Pseudo Inverses

Suppose A is $m \times n$, with $m \geq n$ & rank n.

This means the columns of A are lin. indep.,
so the matrix $A^T A$ is invertible.

Some texts solve $Ax = b$ by computing

$$x = (A^T A)^{-1} A^T b.$$

How does this relate to the "SVD solution"?

It is the same.

To see this, write $A = U \Sigma V^T$.

Then a little algebra shows

$$(A^T A)^{-1} A^T = V\left((\Sigma^T \Sigma)^{-1} \Sigma^T \right) U^T.$$

One finds that $\Sigma^T \Sigma = \begin{pmatrix} \sigma_1^2 & & 0 \\ & \ddots & \\ 0 & & \sigma_n^2 \end{pmatrix}$.

From that one sees that $(\Sigma^T \Sigma)^{-1} \Sigma^T = \frac{1}{\Sigma}$,

so indeed $(A^T A)^{-1} A^T = V\left(\frac{1}{\Sigma}\right) U^T$.