Machine Learning Department
School of Computer Science
Carnegie Mellon University

Neural Potential Functions

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Lecture 11
Oct. 2, 2019

Reminders

- **Homework 2: BP for Syntax Trees**
  - Out: Sat, Sep. 28
  - Due: Sat, Oct. 12 at 11:59pm
- **Last chance to switch between 10-418 / 10-618 is October 7th (drop deadline)**

BACKPROPAGATION AND BELIEF PROPAGATION

Whiteboard:

- Gradient of MRF log-likelihood with respect to log potentials
- Gradient of MRF log-likelihood with respect to potentials

Factor Derivatives

Log-probability:

$$\log p(\mathbf{y}) = \left[\sum_{\alpha} \log \psi_{\alpha}(\mathbf{y}_{\alpha})\right] - \log \sum_{\mathbf{y}' \in \mathcal{Y}} \prod_{\alpha} \psi_{\alpha}(\mathbf{y}'_{\alpha}) \tag{1}$$

Derivatives:

$$\frac{\partial \log p(\mathbf{y})}{\partial \log \psi_{\alpha}(\mathbf{y}'_{\alpha})} = \mathbb{1}(\mathbf{y}_{\alpha} = \mathbf{y}'_{\alpha}) - p(\mathbf{y}'_{\alpha}) \tag{2}$$

$$\frac{\partial \log p(\mathbf{y})}{\partial \psi_{\alpha}(\mathbf{y}'_{\alpha})} = \frac{\mathbb{1}(\mathbf{y}_{\alpha} = \mathbf{y}'_{\alpha}) - p(\mathbf{y}'_{\alpha})}{\psi_{\alpha}(\mathbf{y}'_{\alpha})} \tag{3}$$

Outline of Examples

- **Hybrid NN + HMM**
  - Model: neural net for emissions
  - Learning: backprop for end-to-end training
  - Experiments: phoneme recognition (Bengio et al., 1992)
- **Hybrid RNN + HMM**
  - Model: neural net for emissions
  - Experiments: phoneme recognition (Graves et al., 2013)
- **Hybrid CNN + CRF**
  - Model: neural net for factors
  - Experiments: natural language tasks (Collobert & Weston, 2011)
  - Experiments: pose estimation
- **Tricks of the Trade**

HYBRID:
NEURAL NETWORK + HMM

Recall…

Markov Random Field (MRF)

Joint distribution over tags Y_i <u>and</u> words X_i
The individual factors aren't *necessarily* probabilities.

$$p(\text{n, v, p, d, n, time, flies, like, an, arrow}) = \frac{1}{Z}(4 * 8 * 5 * 3 * \ldots)$$

	v	n	p	d
v	1	6	3	4
n	8	4	2	0.1
p	1	3	1	3
d	0.1	8	0	0

	v	n	p	d
v	1	6	3	4
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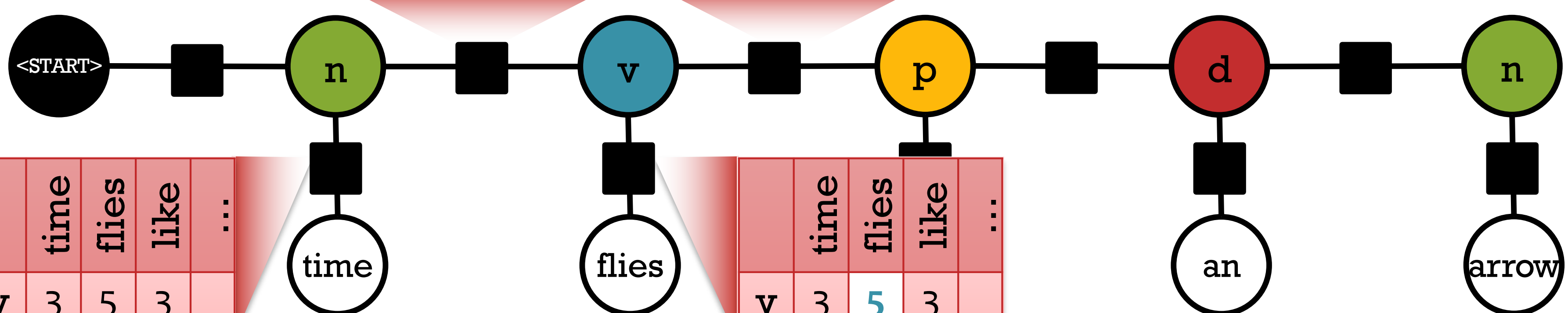

	time	flies	like	…
v	3	5	3	
n	4	5	2	
p	0.1	0.1	3	
d	0.1	0.2	0.1	

	time	flies	like	…
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Recall…

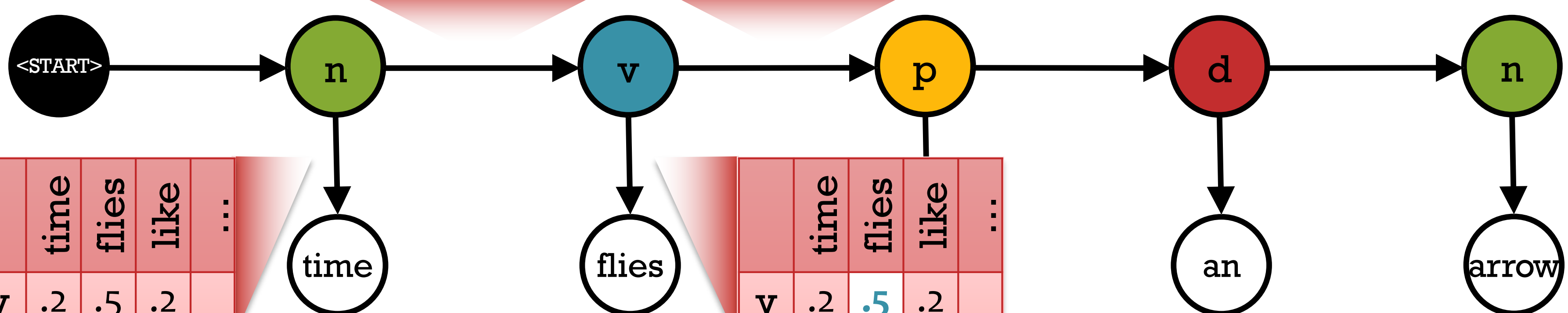
Hidden Markov Model

But sometimes we *choose* to make them probabilities.
Constrain each row of a factor to sum to one. Now $Z = 1$.

$$p(\text{n, v, p, d, n, time, flies, like, an, arrow}) = \cancel{\frac{1}{Z}}(.3 * .8 * .2 * .5 * \ldots)$$

	v	n	p	d
v	.1	.4	.2	.3
n	.8	.1	.1	0
p	.2	.3	.2	.3
d	.2	.8	0	0

	v	n	p	d
v	.1	.4	.2	.3
n	.8	.1	.1	0
p	.2	.3	.2	.3
d	.2	.8	0	0



	time	flies	like	…
v	.2	.5	.2	
n	.3	.4	.2	
p	.1	.1	.3	
d	.1	.2	.1	

	time	flies	like	…
v	.2	.5	.2	
n	.3	.4	.2	
p	.1	.1	.3	
d	.1	.2	.1	

Hybrid: NN + HMM

(Bengio et al., 1992)

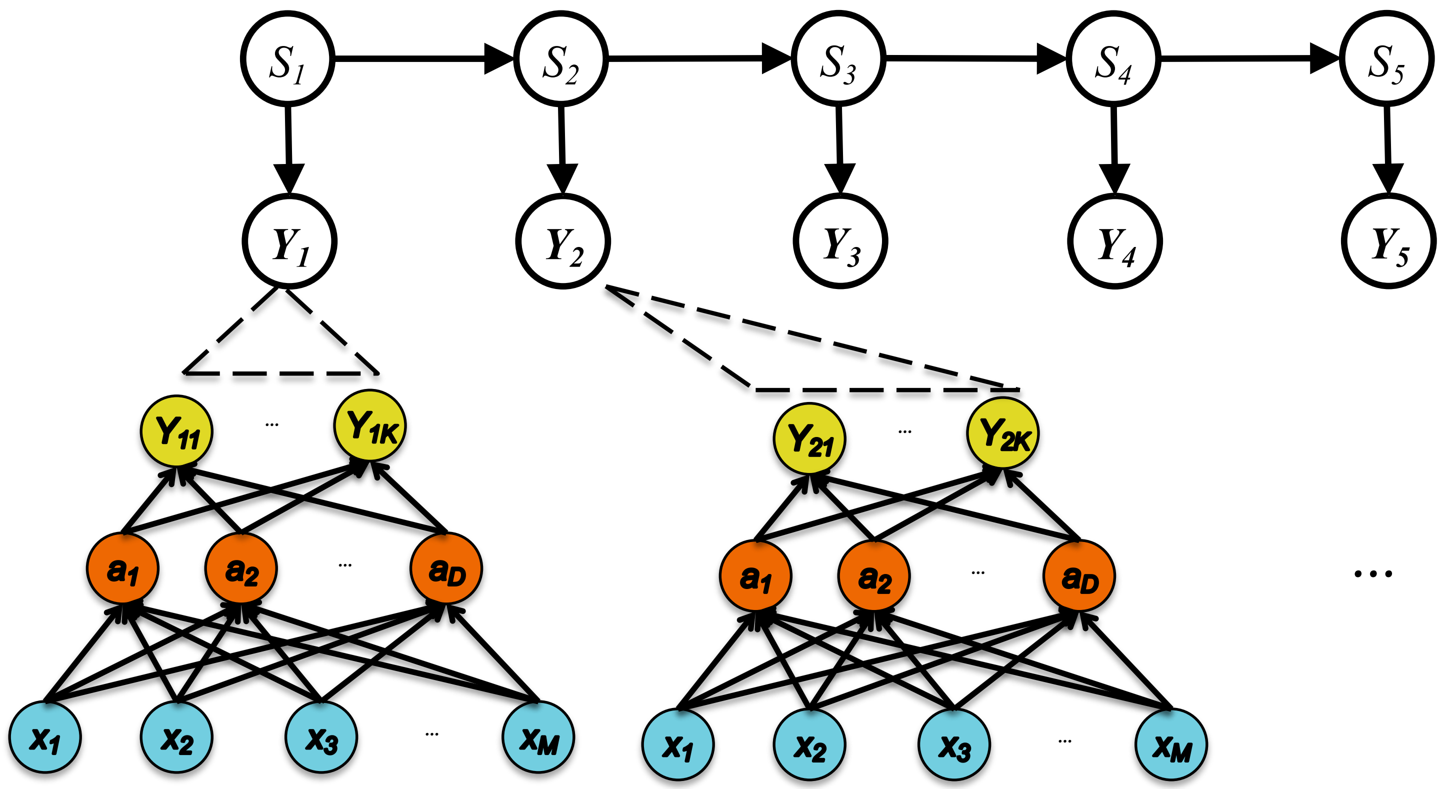
Discrete HMM state: $S_t \in \{/p/, /t/, /k/, /b/, /d/, \ldots, /g/\}$

Continuous HMM emission: $Y_t \in \mathcal{R}^K$

HMM: $p(\mathbf{Y}, \mathbf{S}) = \prod_{t=1}^{T} p(Y_t|S_t) p(S_t|S_{t-1})$

Gaussian emission:

$$p(Y_t|S_t = i) = b_{i,t} = \sum_k \frac{Z_k}{((2\pi)^n \mid \Sigma_k \mid)^{1/2}} \exp(-\frac{1}{2}(Y_t - \mu_k)\Sigma_k^{-1}(Y_t - \mu_k)^T)$$



