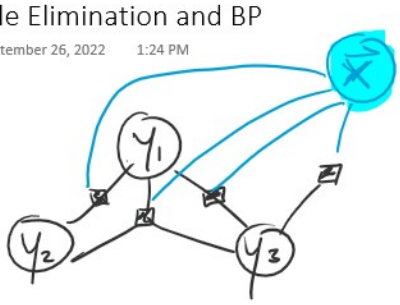
Variable Elimination and BP

Monday, September 26, 2022 1:24 PM



$$p(\vec{y}|\vec{x}) \propto \frac{\text{score}(\vec{y},\vec{x})}{Z(\vec{x})}$$

marginal probability: $p(y_2|\vec{x}) = \sum_{y_1}\sum_{y_3} p(y_1,y_2,y_3)$

$$= \sum_{y_1}\sum_{y_3} \text{score}(y_1,y_2,y_3)/Z(\vec{x})$$

partition function: $Z(\vec{x}) = \sum_{y_1}\sum_{y_2}\sum_{y_3} \text{score}(y_1,y_2,y_3)$

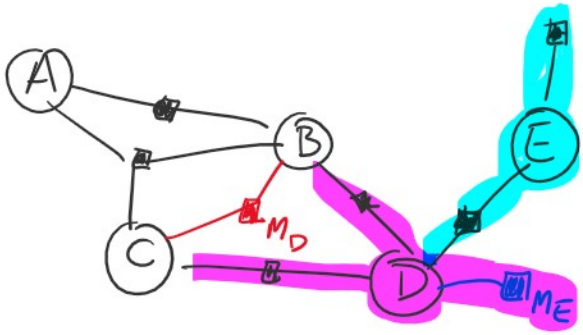
MAP inference: $\hat{y}_1,\hat{y}_2,\hat{y}_3 = \underset{y_1,y_2,y_3}{\text{argmax}}\ p(y_1,y_2,y_3)$

$$= \underset{y_1,y_2,y_3}{\text{argmax}}\ \text{score}(y_1,y_2,y_3)$$

Variable Elimination

Ex:

Assume binary variables



e	$\psi_E(e)$
0	3
1	2

d	e	$\psi_{DE}(d,e)$
0	0	1
0	1	5
1	0	1
1	1	4

$$m_E(d) = \sum_{e\in\{0,1\}} \psi_E(e)\,\psi_{DE}(d,e)$$

d	$m_E(d)$
0	$\psi_E(0)\psi_{DE}(0,0) + \psi_E(1)\psi_{DE}(0,1)$
1	$\psi_E(0)\psi_{DE}(1,0) + \psi_E(1)\psi_{DE}(1,1)$

$$m_D(b,c) = \sum \psi_{BD}(b,d)\,\psi_{CD}(c,d)\,m_E(d)$$

$$m_D(b,c) = \sum_{d \in \{0,1\}} \psi_{BD}(b,d)\, \psi_{CD}(c,d)\, m_E(d)$$

b	c	$m_D(b,c)$
0	0	~
0	1	—
1	0	~
1	1	~