

Reject SamplingMetropolis-HastingsGibbs Sampling

Key Idea: we draw samples one-variable-at-a-time from "full conditionals" and always accept next sample

Full Conditionals: for distribution  $p(x_1, x_2, \dots, x_J)$  we have f.c.s:

$$p(x_i | x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_J) = p(x_i | \{x_j\}_{j \neq i})$$
Algo:

Let  $x^{(1)}$  be initial assignment to vars

$t=1$

while true:

for  $i=1, \dots, J$ :

sample  $x_i^{(t+1)} \sim p(x_i | \{x_j^{(t)}\}_{j \neq i})$

set  $\vec{x}_{-i}^{(t+1)} \leftarrow \vec{x}_{-i}^{(t)}$

increment  $t \leftarrow t+1$

Do not assume access to partition function  $Z$   
e.g.  $p(\vec{x}) = \tilde{p}(\vec{x})/Z$   
we only need  $\tilde{p}(\vec{x})$

Algo: (exactly same)

Initialize  $\vec{x}$

for  $t=1, 2, 3, \dots$

for  $i=1, \dots, J$ :

$x_i \sim p(x_i | \{x_j\}_{j \neq i})$

Two Great Ideas:

①  $p(x_i | \{x_j\}_{j \neq i}) \propto \tilde{p}(x_1, \dots, x_J)$

② we might need even less than  $\tilde{p}(x)$  if we can rely on just Markov boundary

## Metropolis's Algorithm

- ① Choose a proposal  $q(\vec{x}|\vec{x}')$  s.t.  $q(\vec{x}|\vec{x}') = q(\vec{x}'|\vec{x})$
- ② Choose an initial state  $\vec{x}^{(1)}$
- ③ for  $t=1, 2, 3, \dots$

④ propose a new state  $\vec{x} \sim q(\vec{x}|\vec{x}^{(t)})$

⑤ accept  $\vec{x}$  w/probability

$$a = A(\vec{x} \leftarrow \vec{x}^{(t)}) = \min\left(1, \frac{\tilde{p}(\vec{x})}{\tilde{p}(\vec{x}^{(t)})}\right)$$

if new  $\vec{x}$  is more probable than  $\vec{x}^{(t)}$ , then always accept

⑥ if  $\vec{x}$  is accepted:  
 $\vec{x}^{(t+1)} = \vec{x}$

$a \geq u$  where  $u \sim \text{Uniform}(0, 1)$

else:  
 $\vec{x}^{(t+1)} = \vec{x}^{(t)}$

## Metropolis-Hastings Algorithm

- The proposal need not be symmetric
- Identical to Metropolis Algo, but we accept w/prob

$$A(\vec{x} \leftarrow \vec{x}^{(t)}) = \min\left(1, \frac{\tilde{p}(\vec{x})}{\tilde{p}(\vec{x}^{(t)})} \frac{q(\vec{x}^{(t)}|\vec{x})}{q(\vec{x}|\vec{x}^{(t)})}\right)$$