Hybrid: NN + HMM

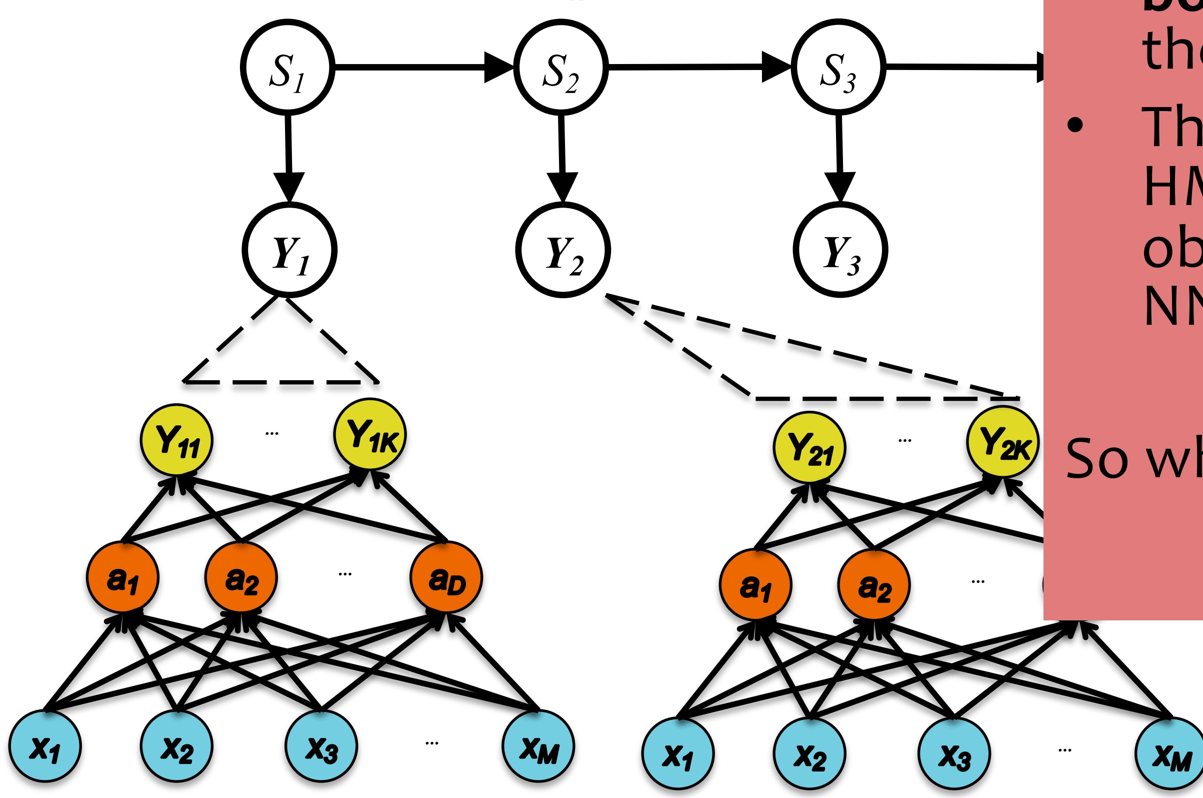
(Bengio et al., 1992)

Discrete HMM state: $S_t \in \{/p/, /t/, /k/, /b/, /d/, \ldots, /g/\}$

Continuous HMM emission: $Y_t \in \mathcal{R}^K$

HMM: $p(\mathbf{Y}, \mathbf{S}) = \prod_{t=1}^{T} p(Y_t|S_t) p(S_t|S_{t-1})$

$$p(Y_t|S_t = i) = b_{i,t} = \sum_k \frac{Z_k}{((2\pi)^n \mid \Sigma_k \mid)^{1/2}} e$$

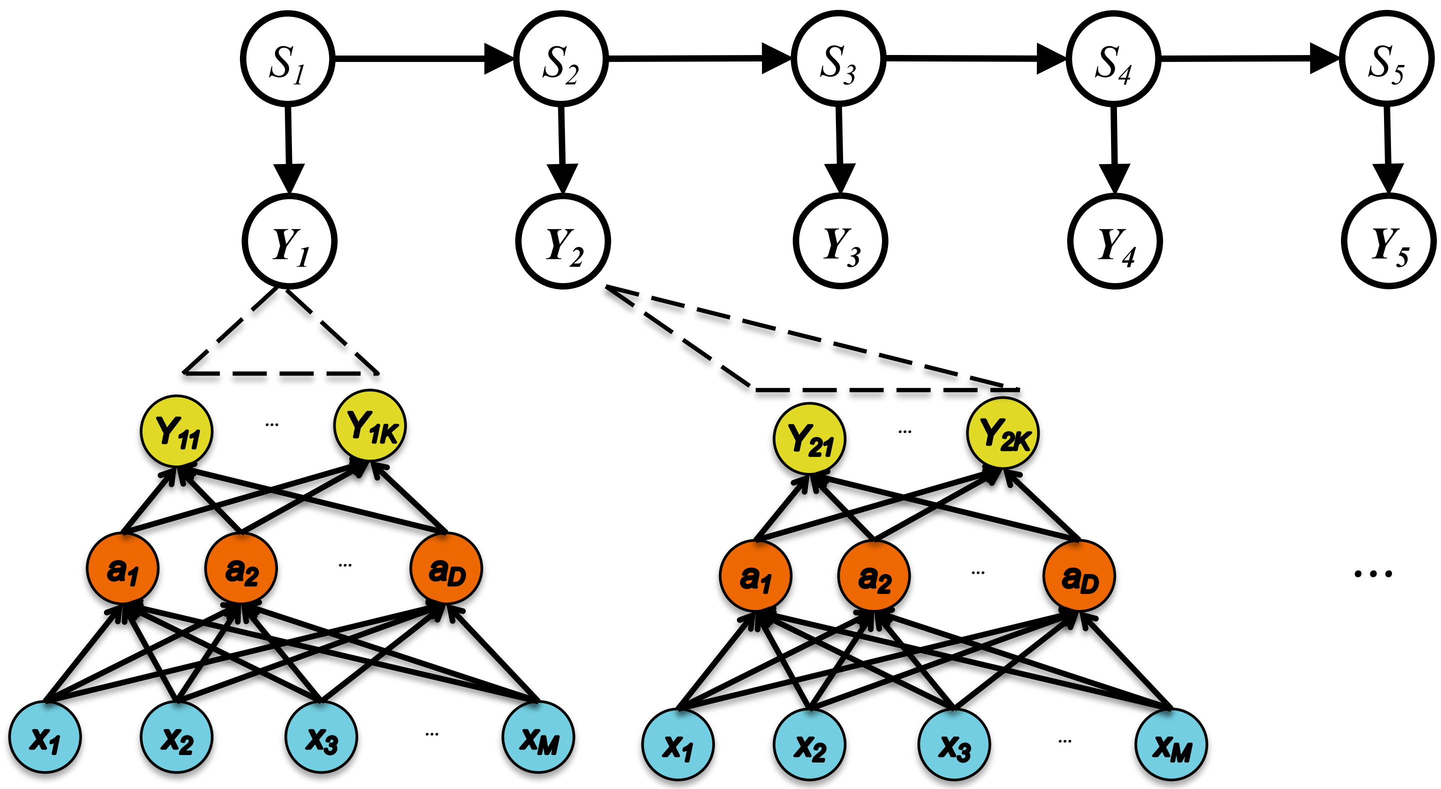


Lots of oddities to this picture:

- **Clashing visual notations** (graphical model vs. neural net)
- HMM generates data **top-down,** NN generates **bottom-up** and they meet in the middle.
- The "observations" of the HMM are not actually observed (i.e. x's appear in NN only)

So what are we missing?

Hybrid: NN + HMM



$a_{i,j} = p(S_t = i | S_{t-1} = j)$

$b_{i,t} = p(Y_t | S_t = i)$

Hybrid: NN + HMM

Forward-backward algorithm: a “feed-forward” algorithm for computing alpha-beta probabilities.

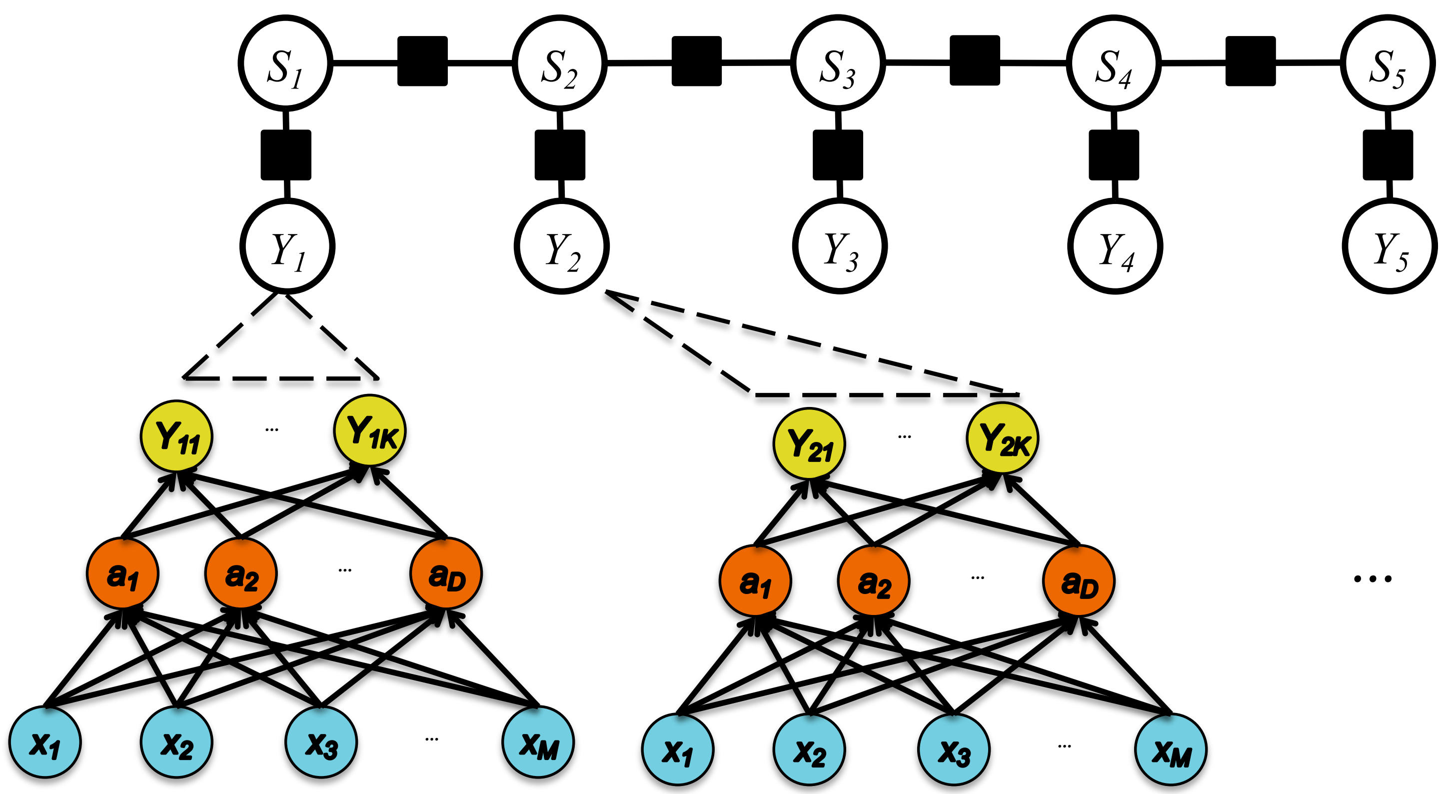
$$\alpha_{i,t} = P(Y_1^t \text{ and } S_t = i \mid model) = b_{i,t} \sum_j a_{ji} \alpha_{j,t-1}$$

$$\beta_{i,t} = P(Y_{t+1}^T \mid S_t = i \text{ and } model) = \sum_j a_{ij} b_{j,t+1} \beta_{j,t+1}$$

$$\gamma_{i,t} = P(S_t = i \mid Y_1^t \text{ and } model) = \alpha_{i,t}\, \beta_{i,t}$$

