10-301/601: Introduction to Machine Learning Lecture 2 – ML as Function Approximation

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Q & A

- In Lecture 1, why did we use the term experience instead of just data?
- Because our concern isn’t just the data itself, but also where the data comes from (e.g., an agent interacting with the world vs. knowledge from a book). As well, the word experience better aligns with the notion of what humans require in order to learn.

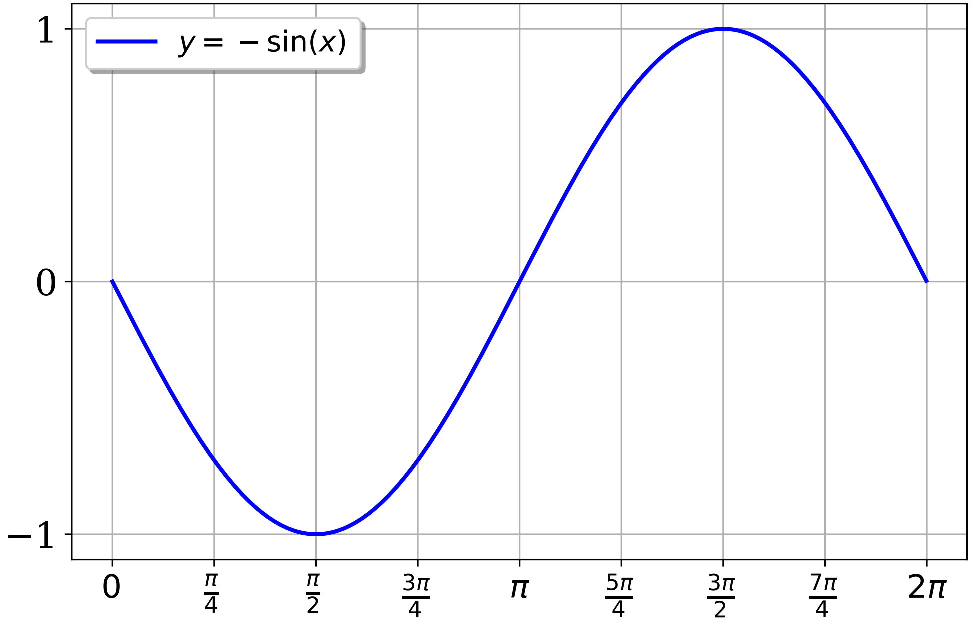
Front Matter

- Announcements:
  - HW1 released 8/29, due 9/7 at 11:59 PM
  - Two components: written and programming
    - Separate submissions on Gradescope
  - Unique policies specific to HW1:
    - Two submissions for the written portion (see write-up for details)
    - Unlimited submissions for the programming portion (really, just keep submitting until you get 100%)
    - We will grant (almost) any extension request

Function Approximation: Example

- Challenge: implement a function that computes

$$-\sin(x) \text{ for } x \in [0, 2\pi]$$



- You may not call any trigonometric functions
- You may call an existing implementation of $\sin(x)$ a few times (e.g., 100) to check your work

Our 2nd Machine Learning Task

- Learning to diagnose heart disease

  as a **(supervised) binary classification task**

features | labels

examples

Family History	Resting Blood Pressure	Cholesterol	Heart Disease?
Yes	Low	Normal	No
No	Medium	Normal	No
No	Low	Abnormal	Yes
Yes	Medium	Normal	Yes
Yes	High	Abnormal	Yes

Our 2nd Machine Learning Task

- Learning to diagnose heart disease

  as a **(<u>supervised</u>) binary classification task**

features

labels

examples

Family History	Resting Blood Pressure	Cholesterol	Heart Disease?
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Our 2nd Machine Learning Task

- Learning to diagnose heart disease

  as a **(supervised)** **binary classification** **task**

features labels

Family History	Resting Blood Pressure	Cholesterol	Heart Disease?
Yes	Low	Normal	No
No	Medium	Normal	No
No	Low	Abnormal	Yes
Yes	Medium	Normal	Yes
Yes	High	Abnormal	Yes

examples

Our 2nd Machine Learning Task

- Learning to diagnose heart disease

  as a **(supervised)** **classification task**

$$\overbrace{\text{Family History} \quad \text{Resting Blood Pressure} \quad \text{Cholesterol}}^{\text{features}} \quad \overbrace{\text{Risk}}^{\text{labels}}$$

