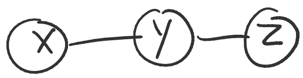
Factor Graphs + Variable Elimination

Friday, September 23, 2022 1:27 PM

Proof of Indep. by Separation



$$p(x,y,z) = \frac{1}{Z_p}\psi_{x,y}(x,y)\,\psi_{y,z}(y,z)$$

Claim: $X \perp\!\!\!\perp Z \mid Y$

Proof: $\boxed{\text{Step 1}}$ show that $p(x|y)$ can be computed with only $\psi_{xy}(x,y)$

$$p(x|y) = \frac{p(x,y)}{p(y)}$$

$$= \frac{\sum_z \cancel{\frac{1}{Z_p}}\psi(x,y)\psi(y,z)}{\sum_x\sum_z \cancel{\frac{1}{Z_p}}\psi(x,y)\psi(y,z)}$$

$$= \frac{\psi(x,y)\cancel{\left[\sum_z \psi(y,z)\right]}}{\sum_x \psi(x,y)\cancel{\left[\sum_z \psi(y,z)\right]}} = \frac{\psi(x,y)}{\sum_x \psi(x,y)}$$

Aside:

$$\sum_{t=1}^{T} 6\cdot t = 6\sum_{t=1}^{T} t$$

$\rightarrow$ $p(z|y)$ only needs $\psi(y,z)$

$\boxed{\text{Step 2}}$ Show that $p(x,z|y) = p(x|y)p(z|y)$

$$p(x,z|y) = \frac{p(x,y,z)}{\sum_x\sum_z p(x,y,z)}$$

$$= \frac{\psi(x,y)\psi(y,z)}{\sum_x\sum_z \psi(x,y)\psi(y,z)}$$

$$= \frac{\psi(x,y)\psi(y,z)}{\left[\sum_x \psi(x,y)\right]\left[\sum_z \psi(y,z)\right]} \rightarrow \left[\sum_x \psi(x,y)\right]\left[\sum_z \psi(y,z)\right]$$

$$= p(x|y)p(z|y)$$

$$= p(x|y)\,p(z|y)$$