Log-likelihood: a “feed-forward” objective function.

$$\log p(\mathbf{S}, \mathbf{Y}) = \alpha_{\text{END},T}$$



A Recipe for Graphical Models

1. Given training data:

$$\{\boldsymbol{x}_i, \boldsymbol{y}_i\}_{i=1}^N$$

2. Choose each of these:
   - Decision function

$$\hat{\boldsymbol{y}} = f_{\boldsymbol{\theta}}(\boldsymbol{x}_i)$$

   - Loss function

$$\ell(\hat{\boldsymbol{y}}, \boldsymbol{y}_i) \in \mathbb{R}$$

Decision / Loss Function for Hybrid NN + HMM

Forward-backward algorithm: a "feed-forward" algorithm for computing alpha-beta probabilities.

$$\alpha_{i,t} = P(Y_1^t \text{ and } S_t = i \mid \mathit{model}) = b_{i,t} \sum_j a_{ji} \alpha_{j,t-1}$$

$$\beta_{i,t} = P(Y_{t+1}^T \mid S_t = i \text{ and } \mathit{model}) = \sum_j a_{ij} b_{j,t+1} \beta_{j,t+1}$$

$$\gamma_{i,t} = P(S_t = i \mid Y_1^t \text{ and } \mathit{model}) = \alpha_{i,t}\, \beta_{i,t}$$

Log-likelihood: a "feed-forward" objective function.

$$\log p(\mathbf{S}, \mathbf{Y}) = \alpha_{\text{END},T}$$

4. Train with SGD:

nt)

$$- \eta_t \nabla \ell(f_{\boldsymbol{\theta}}(\boldsymbol{x}_i), \boldsymbol{y}_i)$$

How do we compute the gradient?

Recall...

Training

Backpropagation

Backpropagation is just repeated application of the **chain rule** from Calculus 101.

Graphical Model and Log-likelihood

Neural Network

$$\boldsymbol{y} = g(\boldsymbol{u}) \text{ and } \boldsymbol{u} = h(\boldsymbol{x}).$$

How to compute these partial derivatives?

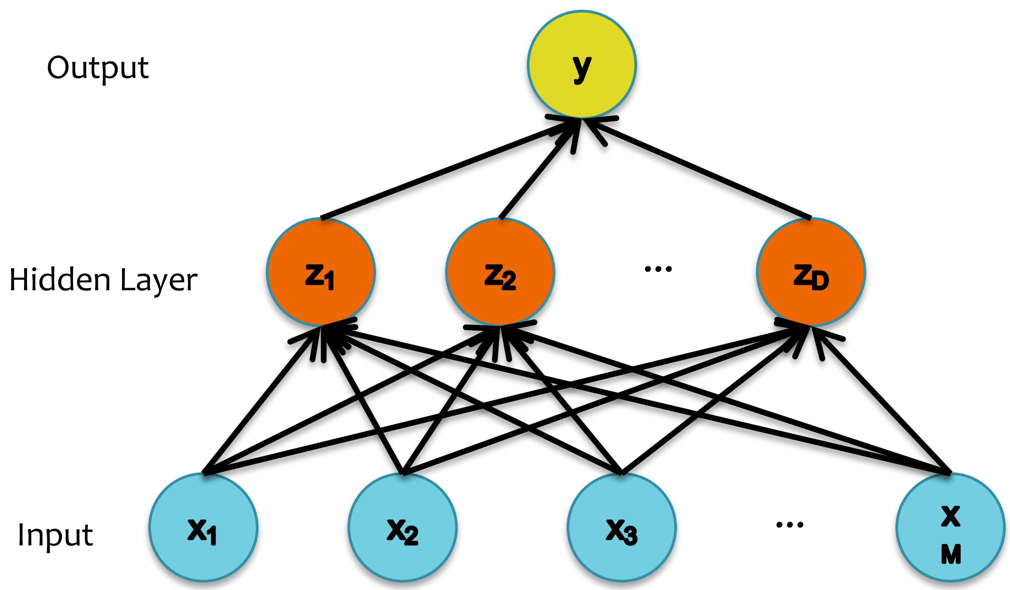
Chain Rule:

$$\frac{dy_i}{dx_k} = \sum_{j=1}^{J} \frac{dy_i}{du_j} \frac{du_j}{dx_k}, \quad \forall i, k$$

Recall...

Training   Backpropagation

What does this picture actually mean?



(F) Loss
$J = \frac{1}{2}(y - y^{(d)})^2$

↑

(E) Output (sigmoid)
$y = \frac{1}{1+\exp(b)}$

↑

(D) Output (linear)
$b = \sum_{j=0}^{D} \beta_j z_j$

↑

(C) Hidden (sigmoid)
$z_j = \frac{1}{1+\exp(a_j)}, \ \forall j$

↑

(B) Hidden (linear)
$a_j = \sum_{i=0}^{M} \alpha_{ji} x_i, \ \forall j$

↑

(A) Input
Given $x_i, \ \forall i$

Recall...

Training Backpropagation

Case 2: Neural Network

Forward

$$J = y^* \log q + (1 - y^*) \log(1 - q)$$

$$q = \frac{1}{1 + \exp(-b)}$$

$$b = \sum_{j=0}^{D} \beta_j z_j$$

$$z_j = \frac{1}{1 + \exp(-a_j)}$$

$$a_j = \sum_{i=0}^{M} \alpha_{ji} x_i$$

Backward

$$\frac{dJ}{dq} = \frac{y^*}{q} + \frac{(1 - y^*)}{q - 1}$$

$$\frac{dJ}{db} = \frac{dJ}{dy}\frac{dy}{db}, \quad \frac{dy}{db} = \frac{\exp(b)}{(\exp(b) + 1)^2}$$

$$\frac{dJ}{d\beta_j} = \frac{dJ}{db}\frac{db}{d\beta_j}, \quad \frac{db}{d\beta_j} = z_j$$

$$\frac{dJ}{dz_j} = \frac{dJ}{db}\frac{db}{dz_j}, \quad \frac{db}{dz_j} = \beta_j$$

$$\frac{dJ}{da_j} = \frac{dJ}{dz_j}\frac{dz_j}{da_j}, \quad \frac{dz_j}{da_j} = \frac{\exp(a_j)}{(\exp(a_j) + 1)^2}$$

$$\frac{dJ}{d\alpha_{ji}} = \frac{dJ}{da_j}\frac{da_j}{d\alpha_{ji}}, \quad \frac{da_j}{d\alpha_{ji}} = x_i$$

$$\frac{dJ}{dx_i} = \frac{dJ}{da_j}\frac{da_j}{dx_i}, \quad \frac{da_j}{dx_i} = \sum_{j=0}^{D} \alpha_{ji}$$

Hybrid: NN + HMM

Computing the Gradient: $\nabla \ell(f_{\boldsymbol{\theta}}(\boldsymbol{x}_i), \boldsymbol{y}_i)$

Forward computation

$$\log p(\mathbf{S}, \mathbf{Y}) = \alpha_{\text{END},T}$$

$\alpha_{i,t} = \ldots$ (forward prob)

$\beta_{i,t} = \ldots$ (backward prop)

$\gamma_{i,t} = \ldots$ (marginals)

$a_{i,j} = \ldots$ (transitions)

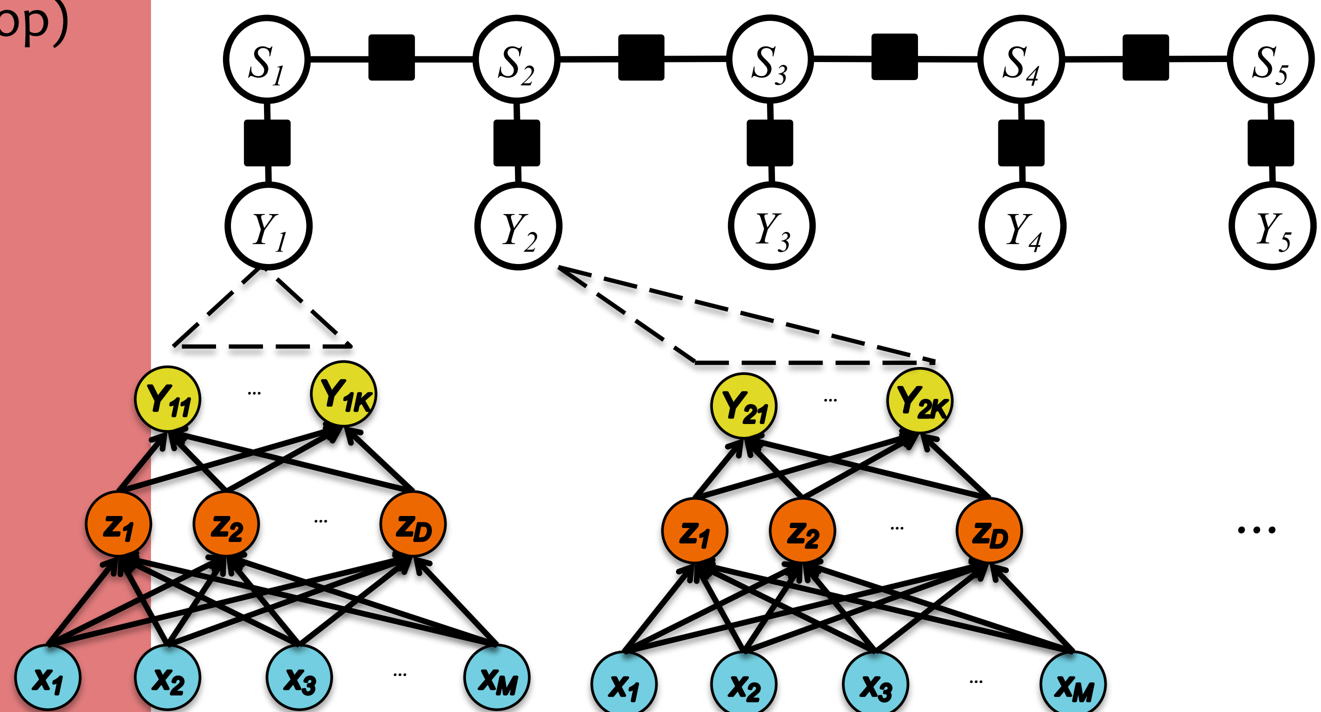
$b_{i,t} = \ldots$ (emissions)

