Proof for Indep. by Sequencing:

\[ p(x, y, z) = \frac{1}{z_p} \psi_{xy}(x, y) \psi_{yz}(y, z) \]

Claim: \( x \perp z \mid y \)

**Proof:**

- **Step 1:** Show that \( p(x \mid y) \) can be computed with only \( \psi_{xy}(x, y) \)

\[
p(x \mid y) = \frac{p(x, y)}{p(y)} = \frac{\sum_z \psi_{xy}(x, y) \psi(y, z)}{\sum_z \psi(y, z)}
\]

\[
= \frac{\psi_{xy}(x, y) \sum_z \psi(y, z)}{\sum_z \psi(y, z)} = \frac{\psi_{xy}(x, y)}{\sum_z \psi(y, z)}
\]

\[ \Rightarrow p(z \mid y) \text{ only needs } \psi(y, z) \]

- **Step 2:** Show that \( p(x, z \mid y) = p(x \mid y) p(z \mid y) \)

\[
p(x, z \mid y) = \frac{p(x, y, z)}{p(y)} = \frac{\sum_x \sum_z p(x, y, z)}{\sum_z \psi(y, z)}
\]

\[
= \frac{\psi_{xy}(x, y) \psi(y, z)}{\sum_z \psi(y, z)}
\]

\[
= \frac{\psi_{xy}(x, y) \psi(y, z)}{\sum_z \psi(y, z)}
\]

\[ \Rightarrow \]

Aside:

\[
\sum_{t=1}^{6} t = \frac{6(6+1)}{2} = 21
\]
\[ f = p(x|y) p(z|y) \]