Key Idea: we draw samples one-variable-at-a-time from "full conditionals" and always accept next sample.

Full Conditionals: for distribution \( p(x_1, x_2, \ldots, x_J) \) we have f.c.s:
\[
p(x_i \mid x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_J) = p(x_i \mid \{x_j : j \neq i\})
\]

Alg2:

Let \( x^{(0)} \) be initial assignment to vars.

\[ t = 1 \]

while true:

for \( i = 1, \ldots, J \):

sample \( x_i^{(t+1)} \sim p(x_i \mid \{x_j^{(t)} : j \neq i\}) \)

set \( x_i^{(t+1)} \leftarrow x_i^{(t)} \)

increment \( t \leftarrow t + 1 \)

Alg2: (exactly same)

Initialize \( x \)

for \( t = 1, 2, 3, \ldots \):

for \( i = 1, \ldots, J \):

\( x_i \sim p(x_i \mid \{x_j^{(t)} : j \neq i\}) \)

Two Great Ideas:

1. \( p(x_i \mid \{x_j : j \neq i\}) \propto p(x_i, \ldots, x_J) \)

2. we might need even less than \( p(x) \) if we can rely on just Markov boundary.
Metropolis Algorithm

1. Choose a proposal $q(\hat{x} | x')$ s.t. $q(\hat{x} | x') = q(x' | x)$
2. Choose an initial state $x^{(1)}$
3. For $t = 1, 2, 3, ...$
   a. propose a new state $\hat{x} \sim q(\hat{x} | x^{(t)})$
   b. accept $\hat{x}$ w/probability
      $$a = \min \left(1, \frac{p(\hat{x})}{p(x^{(t)})} \frac{q(x^{(t)} | \hat{x})}{q(\hat{x} | x^{(t)})} \right)$$
   c. if $\hat{x}$ is accepted: $x^{(t+1)} = \hat{x}$
      else: $x^{(t+1)} = x^{(t)}$

Metropolis - Hastings Algorithm

- The proposal need not be symmetric
- Identical to Metropolis Alg, but we accept w/prob
  $$A(\hat{x} \leftarrow x^{(t)}) = \min \left(1, \frac{p(\hat{x})}{p(x^{(t)})} \frac{q(x^{(t)} | \hat{x})}{q(\hat{x} | x^{(t)})} \right)$$