$$y_{tk} = \frac{1}{1 + \exp(-b)}$$

$$b = \sum_{j=0}^{D} \beta_j z_j$$

$$z_j = \frac{1}{1 + \exp(-a_j)}$$

$$a_j = \sum_{i=0}^{M} \alpha_{ji} x_i$$



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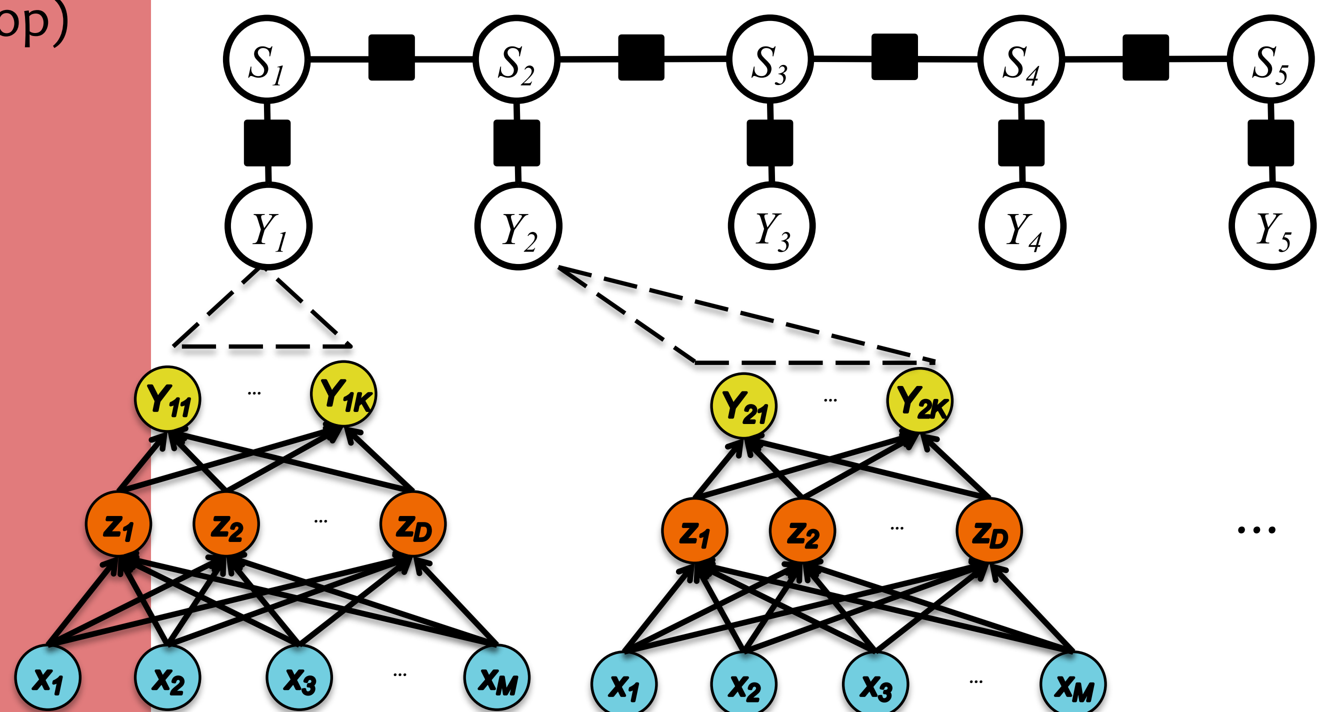

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Backward computation

$$\frac{dJ}{db_{i,t}} = \frac{\partial \alpha_{F_{model},T}}{\partial \alpha_{i,t}} \frac{\partial \alpha_{i,t}}{\partial b_{i,t}} = \left(\sum_j \frac{\partial \alpha_{j,t+1}}{\partial \alpha_{i,t}} \frac{\partial L_{model}}{\partial \alpha_{j,t+1}}\right)\left(\sum_j a_{ji} \alpha_{j,t-1}\right)$$

$$= \left(\sum_j b_{j,t+1} a_{ji} \frac{\partial \alpha_{F_{model},T}}{\partial \alpha_{j,t+1}}\right)\left(\sum_j a_{ji} \alpha_{j,t-1}\right) = \beta_{i,t} \frac{\alpha_{i,t}}{b_{i,t}} = \frac{\gamma_{i,t}}{b_{i,t}}$$

Hybrid: NN + HMM

Computing the Gradient: $\nabla \ell(f_{\boldsymbol{\theta}}(\boldsymbol{x}_i), \boldsymbol{y}_i)$

Forward computation

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Backward computation

$$\frac{dJ}{db_{i,t}} = \frac{\gamma_{i,t}}{b_{i,t}}$$

$$\frac{dJ}{dy_{t,k}} = \sum_{b_{i,t}} \frac{dJ}{db_{i,t}} \frac{db_{i,t}}{dy_{t,k}}$$

$$\frac{\partial b_{i,t}}{\partial Y_{jt}} = \sum_k \frac{Z_k}{((2\pi)^n \mid \Sigma_k \mid)^{1/2}} \left(\sum_l d_{k,lj}(\mu_{kl} - Y_{lt})\right) \exp\left(-\frac{1}{2}(Y_t - \mu_k)\Sigma_k^{-1}(Y_t - \mu_k)^T\right)$$

$$\frac{dJ}{db} = \frac{dJ}{dy}\frac{dy}{db}, \quad \frac{dy}{db} = \frac{\exp(b)}{(\exp(b) + 1)^2}$$

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Forward computation

$$J = \log p(\mathbf{S}, \mathbf{Y}) = \alpha_{\text{END},T}$$

$\alpha_{i,t} = \ldots$ (forward prob)

$\beta_{i,t} = \ldots$ (backward prop)

$\gamma_{i,t} = \ldots$ (marginals)

The derivative of the log-likelihood with respect to the neural network parameters!

$$a_j = \sum_{i=0}^{M} \alpha_{ji} x_i$$

Backward computation

$$\frac{dJ}{db_{i,t}} = \frac{\gamma_{i,t}}{b_{i,t}}$$

$$\frac{dJ}{dy_{t,k}} = \sum_{b_{i,t}} \frac{dJ}{db_{i,t}} \frac{db_{i,t}}{dy_{t,k}}$$

$$\frac{\partial b_{i,t}}{\partial Y_{jt}} = \sum_k \frac{Z_k}{((2\pi)^n \mid \Sigma_k \mid)^{1/2}} (\sum_l d_{k,lj} (\mu_{kl} - Y_{lt})) \exp(-\frac{1}{2}(Y_t - \mu_k) \Sigma_k^{-1} (Y_t - \mu_k)^T)$$

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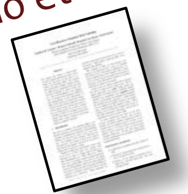
$$\frac{dJ}{d\alpha_{ji}} = \frac{dJ}{da_j} \frac{da_j}{d\alpha_{ji}}, \quad \frac{da_j}{d\alpha_{ji}} = x_i$$

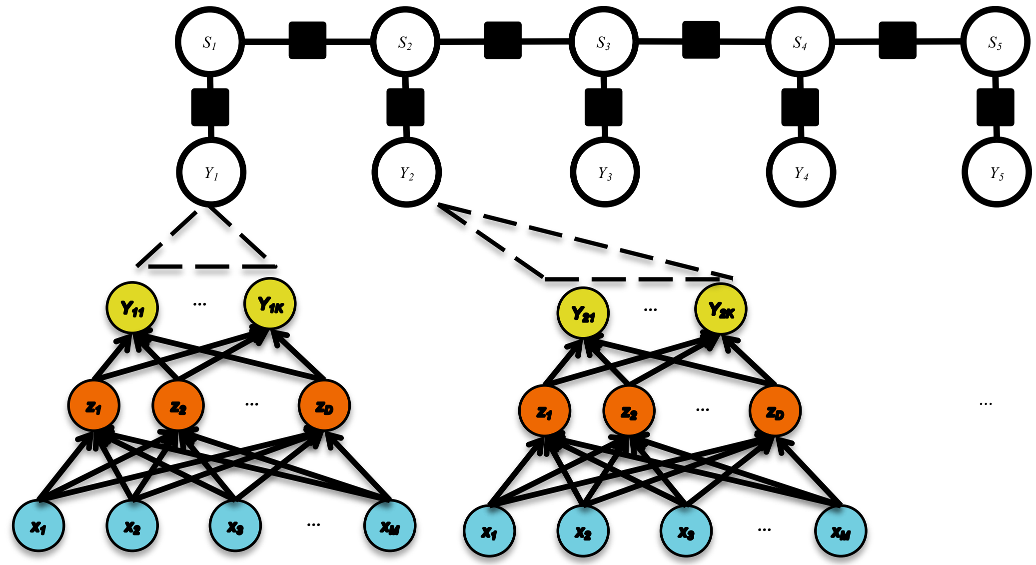
(Bengio et al., 1992)

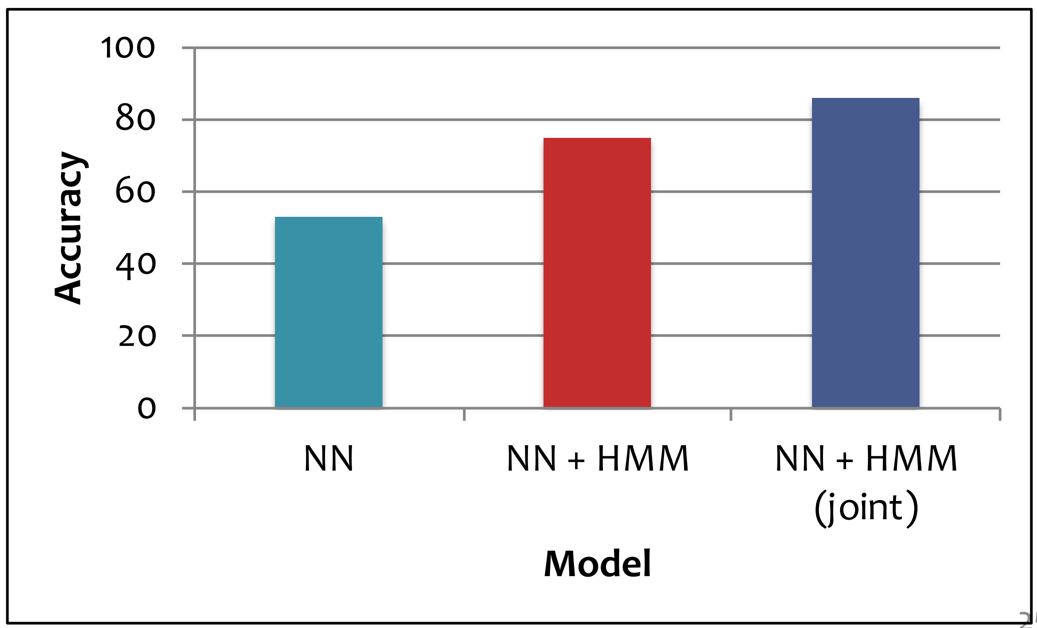


Hybrid: NN + HMM

Experimental Setup:

- **Task:** Phoneme Recognition (aka. speaker independent recognition of plosive sounds)
- **Eight output labels:**
  - /p/, /t/, /k/, /b/, /d/, /g/, /dx/, /all other phonemes/
  - These are the HMM hidden states
- **Metric:** Accuracy
- **3 Models:**
  1. NN only
  2. NN + HMM (trained independently)
  3. NN + HMM (jointly trained)



