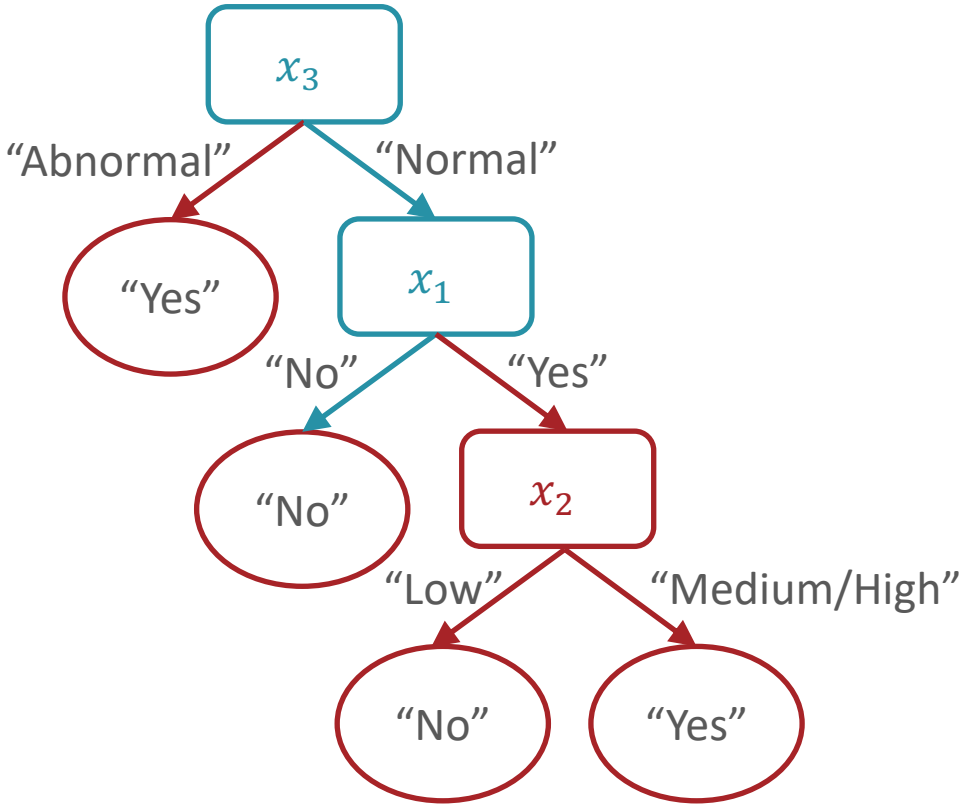

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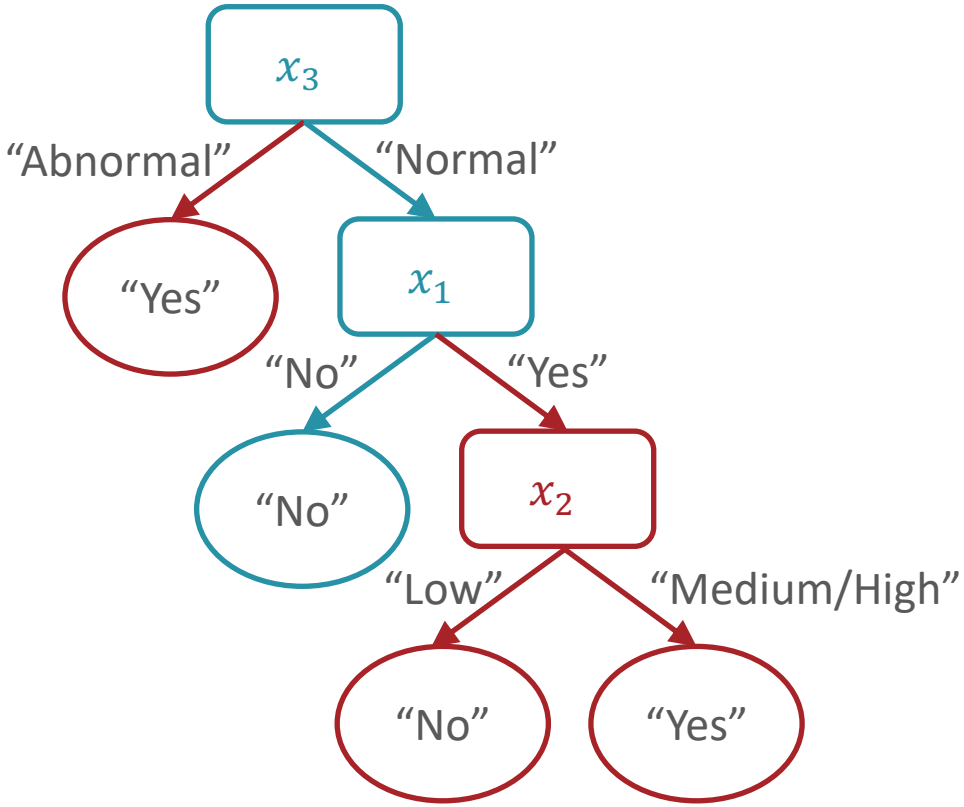