Family History	Resting Blood Pressure	Cholesterol	Risk
Yes	Low	Normal	Low Risk
No	Medium	Normal	Low Risk
No	Low	Abnormal	Medium Risk
Yes	Medium	Normal	High Risk
Yes	High	Abnormal	High Risk

examples

Our 2nd Machine Learning Task

- Learning to diagnose heart disease

  as a **(supervised)** **<u>regression</u> task**

features / targets

Family History	Resting Blood Pressure	Cholesterol	Medical Costs
Yes	Low	Normal	$0
No	Medium	Normal	$20
No	Low	Abnormal	$30
Yes	Medium	Normal	$100
Yes	High	Abnormal	$5000

examples

Our 2nd Machine Learning Classifier

- A **classifier** is a function that takes feature values as input and outputs a label
- Majority vote classifier: always predict the most common label in the training dataset

features

labels

examples

Family History	Resting Blood Pressure	Cholesterol	Heart Disease?	Predictions
Yes	Low	Normal	No	Yes
No	Medium	Normal	No	Yes
No	Low	Abnormal	Yes	Yes
Yes	Medium	Normal	Yes	Yes
Yes	High	Abnormal	Yes	Yes

Notation

- Feature space, $\mathcal{X}$
- Label space, $\mathcal{Y}$
- (Unknown) Target function, $c^*: \mathcal{X} \rightarrow \mathcal{Y}$
- Training dataset:

$$\mathcal{D} = \left\{ \left(\boldsymbol{x}^{(1)}, c^*\left(\boldsymbol{x}^{(1)}\right) = y^{(1)} \right), \left(\boldsymbol{x}^{(2)}, y^{(2)} \right) \dots, \left(\boldsymbol{x}^{(N)}, y^{(N)} \right) \right\}$$

- Example: $\left(\boldsymbol{x}^{(n)}, y^{(n)} \right) = \left(x_1^{(n)}, x_2^{(n)}, \dots, x_D^{(n)}, y^{(n)} \right)$
- Hypothesis space: $\mathcal{H}$
- Goal: find a classifier, $h \in \mathcal{H}$, that best approximates c^*