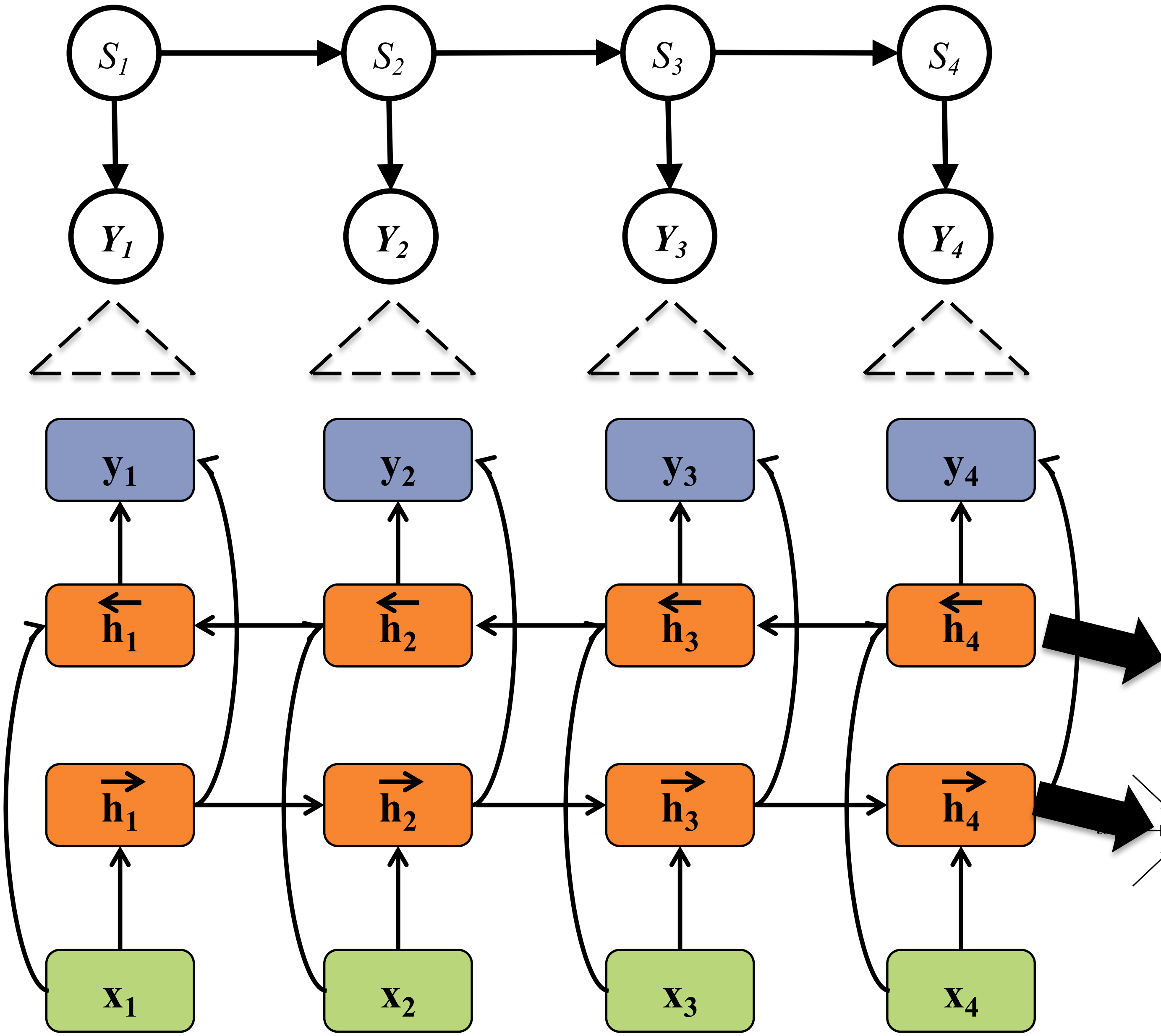


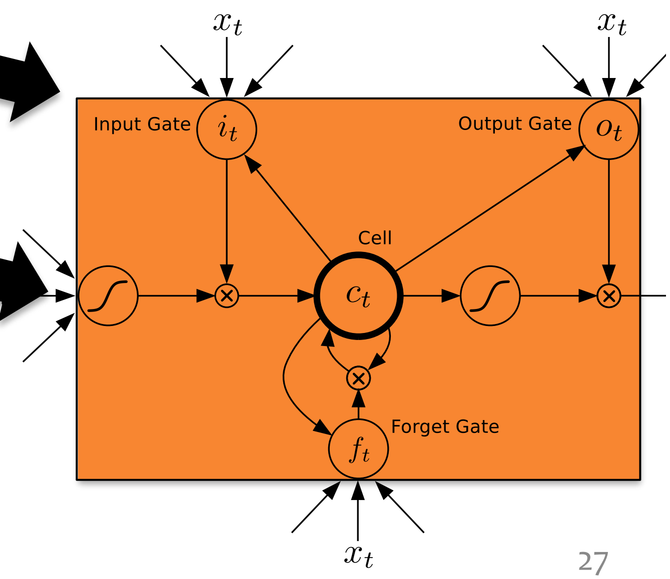
HYBRID:
RNN + HMM

Hybrid: RNN + HMM

(Graves et al., 2013)

- Graves et al. (2013) uses a Deep Bidirectional LSTM
- Each hidden unit is an LSTM
- Deep ➔ More than two layers



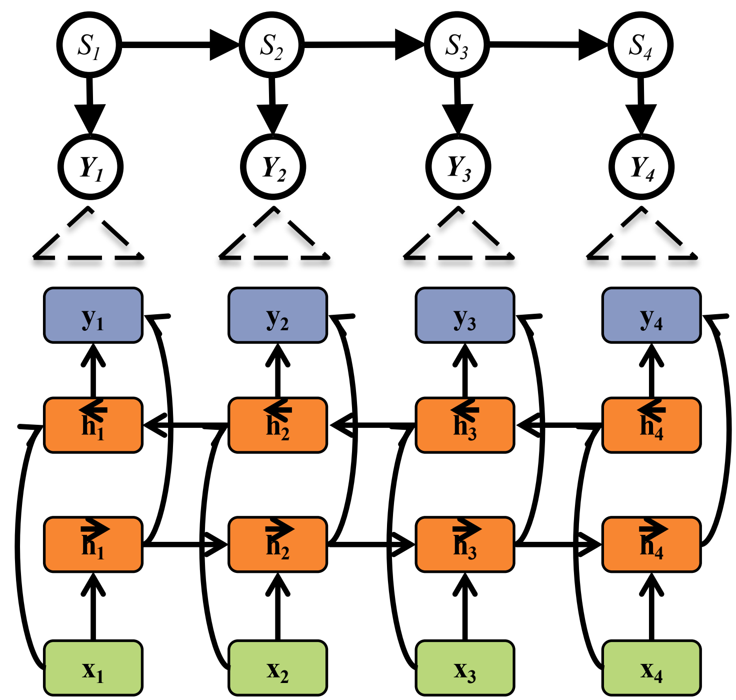


Hybrid: RNN + HMM

(Graves et al., 2013)

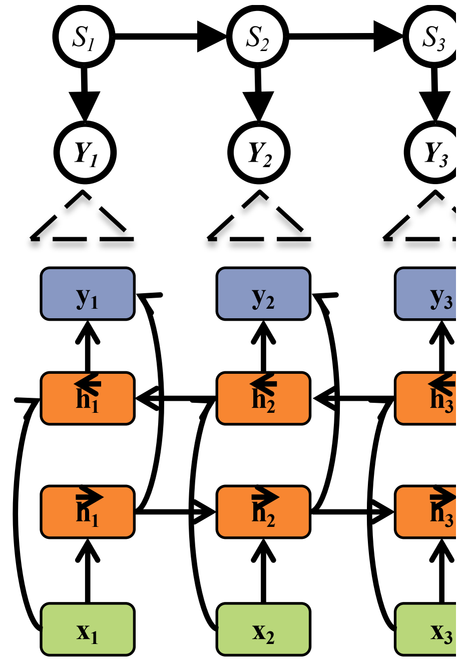
The model, inference, and learning can be **analogous** to our NN + HMM hybrid

- **Objective:** log-likelihood
- **Model:** HMM/Gaussian emissions
- **Inference:** forward-backward algorithm
- **Learning:** SGD with gradient by backpropagation



(Graves et al., 2013)

Hybrid: RNN + HMM



Experimental Setup:

- **Task:** Phoneme Recognition
- **Dataset:** TIMIT
- **Metric:** Phoneme Error Rate
- **Two classes of models:**
  1. Neural Net only
  2. NN + HMM hybrids

Training Method	Test PER
CTC	21.57 ± 0.25
CTC (noise)	18.63 ± 0.16
Transducer	$\mathbf{18.07 \pm 0.24}$

1. Neural Net only

Network	Dev PER Test PER
DBRNN	19.91 ± 0.22 21.92 ± 0.35
DBLSTM	17.44 ± 0.156 19.34 ± 0.15
DBLSTM (noise)	16.11 ± 0.15 $\mathbf{17.99 \pm 0.13}$

2. NN + HMM hybrids

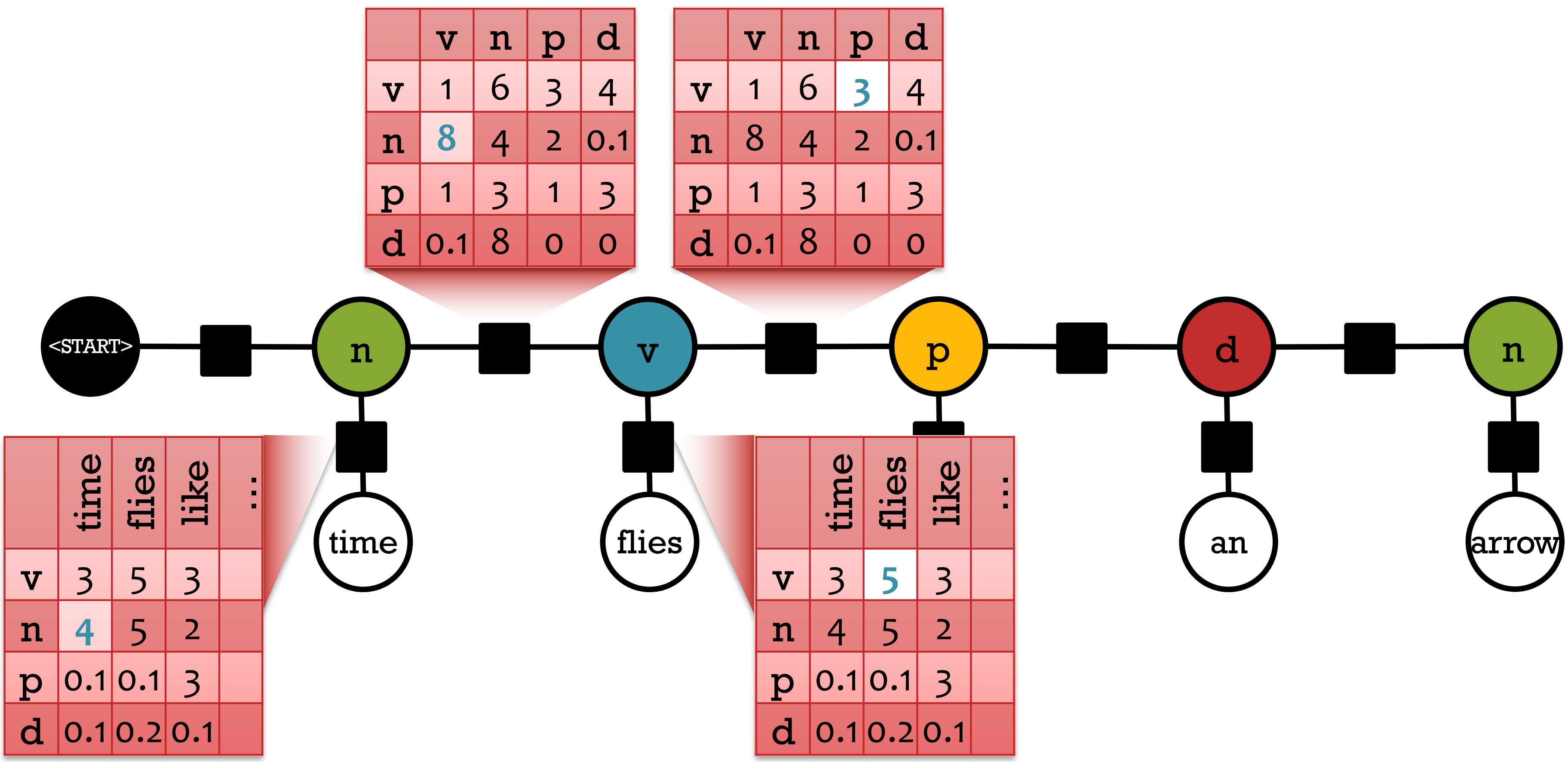
HYBRID:
CNN + CRF

Recall...

