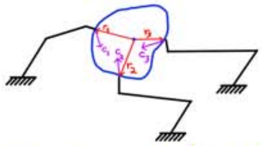

Finger 1 generates force c_1
and torque $r_1 \times c_1$.
We can express these as a 6D vector (a generalized force):

$$f_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -r_{1_z} & r_{1_y} \\ r_{1_z} & 0 & -r_{1_x} \\ -r_{1_y} & r_{1_x} & 0 \end{pmatrix} \begin{pmatrix} c_{1_x} \\ c_{1_y} \\ c_{1_z} \end{pmatrix}$$

$$\downarrow \text{define}$$

$$= W_1 \, c_1$$

All fingers together produce generalized force Wc,
where

$$W = [W_1 \ W_2 \ W_3] \quad \text{wrench matrix}$$

$$\& \quad c = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} \quad . \quad \text{contact forces}$$

If F is a generalized external force acting on the object, can compensate by producing contact forces c that satisfy $-F = Wc$.

F is $6 \times \underline{1}$
c is $3k \times \underline{1}$
(k = number of fingers; 3 here)
W is $6 \times 3k$

General solution will be of the form $c = c_p + c_n$,
with c_p some particular solution
and c_n in the null space of W.
(We will learn more about that decomposition.)

Finally, can determine the joint torques needed at each finger's joints by a Jacobian transpose transformation.

E.g.,

$$\tau_i = J_i^T c_i .$$

Joint torques to command for finger #i.

Jacobian for finger #i.

Contact forces required from finger #i.