Our 2nd Machine Learning Task

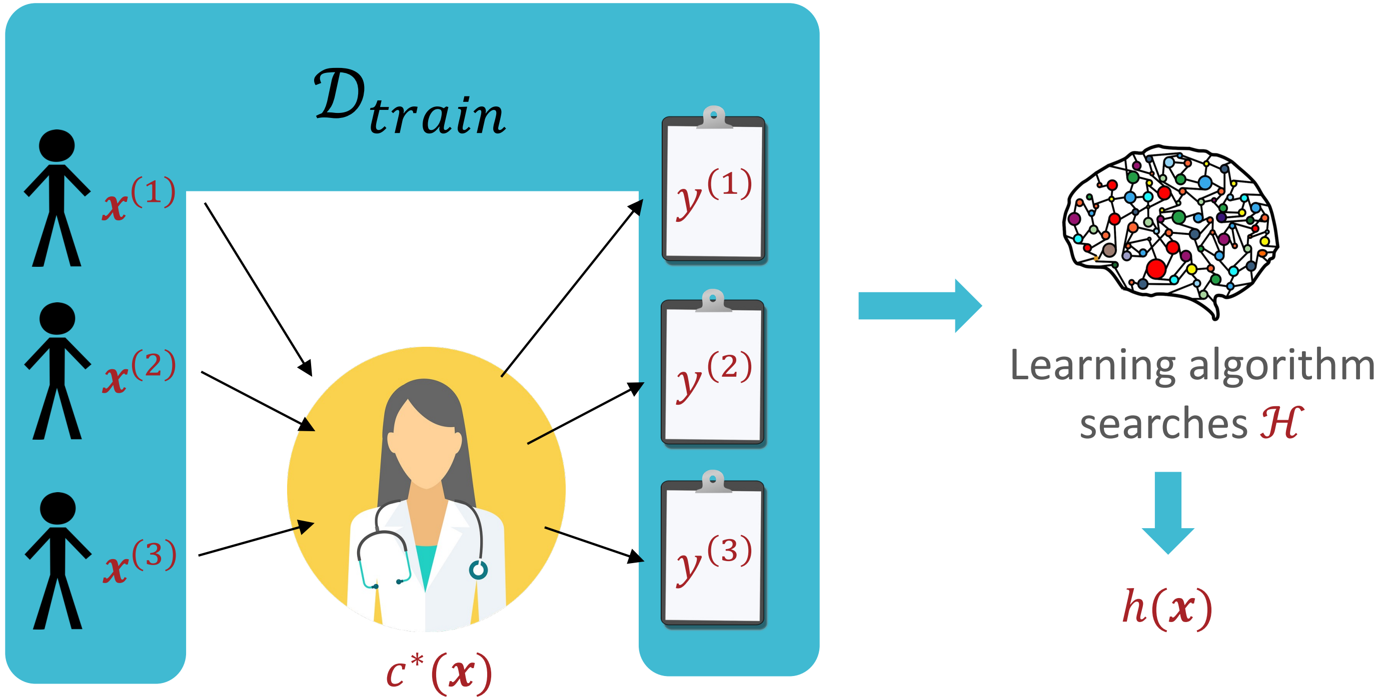


Figure courtesy of Matt Gormley

Our 2nd Machine Learning Classifier

- Majority vote classifier: always predict the most common label in the training dataset

	Family History	Resting Blood Pressure	Cholesterol	Heart Disease?	Predictions
	Yes	Low	Normal	No	Yes
	No	Medium	Normal	No	Yes
$\boldsymbol{x}^{(2)}$	No	Low	Abnormal	Yes	Yes
	Yes	Medium	Normal	Yes	Yes
	Yes	High	Abnormal	Yes	Yes

- $N = 5$ and $D = 3$
- $\boldsymbol{x}^{(2)} = \left(x_1^{(2)} = \text{“No”}, x_2^{(2)} = \text{“Medium”}, x_3^{(2)} = \text{“Normal”}\right)$

Evaluation

- Loss function, $\ell : \mathcal{Y} \times \mathcal{Y} \to \mathbb{R}$
  - Defines how "bad" predictions, $\hat{y} = h(\boldsymbol{x})$, are compared to the true labels, $y = c^*(\boldsymbol{x})$
  - Common choices

1. Squared loss (for regression): $\ell(y, \hat{y}) = (y - \hat{y})^2$
2. Binary or 0-1 loss (for classification):

$$\ell(y, \hat{y}) = \begin{cases} 1 & \text{if } y \neq \hat{y} \\ 0 & \text{otherwise} \end{cases}$$

Evaluation

- Loss function, $\ell : \mathcal{Y} \times \mathcal{Y} \to \mathbb{R}$
  - Defines how "bad" predictions, $\hat{y} = h(\boldsymbol{x})$, are compared to the true labels, $y = c^*(\boldsymbol{x})$
  - Common choices

1. Squared loss (for regression): $\ell(y, \hat{y}) = (y - \hat{y})^2$
2. Binary or 0-1 loss (for classification):

$$\ell(y, \hat{y}) = \mathbb{1}(y \neq \hat{y})$$

- Error rate:

$$err(h, \mathcal{D}) = \frac{1}{N} \sum_{n=1}^{N} \mathbb{1}\left(y^{(n)} \neq \hat{y}^{(n)}\right)$$

Different Kinds of Error

- Training error rate = $err(h, \mathcal{D}_{train})$

- Test error rate = $err(h, \mathcal{D}_{test})$

- True error rate = $err(h)$

  = the error rate of h on all possible examples

  - In machine learning, this is the quantity that we care about but, in most cases, it is unknowable.