Markov Random Field (MRF)

Joint distribution over tags Y_i <u>and</u> words X_i

$$p(\text{n, v, p, d, n, time, flies, like, an, arrow}) = \frac{1}{Z}(4 * 8 * 5 * 3 * \ldots)$$



	v	n	p	d
v	1	6	3	4
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	v	n	p	d
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	time	flies	like	…
v	3	5	3	
n	4	5	2	
p	0.1	0.1	3	
d	0.1	0.2	0.1	

	time	flies	like	…
v	3	5	3	
n	4	5	2	
p	0.1	0.1	3	
d	0.1	0.2	0.1	

Conditional Random Field (CRF)

Recall...

Conditional distribution over tags Y_i given words x_i.
The factors and Z are now specific to the sentence $\boldsymbol{x}$.

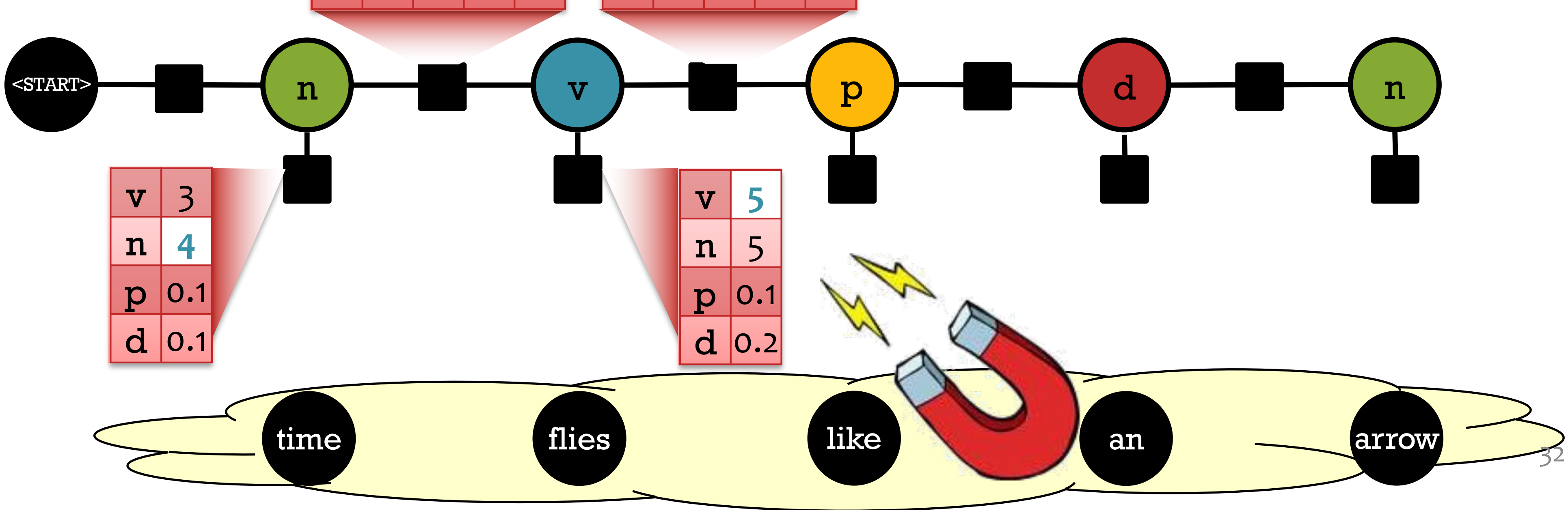
$$p(\text{n, v, p, d, n} \mid \text{time, flies, like, an, arrow}) \quad = \frac{1}{Z}\,(4 * 8 * 5 * 3 * \ldots)$$

	v	n	p	d
v	1	6	3	4
n	8	4	2	0.1
p	1	3	1	3
d	0.1	8	0	0

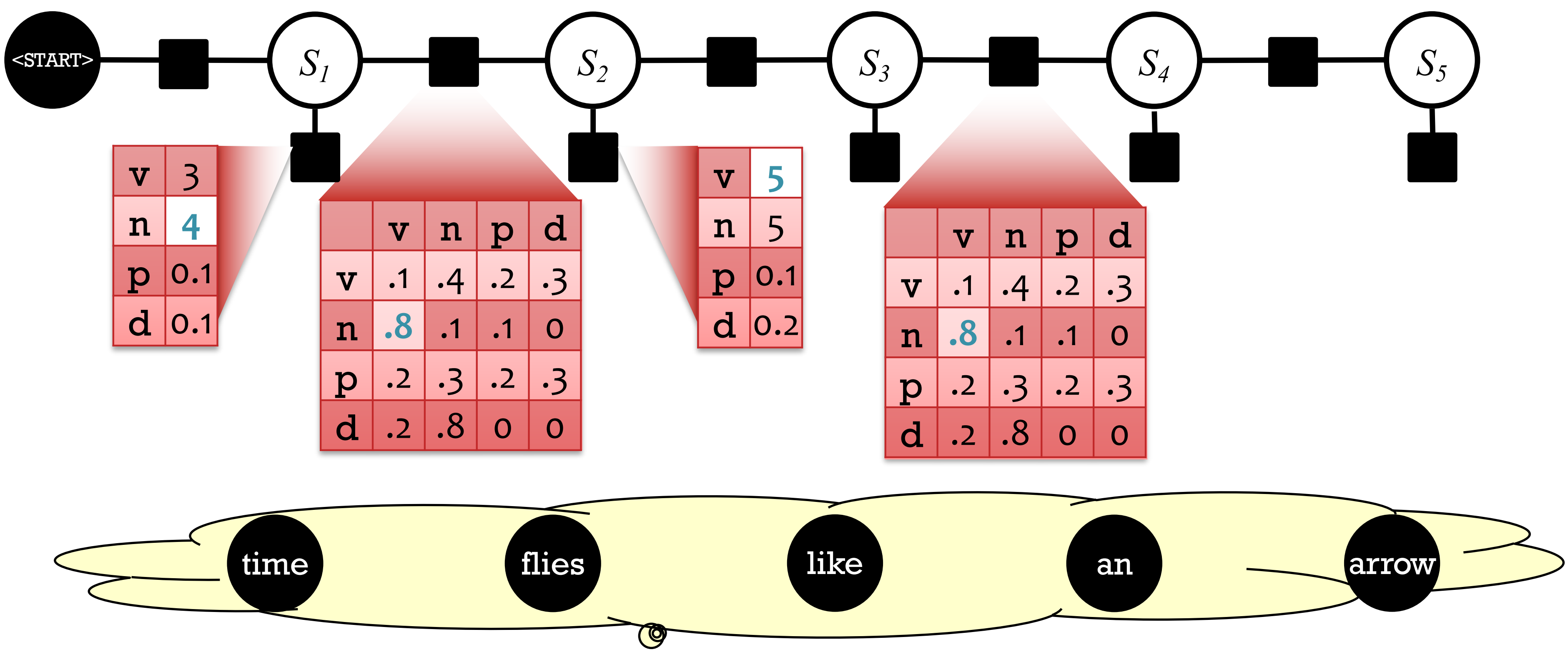
	v	n	p	d
v	1	6	3	4
n	8	4	2	0.1
p	1	3	1	3
d	0.1	8	0	0

v	3
n	4
p	0.1
d	0.1

v	5
n	5
p	0.1
d	0.2



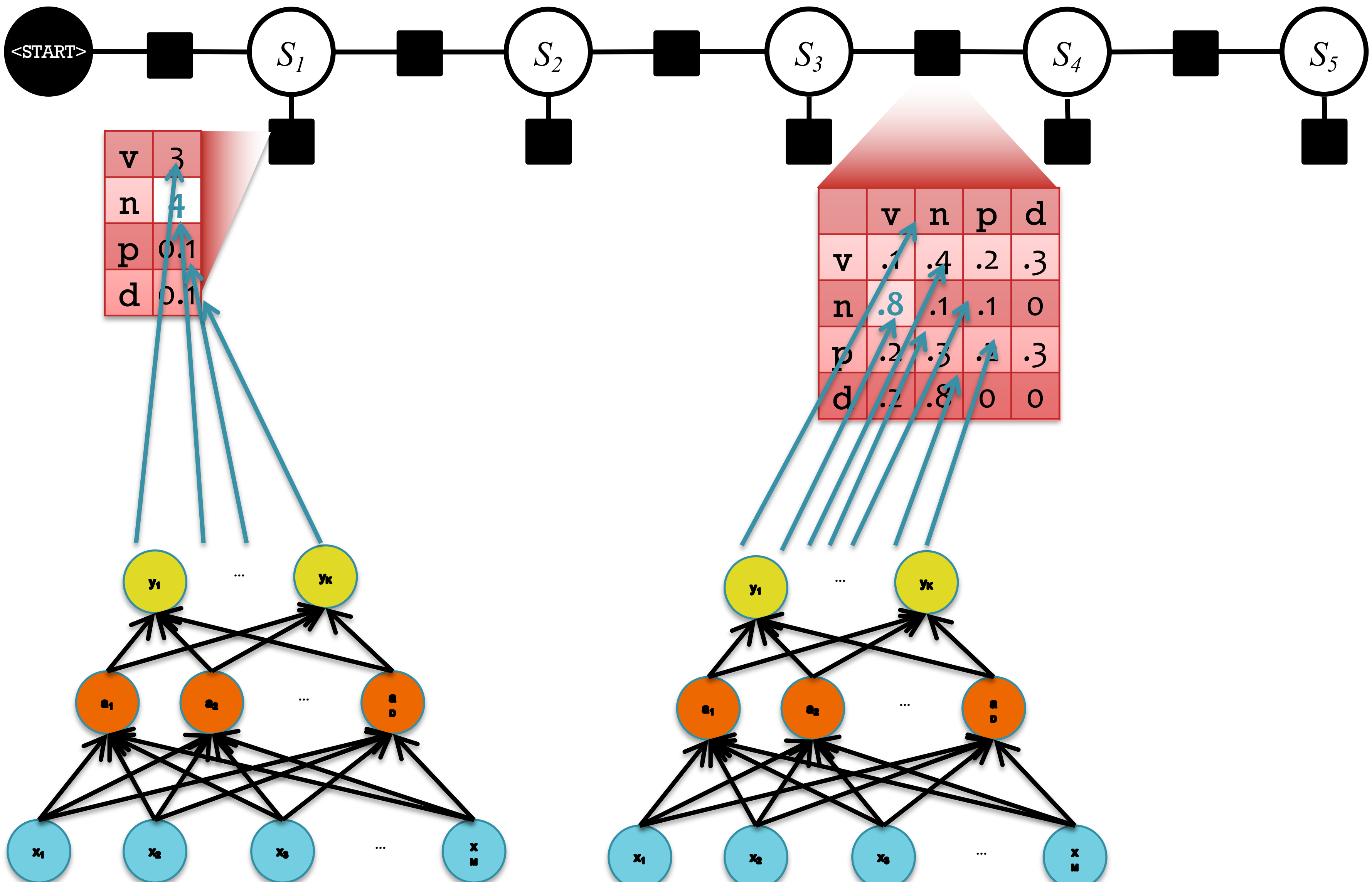
Hybrid: Neural Net + CRF



- In a standard CRF, each of the factor cells is a parameter (e.g. transition or emission)
- In the hybrid model, these values are computed by a neural network with its own parameters

Hybrid: Neural Net + CRF

Forward computation



(Collobert & Weston, 2011)

Hybrid: CNN + CRF

- For **computer vision,** Convolutional Neural Networks are in **2-dimensions**
- For **natural language,** the CNN is **1-dimensional**