Decision Tree: Pseudocode

```
def predict(x′):
  - walk from root node to a leaf node
    while(true):
       if current node is internal (non-leaf):
            check the associated attribute, x_d
            go down branch according to x′_d
       if current node is a leaf node:
            return label stored at that leaf
```

Decision Tree: Example

Learned from medical records of 1000 women

Negative examples are C-sections

```
[833+,167-] .83+ .17-
Fetal_Presentation = 1: [822+,116-] .88+ .12-
| Previous_Csection = 0: [767+,81-] .90+ .10-
| | Primiparous = 0: [399+,13-] .97+ .03-
| | Primiparous = 1: [368+,68-] .84+ .16-
| | | Fetal_Distress = 0: [334+,47-] .88+ .12-
| | | Fetal_Distress = 1: [34+,21-] .62+ .38-
| Previous_Csection = 1: [55+,35-] .61+ .39-
Fetal_Presentation = 2: [3+,29-] .11+ .89-
Fetal_Presentation = 3: [8+,22-] .27+ .73-
```


Figure courtesy of Tom Mitchell

Decision Stumps: Questions

1. Why stop at just one feature?

2. How can we pick which feature to split on as well as the order of the splits?

Splitting Criterion

- A **splitting criterion** is a function that measures how good or useful splitting on a particular feature is *for a specified dataset*
- Idea: when deciding which feature to split on, use the one that optimizes the splitting criterion

Training Error Rate as a Splitting Criterion

x_1 Family History	x_2 Resting Blood Pressure	x_3 Cholesterol	y Heart Disease?
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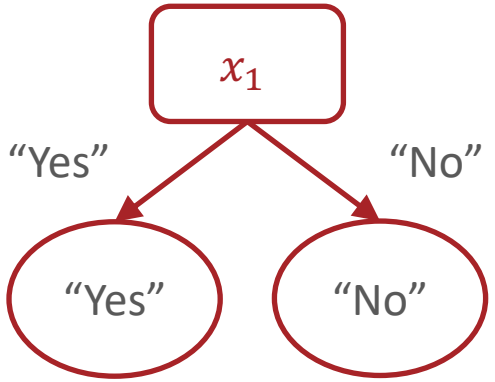
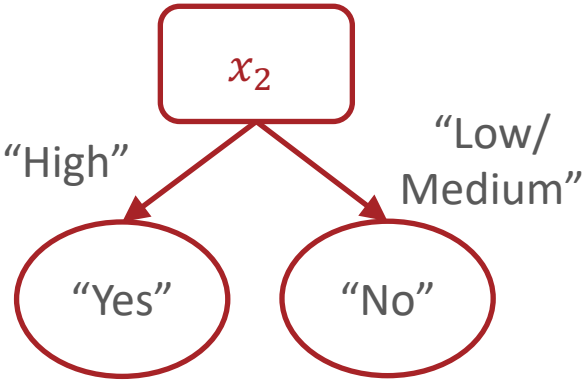
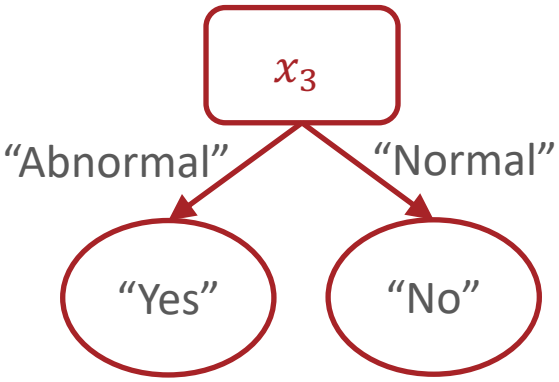


Training error rate: $2/5$



Training error rate: $2/5$



Training error rate: $1/5$

Poll Question 2:

Which feature would you split on using training error rate as the splitting criterion?

x_1	x_2	y
1	0	0
1	0	0
1	0	1
1	0	1
1	1	1
1	1	1
1	1	1
1	1	1

A. x_1

B. x_2

C. Either x_1 or x_2

D. Neither x_1 nor x_2 **(TOXIC)**

Poll Question 2:

Which feature would you split on using training error rate as the splitting criterion?