Our 2nd Machine Learning Task

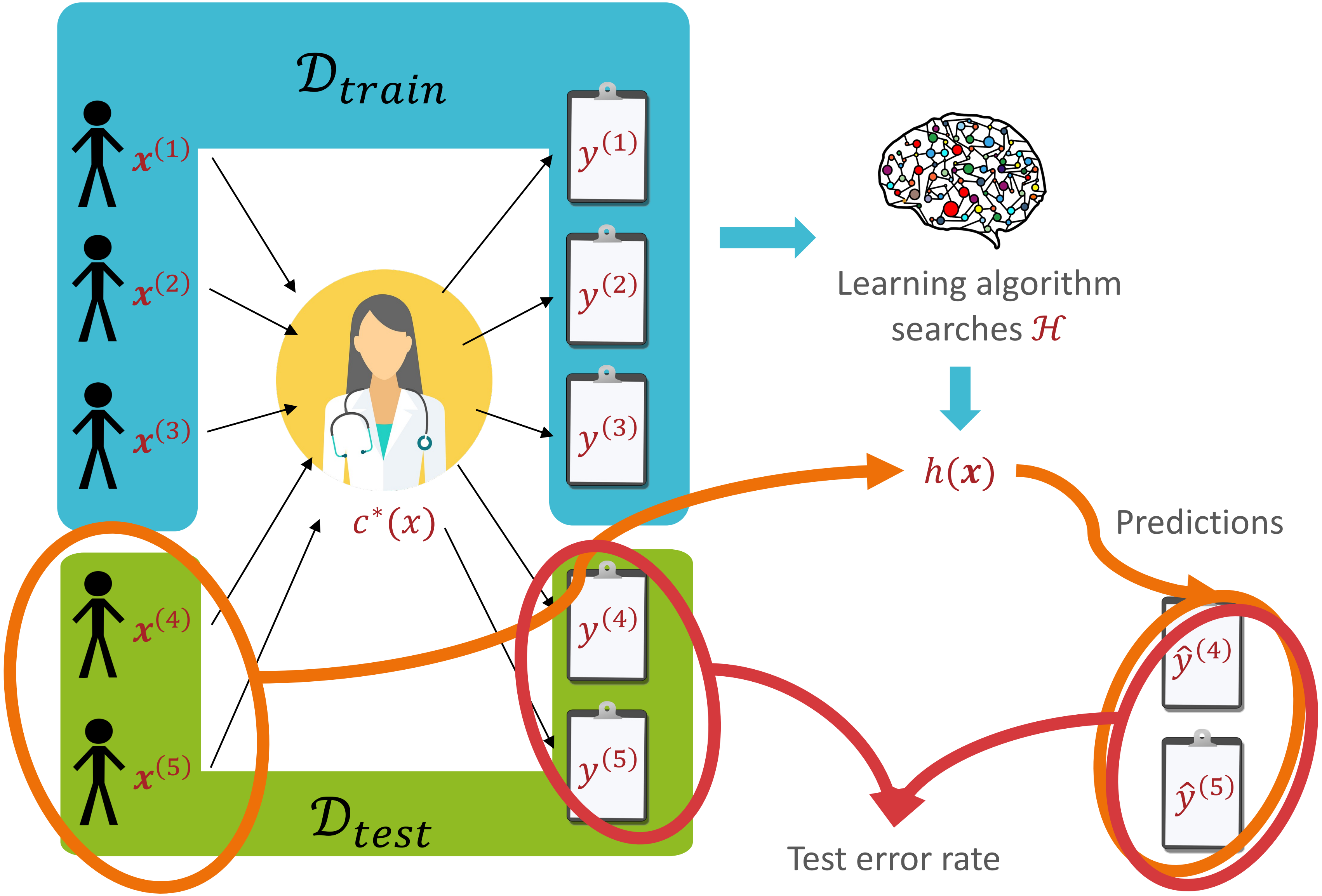


Figure courtesy of Matt Gormley

Our 2nd Machine Learning Classifier

- Majority vote classifier:

Test your understanding

x_1	x_2	y
1	0	-
1	0	-
1	0	+
1	0	+
1	1	+
1	1	+
1	1	+
1	1	+

- What is the **training error** of the **majority vote classifier** on this dataset?