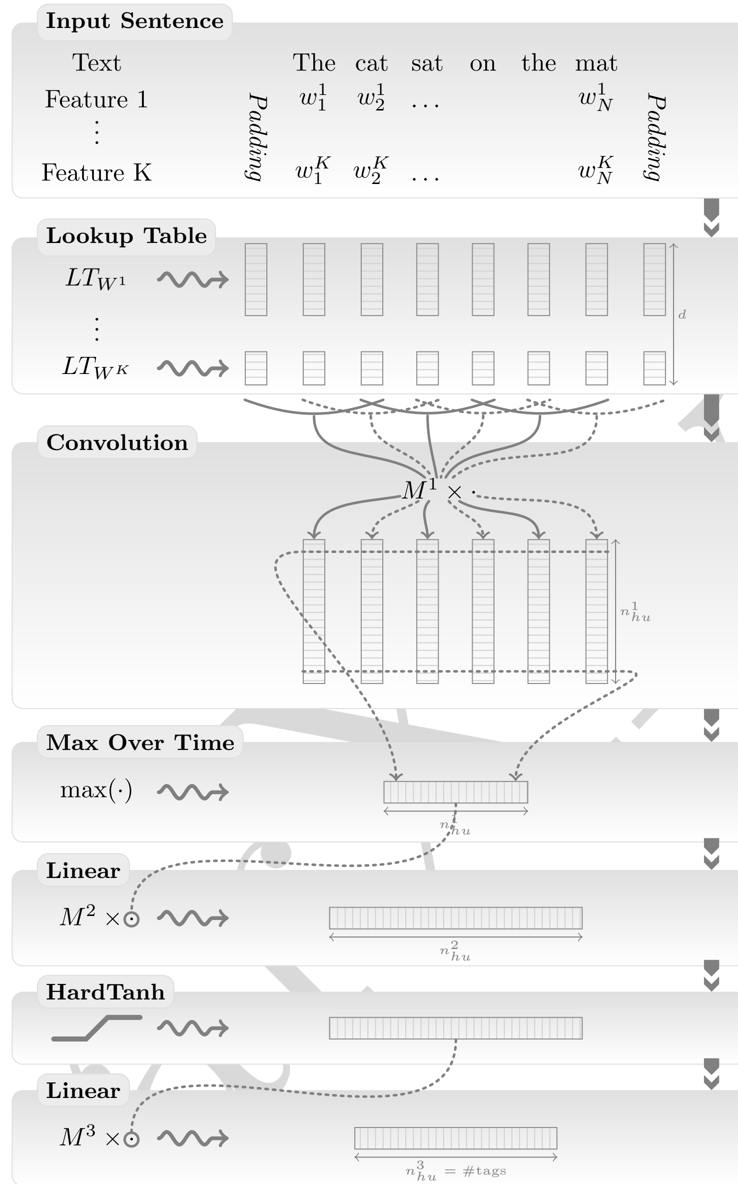


Figure from (Collobert & Weston, 2011)

Hybrid: CNN + CRF

(Collobert & Weston, 2011)

"NN + SLL"

- Model: Convolutional Neural Network (CNN) with **linear-chain CRF**
- Training objective: maximize **sentence-level likelihood** (SLL)

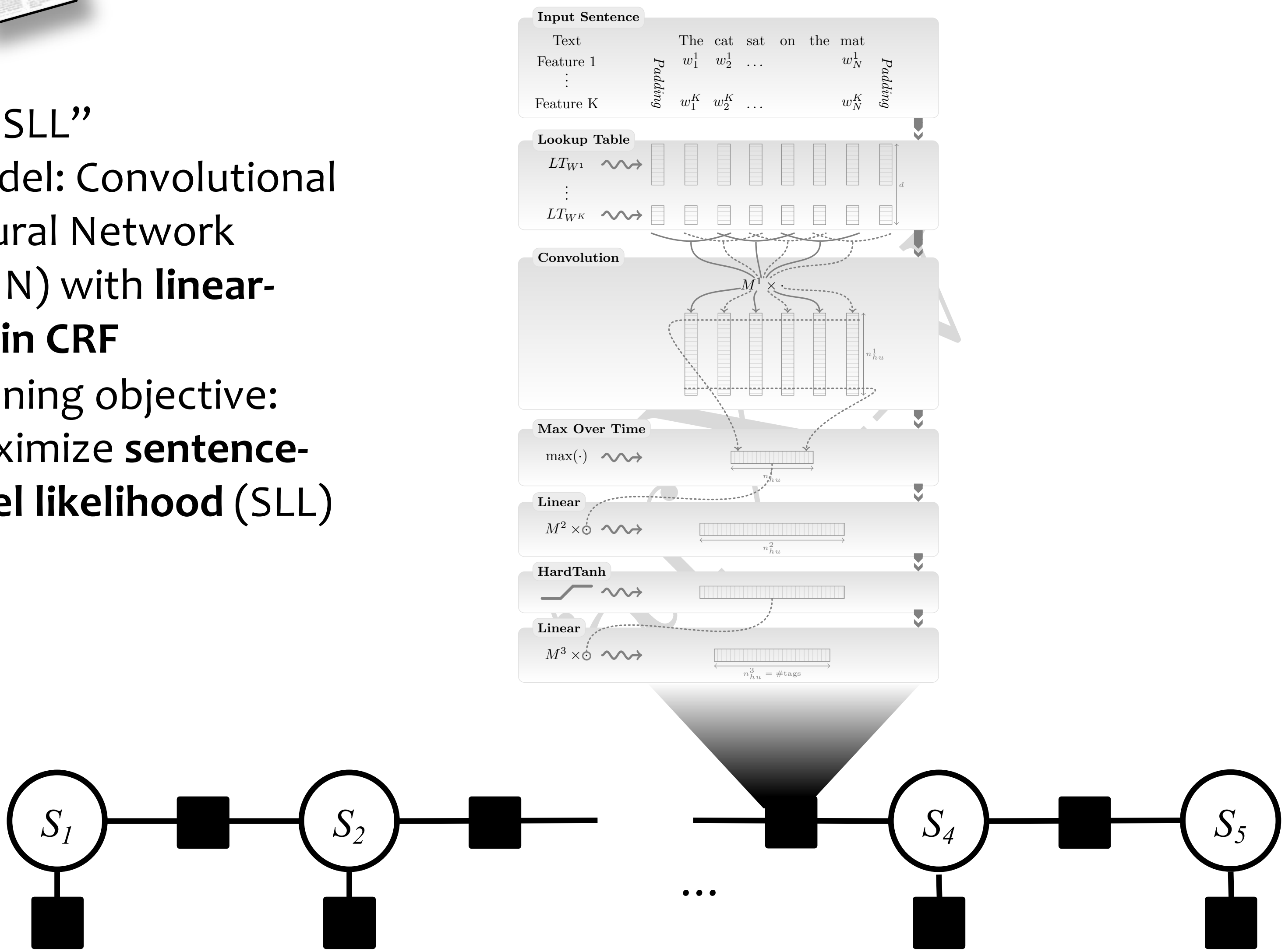
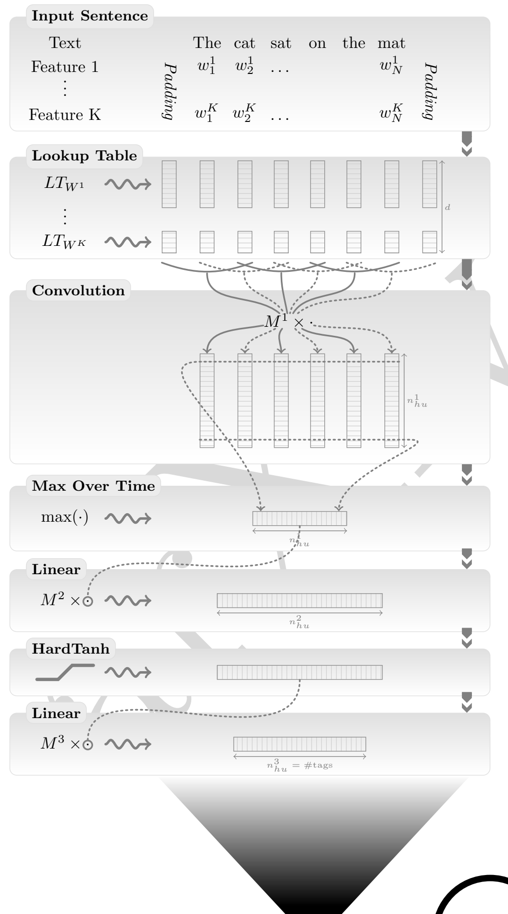


Figure from (Collobert & Weston, 2011)

Hybrid: CNN + CRF

"NN + WLL"

- Model: Convolutional Neural Network (CNN) with **logistic regression**
- Training objective: maximize **word-level likelihood** (WLL)



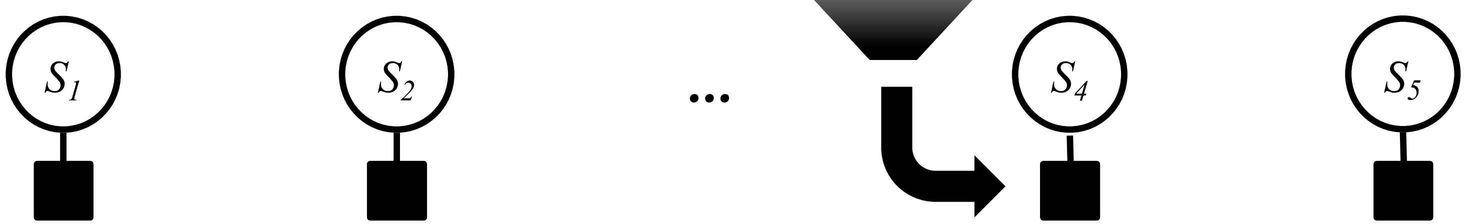


Figure from (Collobert & Weston, 2011)

(Collobert & Weston, 2011)

Hybrid: CNN + CRF

Experimental Setup:

- **Tasks:**
  - Part-of-speech tagging (POS),
  - Noun-phrase and Verb-phrase Chunking,
  - Named-entity recognition (NER)
  - Semantic Role Labeling (SRL)
- **Datasets / Metrics:** Standard setups from NLP literature (higher PWA/F1 is better)
- **Models:**
  - Benchmark systems are typical – non-neural network systems
  - NN+WLL: hybrid CNN with logistic regression
  - NN+SLL: hybrid CNN with linear-chain CRF

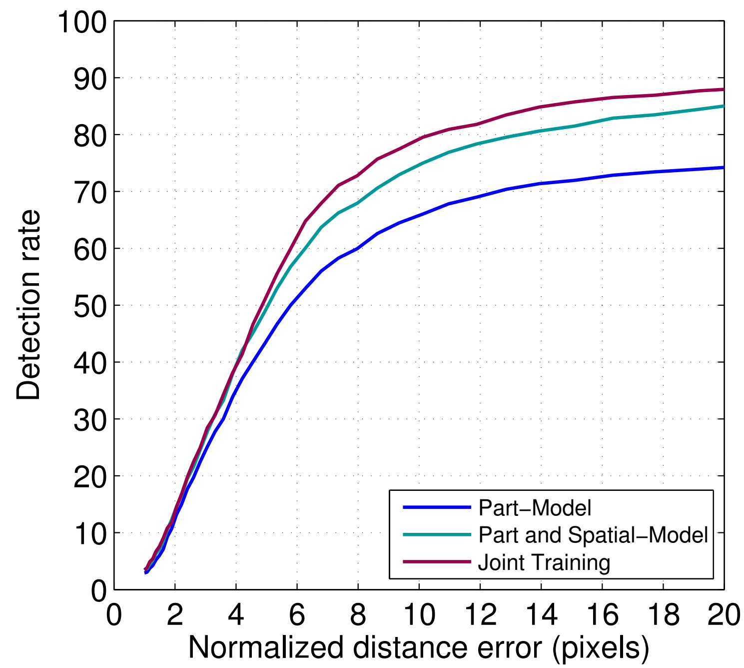
Approach	**POS** (PWA)	**Chunking** (F1)	**NER** (F1)	**SRL** (F1)
Benchmark Systems	97.24	94.29	89.31	77.92
NN+WLL	96.31	89.13	79.53	55.40
NN+SLL	96.37	90.33	81.47	70.99