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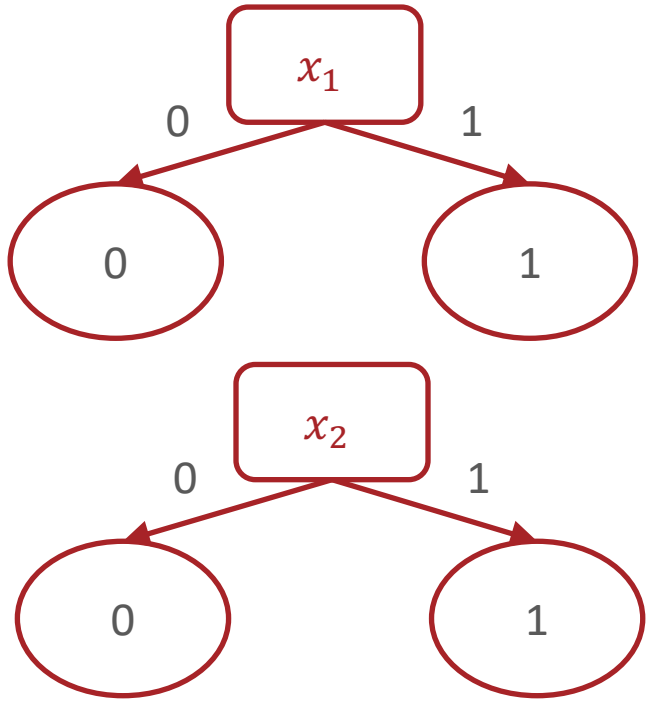

Training error rate: $2/8$

Splitting Criterion

- A **splitting criterion** is a function that measures how good or useful splitting on a particular feature is *for a specified dataset*
- Idea: when deciding which feature to split on, use the one that optimizes the splitting criterion
- Potential splitting criteria:
  - Training error rate (minimize)
  - Gini impurity (minimize) → CART algorithm
  - Mutual information (maximize) → ID3 algorithm

Splitting Criterion

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- Idea: when deciding which feature to split on, use the one that optimizes the splitting criterion
- Potential splitting criteria:
  - Training error rate (minimize)
  - Gini impurity (minimize) → CART algorithm
  - **Mutual information** (maximize) → ID3 algorithm

Entropy

- Entropy describes the purity or uniformity of a collection of values: the lower the entropy, the more pure

$$H(S) = -\sum_{v \in V(S)} \frac{|S_v|}{|S|} \log_2 \left(\frac{|S_v|}{|S|} \right)$$

where S is a collection of values,

$V(S)$ is the set of unique values in S

S_v is the collection of elements in S with value v

- If all the elements in S are the same, then

$$\mathrm{H}(S) = -1 \log_2(1) = 0$$

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where S is a collection of values,

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S_v is the collection of elements in S with value v

- If S is split fifty-fifty between two values, then

$$\mathrm{H}(S) = -\frac{1}{2}\log_2\left(\frac{1}{2}\right) - \frac{1}{2}\log_2\left(\frac{1}{2}\right) = -\log_2\left(\frac{1}{2}\right) = 1$$

Mutual Information

- Mutual information describes how much information or clarity a particular feature provides about the label

$$I(Y; x_d) = H(Y) - \sum_{v \in V(x_d)} (f_v)\left(H\left(Y_{x_d=v}\right)\right)$$

where x_d is a feature

Y is the collection of all labels

$V(x_d)$ is the set of unique values of x_d

f_v is the fraction of inputs where $x_d = v$

$Y_{x_d=v}$ is the collection of labels where $x_d = v$

$H\left(Y_{x_d=v}\right)$ is the conditional entropy of Y given $x_d = v$

Mutual Information: Example

x_d	y
1	1
1	1
0	0
0	0

$$I(Y; x_d) = H(Y) - \sum_{v \in V(x_d)} (f_v)\left(H\left(Y_{x_d=v}\right)\right)$$

$$= 1 - \frac{1}{2} H\left(Y_{x_d=0}\right) - \frac{1}{2} H\left(Y_{x_d=1}\right)$$

$$= 1 - \frac{1}{2}(0) - \frac{1}{2}(0) = 1$$

Mutual Information: Example

x_d	y
1	1
0	1
1	0
0	0

$$I(Y; x_d) = H(Y) - \sum_{v \in V(x_d)} (f_v)\left(H\left(Y_{x_d=v}\right)\right)$$

$$= 1 - \frac{1}{2} H\left(Y_{x_d=0}\right) - \frac{1}{2} H\left(Y_{x_d=1}\right)$$

$$= 1 - \frac{1}{2}(1) - \frac{1}{2}(1) = 0$$

~~Poll Question 3:~~

Which feature would you split on using mutual information as the splitting criterion?

x_1	x_2	y
1	0	0
1	0	0
1	0	1
1	0	1
1	1	1
1	1	1
1	1	1
1	1	1

A. x_1

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