Our 3rd Machine Learning Classifier

- Memorizer: if a set of features exists in the **training** dataset, predict its corresponding label; otherwise, predict a random label.

Family History	Resting Blood Pressure	Cholesterol	Heart Disease?
Yes	Low	Normal	No
No	Medium	Normal	No
No	Low	Abnormal	Yes
Yes	Medium	Normal	Yes
Yes	High	Abnormal	Yes

Our 3rd Machine Learning Classifier

- Memorizer:

Our 3rd Machine Learning Classifier

- Memorizer: if a set of features exists in the **training** dataset, predict its corresponding label; otherwise, predict a random label.

Family History	Resting Blood Pressure	Cholesterol	Heart Disease?	Predictions
Yes	Low	Normal	No	No
No	Medium	Normal	No	No
No	Low	Abnormal	Yes	Yes
Yes	Medium	Normal	Yes	Yes
Yes	High	Abnormal	Yes	Yes

- The training error rate is always 0!

Our 4th Machine Learning Classifier

- Alright, let's actually (try to) extract a pattern from the data

x_1 Family History	x_2 Resting Blood Pressure	x_3 Cholesterol	y Heart Disease?
Yes	Low	Normal	No
No	Medium	Normal	No
No	Low	Abnormal	Yes
Yes	Medium	Normal	Yes
Yes	High	Abnormal	Yes

- Decision stump: based on a single feature, x_d, predict the most common label in the **training** dataset among all data points that have the same value for x_d

Our 4th Machine Learning Classifier: Example

- Alright, let's actually (try to) extract a pattern from the data

x_1 Family History	x_2 Resting Blood Pressure	x_3 Cholesterol	y Heart Disease?
Yes	Low	Normal	No
No	Medium	Normal	No
No	Low	Abnormal	Yes
Yes	Medium	Normal	Yes
Yes	High	Abnormal	Yes

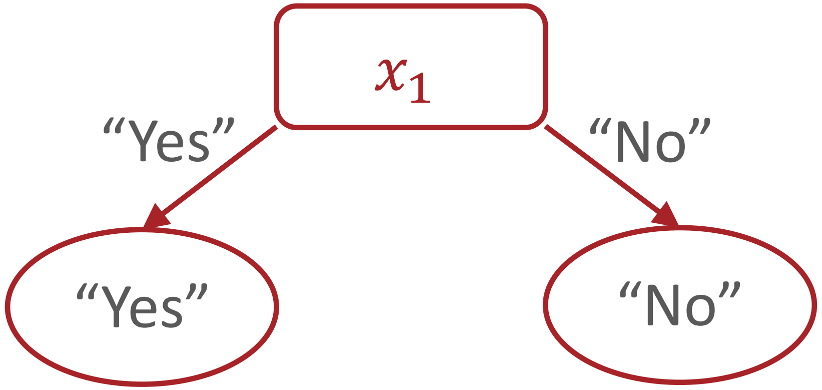
- Decision stump on x_1:

$$h(\boldsymbol{x}') = h(x'_1, \dots, x'_D) = \begin{cases} \text{"Yes" if } x'_1 = \text{"Yes"} \\ \text{"No" otherwise} \end{cases}$$

Our 4th Machine Learning Classifier: Example

- Alright, let's actually (try to) extract a pattern from the data

x_1 Family History	x_2 Resting Blood Pressure	x_3 Cholesterol	y Heart Disease?	$\hat{y}$ Predictions
Yes	Low	Normal	No	Yes
No	Medium	Normal	No	No
No	Low	Abnormal	Yes	No
Yes	Medium	Normal	Yes	Yes
Yes	High	Abnormal	Yes	Yes



Decision Stumps: Pseudocode

Decision Stumps: Questions

1. How can we pick which feature to split on?
2. Why stop at just one feature?