(Thompson et al., 2014)

Hybrid: CNN + MRF

Experimental Setup:

- **Task:** pose estimation
- **Model:** Deep CNN + MRF



TRICKS OF THE TRADE

Backprop in Practice

Y LeCun
MA Ranzato

- Use ReLU non-linearities (tanh and logistic are falling out of favor)
- Use cross-entropy loss for classification
- Use Stochastic Gradient Descent on minibatches
- Shuffle the training samples
- Normalize the input variables (zero mean, unit variance)
- Schedule to decrease the learning rate
- Use a bit of L1 or L2 regularization on the weights (or a combination)
  - But it's best to turn it on after a couple of epochs
- Use "dropout" for regularization
  - Hinton et al 2012 http://arxiv.org/abs/1207.0580
- Lots more in [LeCun et al. "Efficient Backprop" 1998]
- Lots, lots more in "Neural Networks, Tricks of the Trade" (2012 edition) edited by G. Montavon, G. B. Orr, and K-R Müller (Springer)

Deep Learning Tricks of the Trade

- Y. Bengio (2012), "Practical Recommendations for Gradient-Based Training of Deep Architectures"

  - Unsupervised pre-training
  - Stochastic gradient descent and setting learning rates
  - Main hyper-parameters
    - Learning rate schedule & early stopping
    - Minibatches
    - Parameter initialization
    - Number of hidden units
    - L1 or L2 weight decay
    - Sparsity regularization
  - Debugging → use finite difference gradient checks
  - How to efficiently search for hyper-parameter configurations

Slide from Socher, MLSS 2014

Tricks of the Trade

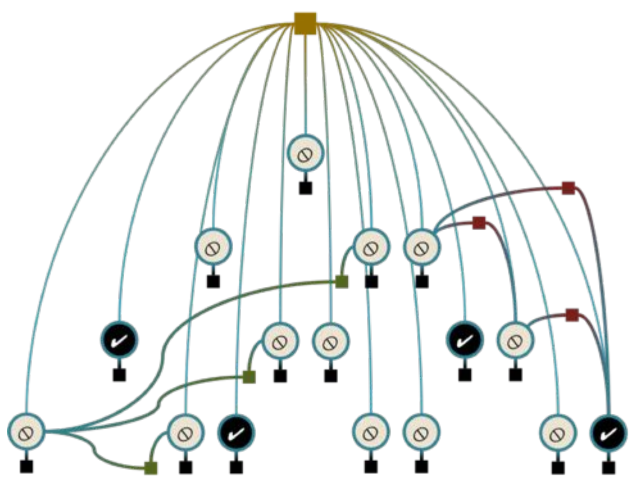
- **Lots of them:**
  - Pre-training helps (but isn't always necessary)
  - Train with adaptive gradient variants of SGD (e.g. Adam)
  - Use max-margin loss function (i.e. hinge loss) – though only sub-differentiable it often gives better results
  - …
- A few years back, they were considered "**poorly documented**" and "requiring great expertise"
- Now there are lots of **good tutorials** that describe (very important) specific implementation details
- Many of them **also apply to training graphical models**!

SUMMARY

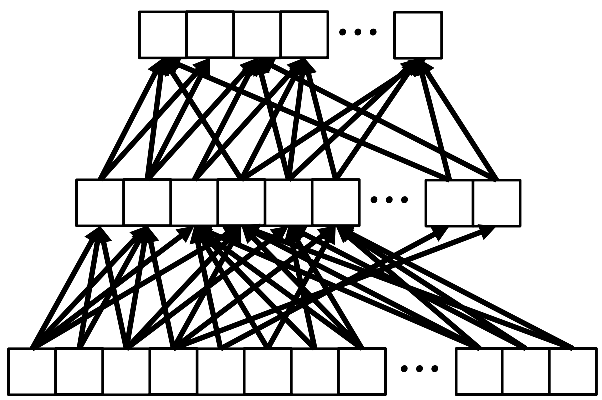
Summary:

Hybrid Models

Graphical models let you encode domain knowledge



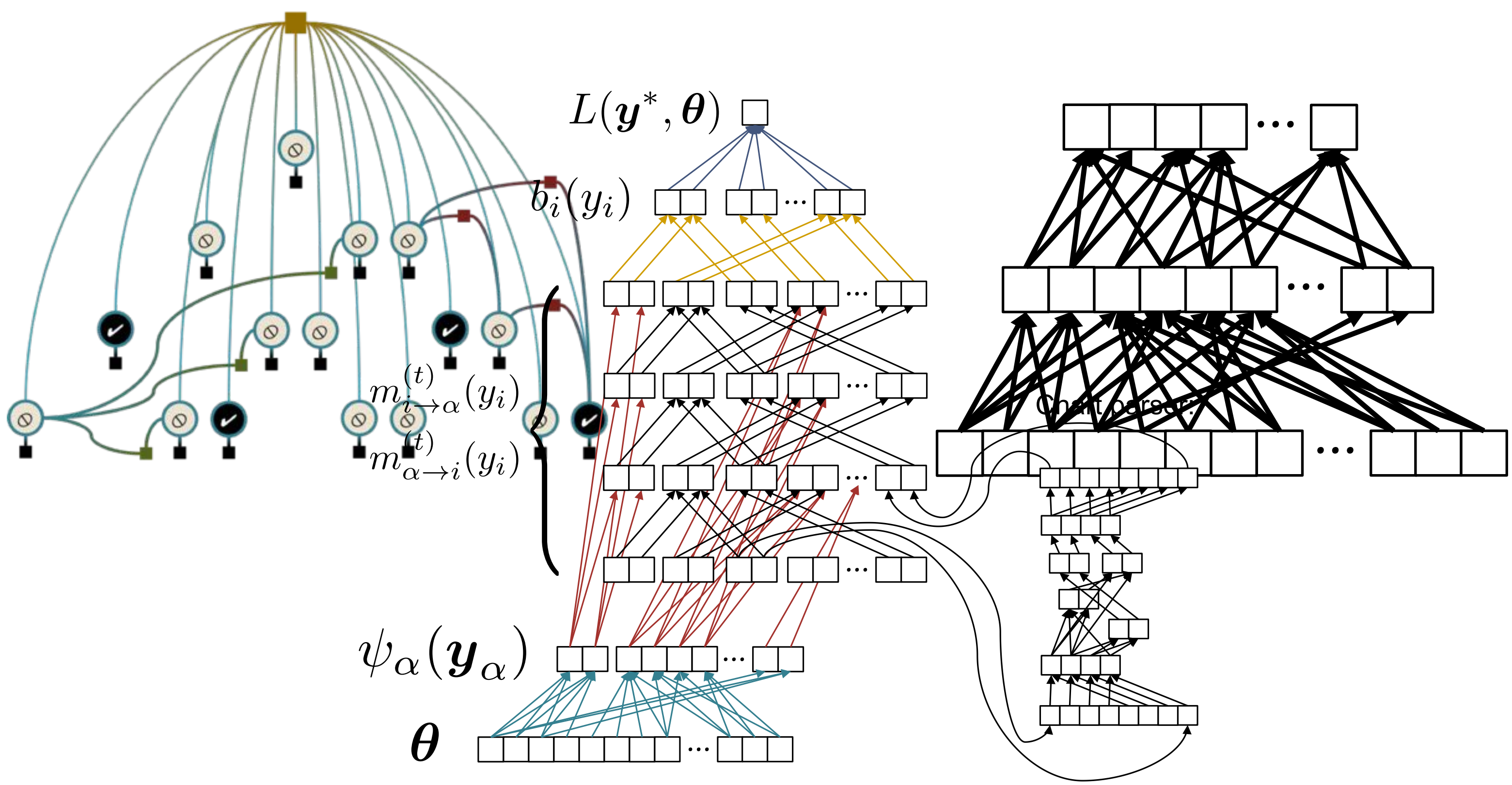
Neural nets are really good at fitting the data discriminatively to make good predictions



Could we define a neural net that incorporates domain knowledge?

Summary:
Hybrid Models

Key idea: Use a NN to learn features for a GM, then train the entire model by backprop



MBR DECODING

Minimum Bayes Risk Decoding

- Suppose we given a loss function $l(\boldsymbol{y}', \boldsymbol{y})$ and are asked for a single tagging
- How should we choose just one from our probability distribution $p(\boldsymbol{y}|\boldsymbol{x})$?
- A minimum Bayes risk (MBR) decoder $h(\boldsymbol{x})$ returns the variable assignment with minimum **expected** loss under the model's distribution

$$
\begin{aligned}
h_{\boldsymbol{\theta}}(\boldsymbol{x}) &= \underset{\hat{\boldsymbol{y}}}{\operatorname{argmin}} \; \mathbb{E}_{\boldsymbol{y} \sim p_{\boldsymbol{\theta}}(\cdot|\boldsymbol{x})}[\ell(\hat{\boldsymbol{y}}, \boldsymbol{y})] \\
&= \underset{\hat{\boldsymbol{y}}}{\operatorname{argmin}} \; \sum_{\boldsymbol{y}} p_{\boldsymbol{\theta}}(\boldsymbol{y} \mid \boldsymbol{x}) \ell(\hat{\boldsymbol{y}}, \boldsymbol{y})
\end{aligned}
$$

Minimum Bayes Risk Decoding

$$h_{\boldsymbol{\theta}}(\boldsymbol{x}) = \underset{\hat{\boldsymbol{y}}}{\text{argmin}} \ \mathbb{E}_{\boldsymbol{y} \sim p_{\boldsymbol{\theta}}(\cdot | \boldsymbol{x})}[\ell(\hat{\boldsymbol{y}}, \boldsymbol{y})]$$

Consider some example loss functions:

The **Hamming loss** corresponds to accuracy and returns the number of incorrect variable assignments:

$$\ell(\hat{\boldsymbol{y}}, \boldsymbol{y}) = \sum_{i=1}^{V} (1 - \mathbb{I}(\hat{y}_i, y_i))$$

The MBR decoder is:

$$\hat{y}_i = h_{\boldsymbol{\theta}}(\boldsymbol{x})_i = \underset{\hat{y}_i}{\text{argmax}} \ p_{\boldsymbol{\theta}}(\hat{y}_i \mid \boldsymbol{x})$$

This decomposes across variables and requires the variable marginals.

Minimum Bayes Risk Decoding

$$h_{\boldsymbol{\theta}}(\boldsymbol{x}) = \underset{\hat{\boldsymbol{y}}}{\text{argmin}} \; \mathbb{E}_{\boldsymbol{y} \sim p_{\boldsymbol{\theta}}(\cdot|\boldsymbol{x})}[\ell(\hat{\boldsymbol{y}}, \boldsymbol{y})]$$

Consider some example loss functions:

The ***0-1* loss function** returns *1* only if the two assignments are identical and *0* otherwise:

$$\ell(\hat{\boldsymbol{y}}, \boldsymbol{y}) = 1 - \mathbb{I}(\hat{\boldsymbol{y}}, \boldsymbol{y})$$

The MBR decoder is:

$$\begin{aligned} h_{\boldsymbol{\theta}}(\boldsymbol{x}) &= \underset{\hat{\boldsymbol{y}}}{\text{argmin}} \; \sum_{\boldsymbol{y}} p_{\boldsymbol{\theta}}(\boldsymbol{y} \mid \boldsymbol{x})(1 - \mathbb{I}(\hat{\boldsymbol{y}}, \boldsymbol{y})) \\ &= \underset{\hat{\boldsymbol{y}}}{\text{argmax}} \; p_{\boldsymbol{\theta}}(\hat{\boldsymbol{y}} \mid \boldsymbol{x}) \end{aligned}$$

which is exactly the MAP inference problem!